American Journal of Engineering Research (AJER)
e-ISSN : 2320-0847 p-ISSN : 2320-0936
Volume-04, Issue-02, pp-60-71
www.ajer.org
Research Paper Open Access

Optimization of Flexural Prediction for Ribbed Floors in Bending, Shear and Deflection

[a]Abejide O. S, [b]Konitufe C.
[a]*Department of Civil Engineering, Abubakar Tafawa Balewa University, P.M.B 248, Bauchi, Nigeria*
[b]*Department of Civil Engineering, Abubakar Tafawa Balewa University, P.M.B 248, Bauchi, Nigeria*

ABSTRACT: The flexural prediction for concrete ribbed floors has been assessed using the minimum weight approach and mathematical techniques for optimization. Results indicate that although both the BS8110 (1997) and EC2 (2008) are reliable, they are quite expensive and cost can be further reduced as they currently encourage abuse. The BS8110 (1997) and EC2 (2008) were found to be under-estimated by about 27 and 19 percent respectively.

Keywords: optimization; ribbed slab; bending; deflection; reinforced concrete

I. INTRODUCTION

Reinforced concrete is a strong durable building material that can be formed into many varied shapes and sizes ranging from a simple rectangular column, to a slender curved dome. Its utility and versatility is achieved by combining the best features of concrete and steel. Thus when they are combined, the steel is able to provide the tensile strength and probably some of the shear strength while the concrete, strong in compression, protects the steel to give durability and fire resistance. The tensile strength of concrete is only about 10 percent of the compressive strength. Hence, nearly all reinforced concrete structures are designed on the assumption that the concrete does not resist any tensile forces. Reinforcement is designed to carry these tensile forces, which are transferred by bond between the interfaces of the two materials (Mosley and Bungney, 1990).

In long span, solid reinforced concrete slabs of lengths greater than 5 meters, the self-weight becomes excessive when compared to the applied dead and imposed loads, resulting in an uneconomic method of construction. One major way of overcoming this problem is to use ribbed slabs. A ribbed slab is a slab which voids have been introduced to the underside to reduce dead weight and increase the efficiency of the concrete section. A slightly deeper section is required but these stiffer floors facilitate longer spans and provision of holes. These longer spans are economic in the range of 8 to 12metres. The saving of materials tends to be offset by some complications in formwork (BS8110, 1997).

When a structure is loaded, it will respond in a manner which depends on the type and magnitude of the load and the strength and stiffness of the structure. The satisfaction of these responses depends on the requirements which must be satisfied. Such requirements might include safety of the structure against collapse, limitation on damage or on deflection or any of a range of other criteria. These requirements are the limit state requirements (Melchers, 1987).

Slabs are major structural elements in structures, other than beams and columns. Standardized and optimized slabs can significantly enhance safety and durability of structures. This requires special techniques to achieve standardized and optimized slabs which can satisfy all the important design standards. These techniques enable the design of the most optimized floors. Structural floor systems made of reinforced concrete are required to efficiently transmit the floor loads to the vertical systems through shear, bending and torsion resisting capacities. In addition to these requirements of strength, they are required to satisfy the deformation criteria also in terms of low deflection and crack width (Melchers, 1987).

This work elaborates the results obtained from the analytical study carried out on ribbed floor system via obtaining the most optimum ribbed slabs design using the BS 8110 (1985; 1997) and EC2 (2008) design requirements and propose a comprehensive design using an optimization technique for one of the most commonly used slabs in building construction i.e ribbed slabs. An objective function was developed for the purpose of achieving an optimum slab design which will fulfill the entire BS 8110 (1985; 1997) and EC2 (2008) design requirements and simultaneously save construction cost.

II. BACKGROUND OF RIBBED/HOLLOW SLABS.

Ribbed slabs used herein refer to singly reinforced concrete slabs with hollow blocks or voids in them. These types of structural plate systems can minimize formwork complexity by using standard modular, reusable formwork. Ribbed slab floors are very adaptable for accommodating a range of service openings. According to BS8110; 1:1997, hollow or solid blocks may be of any suitable material. When required to contribute to the structural strength of a slab, slabs should be made of concrete or burnt clay; Have a characteristic strength of at least 14N/mm^2, measured on the net section, when axially loaded in the direction of compressive stress in the slab. When made of fired brick earth, clay or shale, conform to BS3921 (1985), BS EN772-1 (2000), BS EN 772-3 (1998) and BS EN772-7 (1998).

2.1 Slabs with Permanent Blocks

The clear distance between ribs should not be more than 500mm. The width of the rib will be determined by consideration of cover, bar spacing and fire requirements. But the depth of the rib excluding the topping should not exceed four times the width. If the blocks are suitably manufactured and have adequate strength they can be considered to contribute to the strength of the slab in the design calculations, but in many designs no such allowance is made. These permanent blocks which are capable of contributing to the structural strength if it can be jointed with cement-sand mortar. During construction the hollow tiles should be well soaked in water prior to placing the concrete, otherwise shrinkage cracking of the top concrete flange is liable to occur. This probably develops strength for topping (Mohammed, 2006).

