American Journal of Engineering Research (AJER)
e-ISSN : 2320-0847 p-ISSN : 2320-0936
Volume-4, Issue-1, pp-76-81
www.ajer.org
Research Paper Open Access

A Concept of Input-Output oriented Super-efficiency in Decision Making Units.

Dr.S.Chandrababu[1], Dr.S.Hariprasad[2]
[1,]Lecturer in Statists, N.P.S Govt College, Chittoor
[2,]Assistant Professor in Statistics, P.V.K.N Govt College, Chittoor,
[3,]Corresponding Author: Dr.S.Chandrababu

ABSTRACT : *In data envelopment analysis the input and output decision making units plays a major role to evaluate the efficiency and its super-efficiency. In this paper, super-efficiency can be assessed through graphically with simple notations. The area pointed on the feasibility of decision making units when i) Extremely efficient, ii) Efficient, but not extremely efficient, iii) Weakly efficient and iv) Inefficient.*

KEY WORDS: *Super efficiency, Data Envelopment Analysis, Decision Making Units, Input and output Super efficiency.*

I. INTRODUCTION

The data envelopment analysis as formulated by BCC gives input and output targets to the decision making units. These targets are pro rata. For an inefficient DMU inputs are decreased and outputs are increased pro rata. Pro rata input targets reveal input losses and output targets yield output losses of a decision making unit whose efficiency is under evaluation. A major deficiency of DEA is the DMU_0 under evaluation chooses multiplier weights most advantageous to it. To get out of this problem the multi-objective DEA problems have to be solved. In this approach we minimize the maximum deviation or sum of all the deviations. This approach identifies fewer numbers of DMUs as efficient. The efficiencies are evaluated exposing all the DMUs to the same light.

II. SUPER EFFICIENCY

When efficiency problem is solved for all the DMUs in competition, it is possible that more than one DMU arises with 100% efficiency score. If further analysis is to be performed to rank the efficient units one may solve super-efficient problem for extremely efficient units. In fact efficient DMUs are three types, extremely efficient, efficient but not extremely efficient and weekly efficient. To find super efficiency of extremely efficient DMU, its input and output vectors are removed from the reference technology. Consequently, the production possibility set shrinks which allows DMUs to become super-efficient attaining super efficiency score that exceeds unity. There is no need that a super efficiency problem to be feasible when RTS are variable. If an input based super – efficiency problem is infeasible then the corresponding output based super-efficiency problem is feasible. Similarly, if an output based super –efficiency problem is infeasible then the corresponding input based super-efficiency problem is feasible.However, for CRS production possibility set, both the super efficiency problems are always feasible.

The concept of 'Super-efficiency' arises if (i) the Decision Making Unit (DMU) is extremely efficient and (ii) the DMU under evaluation is removed from the reference set.

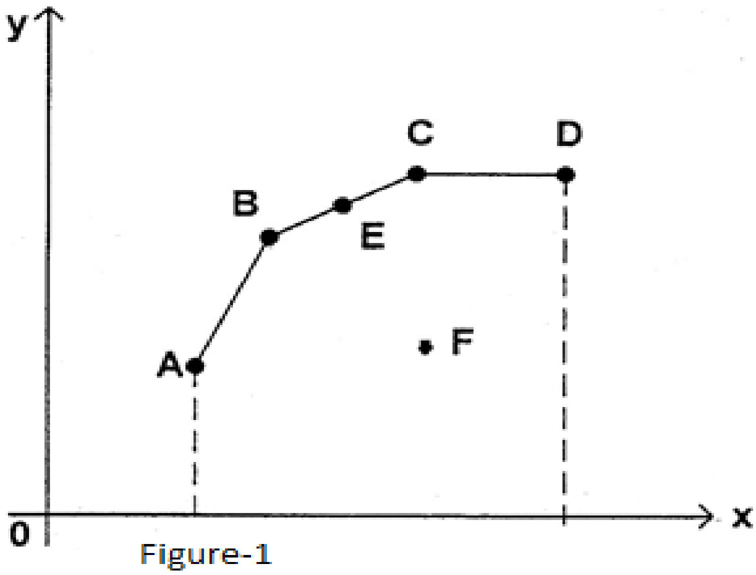

Figure-1

In figure - 1 we find six DMUs A, B, C, D, E and F. The DMUs A, B, C and D are extremely efficient. These DMUs determine the frontier production function. DMU E is efficient but not extremely efficient since, its output, input pair is determined as a convex combination input, output pairs of DMUs B and DMU C. DMU F is inefficient. All the DMUs employ one input and produce one output.