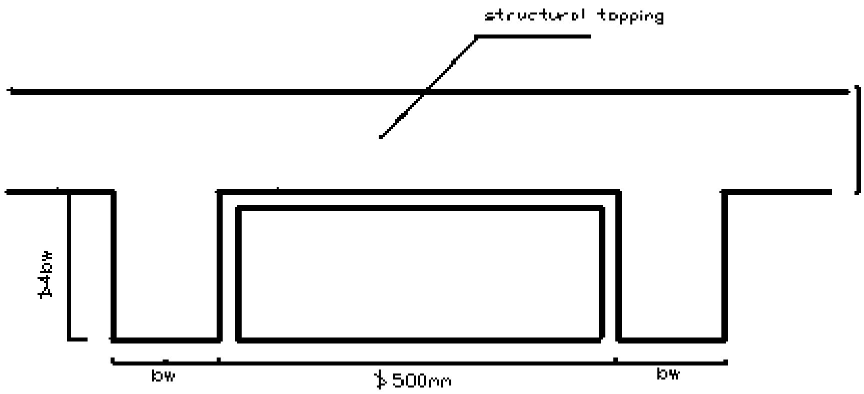

Fig 1: Permanent Blocks Contributing to Structural Strength. (Source: Mohammed, 2006).

2.2 Concept of Optimization

The concept of optimization is basic to much of what we do in our daily lives: a desire to do better or be the best in one field or another. In engineering we wish to produce the best possible result with the available resources. The term optimization has been used in operations management, operations research, and engineering for decades. The idea is to use mathematical techniques to arrive at the best solution, given what is being optimized (cost, profit, or time, for instance). To optimize a manufacturing system means that the effort to find best solutions focuses on finding the most effective use of resources over time. In a highly competitive modern world it is no longer sufficient to design a system whose performance of the required task is just satisfactory, it is essential to design the best system. Thus in modern design we must use tools which provide the desired results in a timely and economical fashion (Vanderplaats, 2009).

Optimization, or Mathematical Programming, refers to choosing the best element from some set of available alternatives (Yang, 2008). In mathematical programming, this means solving problems in which one seeks to minimize or maximize a real function by systematically choosing the values of real or integer variables from within an allowed set. More generally, it means finding best available values of some objective function given a defined domain, including a variety of different types of objective functions and different types of domains.

An optimization problem may be represented in the following way (Avriel, 2003):
Given: a function f: A $\longrightarrow$ R from some set A to the real numbers
Sought: an element x_0 in A such that $f(x_0) \leq f(x)$ for all x in A (minimization) or such that $f(x_0) \geq f(x)$ for all x in A (maximization).

A is some subset of the Euclidean space R^n, often specified by a set of constraints, equalities or inequalities that the members of A have to satisfy. The domain of A of f is called the *search space* or the *choice set*, while the elements of A are called *feasible solutions*. The f function is called an objective function. A feasible solution that maximizes (or minimizes, if that is the goal) the objective function is called an *optimal solution*.

Generally, when the feasible region, or the objective function of the problem does not present convexity, there may be several local minima and maxima, where a *local minimum* x^* is defined as a point for which there exists some $\delta > 0$ so that for all x such that

$$\| x - x^* \| \leq \delta;$$

The expression

$$f(x^*) \leq f(x)$$

Holds; that is to say, on some region around x^* all of the function values are greater than or equal to the value at that point. Local maxima are defined similarly (Avriel, 2003).

2.3 Multi-Objective Optimization

Adding more than one objective to an optimization problem adds complexity. For example, if you wanted to optimize a structural design, you would want a design that is both light and rigid. Since these two objectives conflict, a trade-off exists. There will be one lightest design, one stiffest design, and an infinite number of designs that are some compromise of weight and stiffness. This set of trade-off designs is known as a Pareto set. The curve created plotting weight against stiffness of the best designs is known as the Pareto frontier. A design is judged to be Pareto optimal if it is not dominated by other designs: a Pareto optimal design must be better than another design in at least one aspect. If it is worse than another design in all respects, then it is dominated and is not Pareto optimal (Papalambros and Wilde, 2000).

2.4 Multi-Modal Optimization

Optimization problems are often multi-modal, that is they possess multiple good solutions (Papalambros and Wilde, 2000). They could all be globally good (same cost function value) or there could be a mix of globally good and locally good solutions. Obtaining all (or at least some of) the multiple solutions is the goal of a multi-modal optimizer. Classical optimization techniques due to their iterative approach do not perform satisfactorily when they are used to obtain multiple solutions, since it is not guaranteed that different solutions will be obtained even with different starting points in multiple runs of the algorithm. Evolutionary Algorithms are however a very popular approach to obtain multiple solutions in a multi-modal optimization task (Papalambros and Wilde, 2000).

2.5 Analytical Characterization of Optima

The extreme value theorem of Karl Weierstrass states that if a real-valued function f is continuous in the closed and bounded interval $[a,b]$, then f must attain its maximum and minimum value, each at least once. That is, there exist numbers c and d in $[a,b]$ such that (Jerome, 1986):

$$f(\mathrm{c}) \geq f(x) \geq f(\mathrm{d}) \quad \text{for all } x \in [\,a,b\,].$$

A related theorem is the boundedness theorem which states that a continuous function f in the closed interval $[a,b]$ is bounded on that interval. That is, there exist real numbers m and M such that:

$$m \leq f(x) \leq M \quad \text{for all } x \in [\,a,b\,].$$

The extreme value theorem enriches the boundedness theorem by saying that not only is the function bounded, but it also attains its least upper bound as its maximum and its greatest lower bound as its minimum (Jerome, 1986).

The satisfiability problem, also called the feasibility problem, is just the problem of finding any feasible solution at all without regard to objective value. This can be regarded as the special case of mathematical optimization where the objective value is the same for every solution, and thus any solution is optimal (Elster, 1993).

Many optimization algorithms need to start from a feasible point. One way to obtain such a point is to relax the feasibility conditions using a slack variable; with enough slack, any starting point is feasible. Then, minimize that slack variable until slack is null or negative (Elster, 1993).

Fermat's theorem states that optima of unconstrained problems are found at stationary points, where the first derivative or the gradient of the objective function is zero. More generally, they may be found at critical points, where the first derivative or gradient of the objective function is zero or is undefined, or on the boundary of the choice set. An equation stating that the first derivative equals zero at an interior optimum is sometimes called a 'first-order condition'.

Optima of inequality-constrained problems are instead found by the Lagrange multiplier method. This method calculates a system of inequalities called the 'Karush-Kuhn-Tucker conditions' or 'complementary slackness conditions', which may then be used to calculate the optimum.

While the first derivative test identifies points that might be optima, it cannot distinguish a point which is a minimum from one that is a maximum or one that is neither. When the objective function is twice differentiable, these cases can be distinguished by checking the second derivative or the matrix of second derivatives (called the Hessian matrix) in unconstrained problems, or a matrix of second derivatives of the objective function and the constraints called the bordered Hessian. The conditions that distinguish maxima and minima from other stationary points are sometimes called 'second-order conditions' (Papalambros and Wilde, 2000).

III. METHODOLOGY

3.1 Concept of Lagranges Multipliers Method

This is a popular optimization method. In mathematical optimization, the method of Lagrange multipliers (named after Joseph Louis Lagrange) provides a strategy for finding the maximum/minimum of a function subject to constraints (Arfken, 1985).

Consider the optimization problem

$f(x, y)$

Subject to $g(x, y) = c$

A new variable (λ) called a Lagrange multiplier is introduced, and the Lagrange function is defined by

$$\Lambda(x, y, \lambda) = f(x, y) + \lambda \cdot (g(x, y) - c)$$

(λ may be either added or subtracted). If (x,y) is a maximum for the original constrained problem, then there exists a λ such that (x,y,λ) is a stationary point for the Lagrange function (stationary points are those points where the partial derivatives of Λ are zero). However, not all stationary points yield a solution of the original problem. Thus, the method of Lagrange multipliers yields a necessary condition for optimality in constrained problems (Arfken, 1985).

Consider the two-dimensional problem introduced above:

Maximize $f(x, y)$

Subject to $g(x, y) = c$

We can visualize contours of f given by

$f(x, y) = d$

For various values of d, and the contour of g given by $g(x,y) = c$.

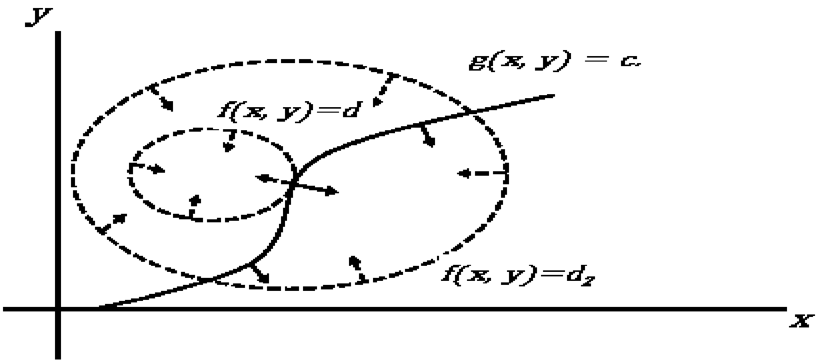

Figure 2: Continuous line showing constraint $g(x, y) = c$. (Source: Arfken, 1985)

The dotted lines are the contours of $f(x, y)$. The point where the continuous line touches the dotted line tangentially is our solution.

Suppose we walk along the contour line with $g = c$. In general the contour lines of f and g may be distinct, so following the contour line for $g = c$ one could intersect with or cross the contour lines of f. This is equivalent to saying that while moving along the contour line for $g = c$ the value of f can vary. Only when the contour line for $g = c$ meets contour lines of f tangentially, we do not increase or decrease the value of f — that is, when the contour lines touch but do not cross. The contour lines of f and g touch when the tangent vectors of the contour lines are parallel. Since the gradient of a function is perpendicular to the contour lines, this is the same as saying that the gradients of f and g are parallel (Arfken, 1985).

Thus we want points (x,y) where $g(x,y) = c$ and

$$\nabla_{x,y} f = -\lambda \nabla_{x,y} g,$$

Where

$$\nabla_{x,y} f = (\partial f/\partial x, \partial f/\partial y)$$

And

$$\nabla_{x,y} g = (\partial g/\partial x, \partial g/\partial y)$$

are the respective gradients. The constant λ is required because although the two gradient vectors are parallel, the magnitudes of the gradient vectors are generally not equal.

To incorporate these conditions into one equation, we introduce an auxiliary function:

$$\Lambda(x, y, \lambda) = f(x, y) + \lambda \cdot (g(x, y) - c)$$

and solve

$$\nabla_{x,y,\lambda} \Lambda(x, y, \lambda) = 0.$$

This is the method of Lagrange multipliers. Note that $\nabla_{\lambda} \Lambda(x, y, \lambda) = 0$ implies $g(x, y) = c$.

3.2 Design Procedure

A ribbed slab floor will be adequately designed to comfortably support a given design load using the BS8110 (1985; 1997) and EC2 (2008) code. Using the ultimate moment of resistance of singly reinforced concrete slabs, the optimization technique will thus be formulated. The goal here is to obtain the most cost effective, smallest and most reliable ribbed concrete slab section. It is important to note here that in order to achieve the optimum section, the major variables to be taken into consideration will be the height of slab, effective depth, area of reinforcing steel, and depth of concrete in compression. Also, member size restrictions will not be imposed.