The production possibility set is enveloped by a piecewise linear frontier constituted by the line segments AB, BC and CD. If T stands for graph (production possibility set) of the technology, then it takes the from

$$T = \left\{ (x, y) \sum_{j=1}^{n} \theta_j x_j \le x, \sum_{j=1}^{n} \theta_j y_j \le y, \theta_j \ge 0 \sum_{j=1}^{n} \theta_j = 1 \right\}$$

Where $x \in R_+^m$, $y \in R_+^s$ x_j and y_j are respectively the input and output vectors of DMU_j.

To estimate input technical efficiency of DMU_0 we solve the following linear programming problem.

$$\lambda_0^* = \text{Min} \quad \lambda_0$$

Subject to

$$\left.\begin{array}{ll} \sum_{j=1}^{n} \theta_j x_{ij} \le \lambda_0 x_{i0}, & i = 1, 2, \dots\dots\dots \quad \dots\dots, \quad m \\ \sum_{j=1}^{n} \theta_j y_{ij} \ge y_{r0} & r = 1, 2, \dots\dots\dots \quad \dots\dots, \quad s \\ \sum_{j=1}^{n} \theta_j = 1 \quad , & \theta_j \ge 0 \end{array}\right\} \dots\dots\dots\dots\dots\dots [1]$$

There arise four cases

[1] DMU_0 is extremely efficient
[2] DMU_0 is efficient, but not extremely efficient
[3] DMU_0 is weakly efficient
[4] DMUs is inefficient

If a DMU is extremely efficient we obtain, $\lambda_0^* = 1, \theta_j^* = 0, \forall \ j \neq 0, \theta_0^* = 1$

$$s_i^{-*} = s_r^{+*} = 0, \ \forall \ i \wedge r$$

If DMU_0 is weakly efficient the optimal solution is of the form

$$\lambda_0^* = 1, \theta_j^* = 0, \forall j \neq 0, \theta_0^* = 1$$

$s_i^{-*} \neq 0$ For some 'i' and / or $s_r^{+*} \neq 0$, for some 'r'

If the DMU under evaluation is inefficient then we have the following optimal solution:

$\lambda_0^* < 1, \theta_0^* = 0, \sum_{j \neq 0} \theta_j^* = 1$, some input and / or output slacks are non-zero.

If an extremely efficient DMU is removed from the references set we obtain.

$$\left(\sum_{j \neq o} \theta_j x_{ij}, \sum_{j \neq o} \theta_j y_{rj} \right), \qquad i = 1, 2, \ldots\ldots\ldots, m \ \text{ And } r = 1, 2 \ldots\ldots\ldots, s$$

Consequently the production possibility set shrinks.

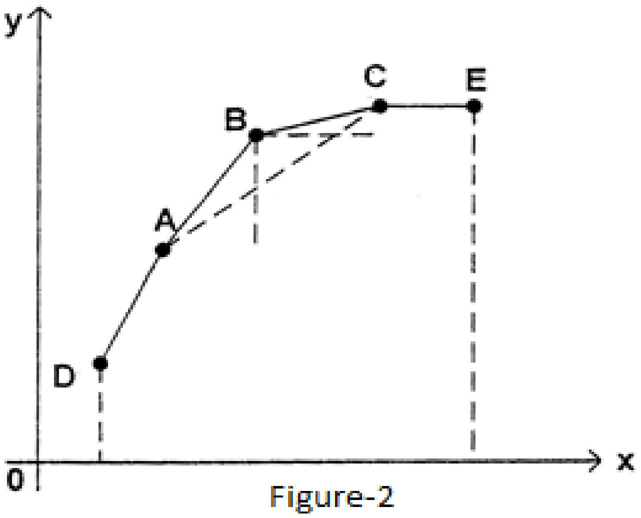

Figure-2

The frontier production function is determined by the decision making units A, B, C, D and E. DMU B is extremely efficient[1].