IV. ANALYSIS AND DESIGN OF A CONTINOUS RIBBED/HOLLOW SLAB

A sample floor slab, consists of several units of ribbed slab, is simply supported at the ends. The effective span is 5.0 m, while the chosen characteristic dead load including finishes and partition is $1.5KN/m^2$ and the characteristic live load is $2.0\ kN/m^2$. The distance of the center to the center of the ribs is 300 mm, concrete strength $f_{cu} = 30N/mm^2$, and characteristic reinforcement, $f_y = 460N/mm^2$

4.1 Optimization Process

The optimization of the above designed ribbed floor will thus be commenced using minimum weight approach and the Lagrange's Multipliers Method. The target or aim as earlier mentioned will be to choose the most cost effective, smallest, and most effective concrete section and reinforcement. The major variables to be considered will be the height of slab, effective depth, area of reinforcing steel, and depth of concrete in compression.

Since optimization definitely focuses on quantity of materials to be used, the cost function will consist of the total sum of the cost of each material multiplied by its unit volume.

$$\text{Cost } (f) = C_c V_c + C_s V_s \tag{1}$$

Where V_c is total volume of section minus volume of steel

$$V_c = b \times d \times h - A_s \tag{2}$$

$$= b \times d \times h - A_s \times h$$

Since a unit volume is being considered, $h = 1$

$$= bd - A_s$$

$$\text{But } A_s = \rho bd$$

$$= bd - \rho bd \tag{3}$$

$$\text{Since } d_s = \rho d$$

$$= b(d - \rho d) = b(d - d_s)V_s = \rho bd = A_s \times 1 \tag{4}$$

$$\text{Cost} = C_c V_c + C_s V_s$$

$$= C_c\, b(d - d_s) + C_s\, \rho bd \tag{5}$$

$$\text{Let } D = -d_s / d \tag{6}$$

Dividing through by C_c

$$f = C_c\, b(1 + D)d + C_s\, \rho bd \tag{7}$$

$$\text{But } d = \sqrt{\frac{M_U}{Kb}} \tag{8}$$

$$f = C_c\, b(1 + D)d + C_s\, \rho b\sqrt{\frac{M_U}{Kb}}$$

$$f = C_c\, b(1 + D)\sqrt{\frac{M_U}{Kb}} + C_s\, \rho b\sqrt{\frac{M_U}{Kb}} \tag{9}$$

$$f = b\sqrt{\frac{M_U}{Kb}}\ [\,(C_c(1 + D)/C_c) + C_s\rho / C_c\,]\ C_c$$
(10)

Let $R = C_s / C_c$
(11)

$$f = b\sqrt{\frac{M_U}{Kb}}\ [\,(1 + D) + R\rho\,]\ C_c$$
(12)

$$f = \sqrt{\frac{b\,M_U}{K}}\ [\,(1 + D) + R\rho\,]\ C_c$$
(13)

C_c , b and M_U are considered constants, since C_c , b and d will remain constant for the slab under consideration. Therefore:

$$f = \left(\frac{1}{\sqrt{K}}\right) [\,(1 + D) + R\rho\,]$$
(14)

Problem is then presented thus :

Minimize $f = \left(\frac{1}{\sqrt{K}}\right) [\,(1 + D) + R\rho\,]$

Subject to

$$0.4 f_{cu}\ bx\,(d - \tfrac{x}{2}) = M_U$$
(15)

$$0.95 f_y\ A_s(d - \tfrac{x}{2}) = M_U \qquad \text{(BS8110, 1997)}$$
(16)

$$0.87 f_y\ A_s(d - \tfrac{x}{2}) = M_U \qquad \text{(EC2, 2008)}$$
(16a)

Where the value of ρ is restricted by BS8110 (1997) code and EC2 (2008) specifications for singly reinforced concrete slabs, the constraint is presented as:

$\rho_1 < \rho < \rho_U$

Equations (15), (16) and (17) are the ultimate moment of resistance of singly reinforced concrete sections presented in terms of concrete and steel strengths respectively according to BS8110 (1997) and EC (2008).

Since $M_U = K\,bd^2$
(17)

$$\text{Where}\ K = \left(\frac{0.4 f_{cu}\, x}{d} - \frac{0.4 f_{cu}\, x^2}{2d^2}\right)$$

Substituting equation (15)into (16)

$$0.4 f_{cu}\ bx\,(d - \tfrac{x}{2}) = 0.95 f_y\ A_s(d - \tfrac{x}{2}) \qquad \text{(BS8110, 1997)}$$

$$0.4 f_{cu}\ bx\,(d - \tfrac{x}{2}) = 0.87 f_y\ A_s(d - \tfrac{x}{2}) \qquad \text{(EC2, 2008)}$$

$$0.4 f_{cu}\ bx = 0.95 f_y\ A_s \qquad \text{(BS8110, 1997)}$$

$$0.4 f_{cu}\ bx = 0.87 f_y\ A_s \qquad \text{(EC2, 2008)}$$

$$x = \frac{0.95 f_y A_s}{0.4 f_{cu}\, b} \qquad \text{(BS8110, 1997)}$$
(18)

$$x = \frac{0.87 f_y A_s}{0.4 f_{cu}\, b} \qquad \text{(EC2, 2008)}$$
(18a)

$$x = \frac{2.37 f_y\, \rho d}{f_{cu}} \qquad \text{(BS8110, 1997)}$$
(19)

$$x = \frac{2.17\, f_y\, \rho d}{f_{cu}} \qquad \text{(EC2, 2008)}$$

(19a)

Substituting equation (19)into (16)

$$M_U = 0.95 f_y\, \rho bd \left(d - \frac{2.37\, f_y\, \rho d}{2 \times f_{cu}}\right) \qquad \text{(BS8110, 1997)} \qquad (20)$$

$$M_U = 0.87 f_y\, \rho bd \left(d - \frac{2.175\, f_y\, \rho d}{2 \times f_{cu}}\right) \qquad \text{(EC2, 2008)} \qquad (20a)$$

$$M_U = 0.95 f_y\, \rho bd \left(d - \frac{1.1875\, f_y\, \rho d}{f_{cu}}\right) \qquad \text{(BS8110, 1997)} \qquad (21)$$

$$M_U = 0.87 f_y\, \rho bd \left(d - \frac{1.0875\, f_y\, \rho d}{f_{cu}}\right) \qquad \text{(EC2, 2008)} \qquad (21a)$$

Since $M_U = K\, bd^2$, Therefore

$$K\, bd^2 = 0.95 f_y\, \rho bd^2 \left(1 - \frac{1.1875\, f_y\, \rho}{f_{cu}}\right) \qquad \text{(BS8110, 1997)}$$

$$K\, bd^2 = 0.87 f_y\, \rho bd^2 \left(1 - \frac{1.0875\, f_y\, \rho}{f_{cu}}\right) \qquad \text{(EC2, 2008)}$$

$$K = 0.95 f_y\, \rho \left(1 - \frac{1.1875\, f_y\, \rho}{f_{cu}}\right) \qquad \text{(BS8110, 1997)} \qquad (22)$$

$$K = 0.87 f_y\, \rho \left(1 - \frac{1.0875\, f_y\, \rho}{f_{cu}}\right) \qquad \text{(EC2, 2008)} \qquad (22a)$$

$$K = 0.95 f_y\, \rho - \frac{1.128\, f_y^2\, \rho^2}{f_{cu}} \qquad \text{(BS8110, 1997)} \qquad (23)$$

$$K = 0.87 f_y\, \rho - \frac{0.946\, f_y^2\, \rho^2}{f_{cu}} \qquad \text{(EC2, 2008)} \qquad (23a)$$

Dividing both sides by K

$$1 = \frac{0.95 f_y\, \rho}{K} - \frac{1.128\, f_y^2\, \rho^2}{K\, f_{cu}} \qquad \text{(BS8110, 1997)}$$

$$1 = \frac{0.87\, f_y\, \rho}{K} - \frac{0.946\, f_y^2\, \rho^2}{K\, f_{cu}} \qquad \text{(EC2, 2008)}$$

$$1 - \frac{0.95\, f_y\, \rho}{K} + \frac{1.128\, f_y^2\, \rho^2}{K\, f_{cu}} = 0 \qquad \text{(BS8110, 1997)} \qquad (24)$$

$$1 - \frac{0.87\, f_y\, \rho}{K} + \frac{0.946\, f_y^2\, \rho^2}{K\, f_{cu}} = 0 \qquad \text{(EC2, 2008)}$$

(24a)

Now let $\gamma_1 = 0.95 f_y$ (BS8110, 1997)

(25)

$$\gamma_1 = 0.87\, f_y \qquad \text{(EC2, 2008)}$$

(25a)

$$\gamma_2 = \frac{1.128\, f_y^2}{f_{cu}} \qquad \text{(BS8110, 1997)}$$

(26)

$$\square_2 = \frac{0.946\, \square_{\square}^2}{\square_{\square\square}} \qquad \text{(EC2, 2008)}$$

(26a)

Thus equation (24) becomes:

$$1 - \frac{\square_{\square}\, \rho}{K} + \frac{\square_{\square}\, \rho^2}{K} = 0$$

(27)

4.2 Applying Lagranges Multipliers Method

$$L = \left(\frac{1}{\sqrt{K}}\right) [\, (1 + D) + R\rho\,] - \lambda \left[1 - \frac{\square_{\square}\, \rho}{K} + \frac{\square_{\square}\, \rho^2}{K}\right]$$

(28)

Now let $(1 + D) = Q$

$$L = \left(\frac{1}{\sqrt{K}}\right)[Q + R\rho] - \lambda\left[1 - \frac{\square_\square \rho}{K} + \frac{\square_\square \rho^2}{K}\right]$$
(29)

L is then partially differentiated separately by ρ and K

$$\frac{\partial L}{\partial \rho} = \left(\frac{1}{\sqrt{K}}\right) - \lambda\left[0 - \frac{\square_\square}{K} + \frac{2\square_\square \rho}{K}\right] = 0$$
(30)

$$\frac{\partial L}{\partial K} = -\frac{Q + R\rho}{2K\sqrt{K}} - \lambda\left[0 + \frac{\square_\square \rho}{K^2} - \frac{\square_\square \rho^2}{K^2}\right] = 0$$
(31)

From equation (30)

$$\left(\frac{1}{\sqrt{K}}\right) = \lambda\left[0 - \frac{\square_\square}{K} + \frac{2\square_\square \rho}{K}\right]$$

$$\left(\frac{1}{\sqrt{K}}\right) = \frac{\lambda}{K(2\square_\square \rho - \square_\square)}$$
(32)

$$\lambda = \left(\frac{K}{\sqrt{K}}\right)(2\square_\square \rho - \square_\square)^{-1}$$

$$\lambda = \frac{\sqrt{K}}{2\square_\square \rho - \square_\square}$$
(33)

From equation (31)

$$\frac{Q + R\rho}{2K\sqrt{K}} = -\lambda\left[\frac{\square_\square \rho}{K^2} - \frac{\square_\square \rho^2}{K^2}\right]$$

$$\frac{Q + R\rho}{2K\sqrt{K}} = \frac{\lambda}{K^2(\square_\square \rho^2 - \square_\square \rho)}$$
(34)

$$\frac{(Q + R\rho)\sqrt{K}}{2} = \lambda(\square_\square \rho^2 - \square_\square \rho)$$
(35)

Substituting λ from equation (33) into (35)

$$\frac{(Q + R\rho)\sqrt{K}}{2} = \frac{\sqrt{K}(\square_\square \rho^2 - \square_\square \rho)}{(2\square_\square \rho - \square_\square)}$$
(36)

$$(Q + R\rho)(2\square_\square \rho - \square_\square) = 2(\square_\square \rho^2 - \square_\square \rho)$$
(37)

$$2\square_\square Q\rho - \square_\square Q + 2\square_\square R\rho^2 - \square_\square R\rho = 2\square_\square \rho^2 - 2\square_\square \rho$$

$$2\square_\square Q\rho - \square_\square Q + 2\square_\square R\rho^2 - \square_\square R\rho - 2\square_\square \rho^2 + 2\square_\square \rho = 0$$
(38)

$$\rho = \frac{\square_\square Q}{(2Q\square_\square + R\square_\square)}$$
(39)

Dividing through by Q $\square_\square$

$$\rho = \frac{1}{\left[\frac{R}{Q} + 2\frac{\square_\square}{\square_\square}\right]}$$
(40)

Recall $\square_\square = 0.95\square_\square$ (BS8110, 1997)

$\square_\square = 0.87\square_\square$ (EC2, 2008)

$\square_\square = \frac{1.128\,\square_\square^2}{\square_{\square\square}}$ (BS8110, 1997)

$\square_\square = \frac{0.946\,\square_\square^2}{\square_{\square\square}}$ (EC2, 2008)

Therefore,

$$\rho^{m}_{opt} = \left[\frac{R}{(1+D)} + \left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-1}$$ (BS8110,1997)

(41)

$$\rho^{m}_{opt} = \left[\frac{R}{(1+D)} + \left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-1}$$ (EC2, 2008)

(41a)

From equation (27)

$$1 = \frac{\square_{\square}\,\rho - \square_{\square}\,\rho^2}{K}$$

$$K^{m}_{opt} = \square_{\square}\,\rho - \square_{\square}\,\rho^2$$

(42)

To optimize K, we obtain its partial derivative with respect to its principal variables R and D

$$\frac{\partial K^{m}_{opt}}{\partial R} = \square_{\square}\frac{\partial \rho^{m}_{opt}}{\partial R} - 2\,\square_{\square}\,\rho^{m}_{opt}\frac{\partial \rho^{m}_{opt}}{\partial R} = 0$$

$$\frac{\partial K^{m}_{opt}}{\partial R} = \left(\square_{\square} - 2\,\square_{\square}\,\rho^{m}_{opt}\right)\frac{\square\rho^{\square}_{\square\square\square}}{\square R}$$

(43)

$$\frac{\square K^{\square}_{\square\square\square}}{\square D} = \square_{1}\frac{\square\rho^{\square}_{\square\square\square}}{\square D} - 2\,\square_{2}\,\rho^{\square}_{\square\square\square}\frac{\square\rho^{\square}_{\square\square\square}}{\square D} = 0$$

$$\frac{\square K^{\square}_{\square\square\square}}{\square D} = \left(\square_{\square} - 2\,\square_{\square}\,\rho^{m}_{opt}\right)\frac{\partial \rho^{m}_{opt}}{\partial D}$$

(44)

From equation (41)

$$\frac{\partial \rho^{m}_{opt}}{\partial R} = \frac{\partial\left[\frac{R}{(1+D)} + \left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-1}}{\partial R}$$ (BS8110,1997)

$$\frac{\partial \rho^{m}_{opt}}{\partial R} = \frac{\partial\left[\frac{R}{(1+D)} + \left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-1}}{\partial R}$$ (EC2, 2008)

$$\frac{\partial \rho^{m}_{opt}}{\partial R} = -\left[\frac{R}{(1+D)} + \left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}(1+D)^{-1} = 0$$ (BS8110,1997)

(45)

$$\frac{\partial \rho^{m}_{opt}}{\partial R} = -\left[\frac{R}{(1+D)} + \left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}(1+D)^{-1} = 0$$ (EC2, 2008)

(45a)

$$\frac{\square\rho^{\square}_{\square\square\square}}{\square D} = \frac{\partial\left[\frac{R}{(1+D)} + \left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-1}}{\partial D}$$ (BS8110,1997)

$$\frac{\square\rho^{\square}_{\square\square\square}}{\square D} = \frac{\partial\left[\frac{R}{(1+D)} + \left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-1}}{\partial D}$$ (EC2, 2008)

$$\frac{\square\rho^{\square}_{\square\square\square}}{\square D} = +\left[\frac{R}{(1+D)} + \left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}\frac{R}{(1+D)^2} = 0$$ (BS8110,1997)

(46)

$$\frac{\square\rho^{\square}_{\square\square\square}}{\square D} = +\left[\frac{R}{(1+D)} + \left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}\frac{R}{(1+D)^2} = 0$$ (EC2, 2008)

(46a)

But $\frac{\partial \rho^{m}_{opt}}{\partial R} = \frac{\partial \rho^{m}_{opt}}{\partial D} = 0$

Therefore equating equation (45) to (46)

For (BS8110, 1997),

$$-\left[\frac{R}{(1+D)}+\left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}(1+D)^{-1}=+\left[\frac{R}{(1+D)}+\left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}\frac{R}{(1+D)^{2}}$$

For (EC2, 2008),

$$-\left[\frac{R}{(1+D)}+\left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}(1+D)^{-1}=+\left[\frac{R}{(1+D)}+\left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}\right)\right]^{-2}\frac{R}{(1+D)^{2}}$$

$$-(1+D)^{-1}=\frac{R}{(1+D)^{2}}$$

(47)

$$R(1+D)=-(1+D)^{2}$$

$$R=-(1+D)$$

(48)

Substituting equation (48) into (41)

$$\rho_{opt}^{m}=\left(\frac{2.37\,\square_{\square}}{\square_{\square\square}}-1\right)^{-1} \qquad \text{(BS8110, 1997)}$$

(49)

$$\rho_{opt}^{m}=\left(\frac{2.17\,\square_{\square}}{\square_{\square\square}}-1\right)^{-1} \qquad \text{(EC2, 2008)}$$

(49a)

$$\rho_{opt}^{m}=\frac{\square_{\square\square}}{(2.37\,\square_{\square}-\square_{\square\square})} \qquad \text{(BS8110, 1997)}$$

(50)

$$\rho_{opt}^{m}=\frac{\square_{\square\square}}{(2.17\,\square_{\square}-\square_{\square\square})} \qquad \text{(EC2, 2008)}$$

(50a)

Substituting equation (50) into (42)

$$K_{\square\square\square}^{\square}=\frac{0.95\square_{\square}\,\square_{\square\square}}{(2.37\,\square_{\square}-\square_{\square\square})}-\frac{1.128\,\square_{\square}^{2}\,\square_{\square\square}^{2}}{\square_{\square\square}\,(2.37\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(BS8110, 1997)}$$

(51)

$$K_{\square\square\square}^{\square}=\frac{0.87\square_{\square}\,\square_{\square\square}}{(2.17\,\square_{\square}-\square_{\square\square})}-\frac{0.946\,\square_{\square}^{2}\,\square_{\square\square}^{2}}{\square_{\square\square}\,(2.17\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(EC2, 2008)}$$

(51a)

$$K_{\square\square\square}^{\square}=\frac{0.95\square_{\square}\,\square_{\square\square}\,(2.37\,\square_{\square}-\square_{\square\square})-1.128\,\square_{\square}^{2}\,\square_{\square\square}}{(2.37\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(BS8110, 1997)} \qquad (52)$$

$$K_{\square\square\square}^{\square}=\frac{0.87\square_{\square}\,\square_{\square\square}\,(2.17\,\square_{\square}-\square_{\square\square})-0.87\square_{\square}^{2}\,\square_{\square\square}}{(2.17\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(EC2, 2008)} \qquad (52a)$$

$$K_{\square\square\square}^{\square}=\frac{2.25\,\square_{\square}^{2}\,\square_{\square\square}-0.95\,\square_{\square}\,\square_{\square\square}^{2}-1.128\,\square_{\square}^{2}\,\square_{\square\square}}{(2.37\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(BS8110, 1997)}$$

$$K_{\square\square\square}^{\square}=\frac{1.8879\,\square_{\square}^{2}\,\square_{\square\square}-0.87\,\square_{\square}\,\square_{\square\square}^{2}-0.946\,\square_{\square}^{2}\,\square_{\square\square}}{(2.17\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(EC2, 2008)}$$

$$K_{\square\square\square}^{\square}=\frac{\square_{\square}\,f_{\square\square}\,(1.12\,\square_{\square}-0.95\square_{\square\square})}{(2.37\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(BS8110, 1997)}$$

(53)

$$K_{\square\square\square}^{\square}=\frac{\square_{\square}\,f_{\square\square}\,(0.942\,\square_{\square}-0.87\square_{\square\square})}{(2.17\,\square_{\square}-\square_{\square\square})^{2}} \qquad \text{(EC2, 2008)}$$

(53a)

Thus the optimum values of the reinforcement ratio and the parameter K can be derived for any combination of reinforcement and concrete strengths from equations (50) and (53) respectively using BS8110, (1997) and EC2 (2008).

4.3 Results of Lagranges Multipliers Optimization Technique

Having successfully derived the formula for K, it is now obvious that the under-estimations for the various combinations of concrete strength and steel strength can be obtained by substituting each value of K into the ultimate moment of resistance formulae. Using the above derived formulae, the ribbed floor slab designed on page (25) can now be optimized. From that problem, the respective steel and concrete strengths were:-

$$f_y = 460\text{N/mm}^2 \text{ and,}$$
$$f_{cu} = 30\text{N/mm}^2$$

Therefore from equation (53)

$$K^{\square}_{\square\square\square} = \frac{(460\times 30)\left((1.12\times 460)-(0.95\times 30)\right)}{\left((2.37\times 460)-30\right)^2} \qquad \text{(BS8110, 1997)}$$

$$= 5.97537042$$

From equation (17)

$$M_u = Kbd^2$$
$$M_u = 5.97537042bd^2 \qquad (54)$$

The ultimate moment for a singly reinforced concrete section is given by BS8110 (1997) as :

$$M_u = 0.156\, bd^2 \square_{\square\square}$$
(55)

In order to evaluate the resistance moment, equation (55) is represented as :

$$M_u = 0.156\, bd^2 \square_{\square\square}\, \square_{\square}$$
(56)

Equating equation (56) to (54)

$$0.156\, bd^2 \square_{\square\square}\, \square_{\square} = 5.97537042bd^2$$
$$\square_{\square} = \frac{5.97537042}{0.156\, \square_{\square\square}} \qquad (57)$$

But $\square_{\square\square} = 30\text{N/mm}^2$, therefore

$$\square_{\square} = \frac{5.97537042}{0.156\times 30}$$
$$\square_{\square} = 1.276788551$$
(58)

Therefore, for a singly reinforced concrete section made up of grade 30 concrete and steel strength of 460N/mm^2, the ultimate method of resistance is under-estimated by about 27.679 percent using the BS8110 (1997).

The amount of reduction in the value of under-estimation as indicated in BS8110 (1997) simply shows the quantitative value of the quality control and cost savings associated with it. The value of $\square_o$ can be obtained for the other reinforced ribbed floor slab concrete sections with various combinations of steel and concrete strengths. An objective function value of less than 1 signifies the degree of effectiveness of the optimization technique used. The value of D has been taken as 0.06 as an example.

Also, steel strengths of 250N/mm^2 and 460N/mm^2 will be used by BS8110 (1997) as they are more recognized by the code though EC2 (2008) will use steel strengths of 250N/mm^2 and 500N/mm^2. Other steel strengths may also be used.

Table 1: Data for Percentage Under-Estimation for Various Singly Reinforced Ribbed Slab Concrete Sections Using BS8110 (1997).

S/NO	Steel strength (f_y) N/mm^2	Concrete strength (f_{cu}) N/mm^2	K^m_{opt}	$\square_o$	Percentage Under-estimation	Objective Function (f)
		20	3.981617437	1.276159435	27.6159	0.51266
		25	4.972927866	1.275109709	27.5110	0.45440

1	250	30	5.961481481	1.273820829	27.3821	0.41098
		40	7.927765607	1.270475258	27.0475	0.34921
		50	9.874917709	1.266015091	26.6015	0.30623
		60	11.7966012	1.260320641	26.0321	0.27385
2	460	20	3.985792558	1.277497615	27.7498	0.52102
		25	4.98098076	1.277174554	27.7175	0.46380
		30	5.97537042	1.276788511	27.6789	0.42136
		40	7.961121098	1.275820689	27.5821	0.36137
		50	9.941719533	1.274579427	27.4579	0.32002
		60	11.91574158	1.273049314	27.3049	0.28919

Table 2: Data for Percentage Under-Estimation for Various Singly Reinforced Ribbed Slab Concrete Sections Using EC2 (2008).

S/NO	Steel strength (f_y) N/mm^2	Concrete strength (f_{cu}) N/mm^2	$K^m{}_{opt}$	$\square_o$	Percentage Under-estimation	Objective Function (f)
1	250	20	3.985256748	1.193190643	19.3191	0.51065
		25	4.976778921	1.192042855	19.2043	0.45220
		30	5.965020821	1.190622918	19.0623	0.40860
		40	7.928516621	1.186903686	18.6904	0.34649
		50	9.868844856	1.181897588	18.1898	0.30317
		60	11.77803431	1.175452526	17.5453	0.27046
2	500	20	3.990389914	1.194727519	19.4728	0.52067
		25	4.986761303	1.194433845	19.4434	0.46348
		30	5.98234541	1.19408092	19.4081	0.42106
		40	7.970513496	1.193190643	19.3191	0.36109
		50	9.953557843	1.192042855	19.2043	0.31975
		60	11.93004164	1.190622918	19.0623	0.28893

V. CONCLUSION AND RECOMMENDATION

From the results, it is obvious that optimization is possible as the current ultimate limit allowed by BS8110 (1997) and EC2 (2008) favors structure reliability over cost minimization. As observed in table 1 and 2 above, an under-estimation of averagely about 27 percent was deduced using BS8110 (1997) while the EC2 (2008) had a lesser under-estimation percentage value of about 19 percent. For a major construction project, optimization could greatly reduce cost in terms of material usage.

It is also important to note that the under-estimation allowed by the British standard code and the EC2 (2008) create accommodation for uncertainties in engineering design like human errors, variations in material strength and variations in wind loading. It also ensures that high expense is traded by safety and reliability of the structure.

REFERENCES

[1] Arfken G.: “Lagrange Multipliers in Mathematical Methods for Physicists”, 3rd edition, Orlando, FL: Academic Press, pp. 945-950, 1985.

[2] Avriel M.: “Nonlinear Programming”, *Analysis and Methods,* Dover Publishing, 2003.

[3] BS8110: “British Standard Institution: The structural use of concrete:” *Parts 1, 2, and 3*. Her Majesty’s stationary office, London, 1997.

[4] EC 2 Eurocode 2: “Part 1.1”: *Design of Concrete Structures*. European Committee for Standardization, Brussels, 2008.

[5] Elster K. H.: “Modern Mathematical Methods of Optimization”, *Vch Publishing*, 1993.

[6] Fischer S. D., Jerome J. W.: “The existence, characterization and essential uniqueness of L_∞ external problems”, Trans. Amer. Math. Soc., 187 (1974), pp. 391–404

[7] Melchers R. E.: “Structural Reliability Analysis and Prediction”. *Ellis Horwood Series in Engineering, Cooper Strut,* West Sussex, England, 1987.

[8] Mohammed A. A.: “How high could buildings made of Ribbed or Flat slab construction without shear walls be built?” *Addis Ababa University Press*. 2006, pp 5-7.

[9] Mosley W. H. and Bungey J. H.: “Reinforced concrete design Fourth edition”, *Macmillan press limited*, 1990, pp 193-195.

[10] Papalambros P. S, and D. J. Wilde D. J.: “Principles of Optimal Design”, *Modeling and Computation*, Cambridge University Press, 2000.

[11] Vanderplaats G. N.: Numerical Optimization techniques for Engineering Designs with Applications”, VMA Engineering, McGraw-Hill Companies January 1st 1984.

[12] Yang X. S.: “Introduction to Mathematical Optimization”, *From Linear Programming to Metaheuristics*, Cambridge Int. Science Publishing, 2008.