If DMU B is removed from the reference set the production possibility set is found to be enveloped by the line segments DA, AC and CE. Consequently, the resultant production possibility set is a subset of the original production possibility set.

[1] A DMU is said to be extremely efficient if its input and output vectors cannot be expressed as a convex combination of the input, output vectors of two or more than two points of the production possibility set, frontier.

The output oriented technical efficiency of DMU_0 can be computed by solving the following linear programming problem.

$$\gamma_0 = \text{Max} \quad \gamma_0$$

Subject to

$$\left.\begin{aligned} &\sum_{j=1}^{n} \theta_j x_{ij} \le x_{i0}, && i = 1, 2, \ldots\ldots, m \\ &\sum_{j=1}^{n} \theta_j y_{nj} \le \gamma_0 \; y_{r0}, && r = 1, 2, \ldots\ldots, s \\ &\sum_{j=1}^{n} \theta_j = 1 && \theta_j = 0 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \quad [2]$$

If DMU_0 is extremely efficient, we must have the optimal solution as follows.

$$\gamma_0^* = 1, \; \theta_0^* = 1, \; \theta_j^* = 0, \; \forall \; j \ne 0$$

$$s_i^{-*} = s_r^{+*} = 0$$

Super efficiency refers to extremely efficient DMUs, since if an efficient DMU that is not extremely efficient or weakly efficient or an inefficient DMU is removed from the reference set the production possibility set remains to be the same.

Input super efficiency:

DMU B below is extremely efficient and it is removed from the reference set. The following input super – efficiency problem is solved for

DMU B = DMU_0.

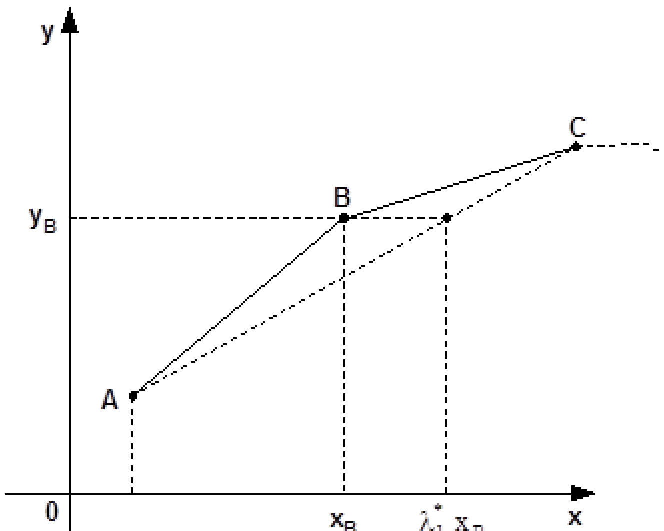

Figure-3

$$\lambda_1^* = \text{Min} \quad \lambda$$

Subject to

$$\left.\begin{aligned} &\sum_{j\neq 0}^{n} \theta_j x_{ij} \leq \lambda_1 x_{i0}, \quad i = 1, 2, \ldots\ldots\ldots \ \ldots\ldots, \quad m \\ &\sum_{j\neq 0}^{n} \theta_j y_{rj} \geq y_{r0}, \\ &\sum_{j\neq 0}^{n} \theta_j = 1 \quad , \quad \theta_j \geq 0, \forall j \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots[3]$$

The super – efficiency score of DMU_0 is λ_1^*. It can be seen that $\lambda_1^* > 1$. DMU_0 is said to be input super-efficient, because to produce the output vector of DMU_0, the remaining DMUs or a convex combination of them need to utilize more inputs than the DMU under evaluation viz., DMU_0. The input saving of the super-efficient DMU is $(\lambda_1^* - 1)x_0$.

Thus, larger is λ_1^* above unity greater is the super-efficiency of the DMU.

Output Super Efficiency

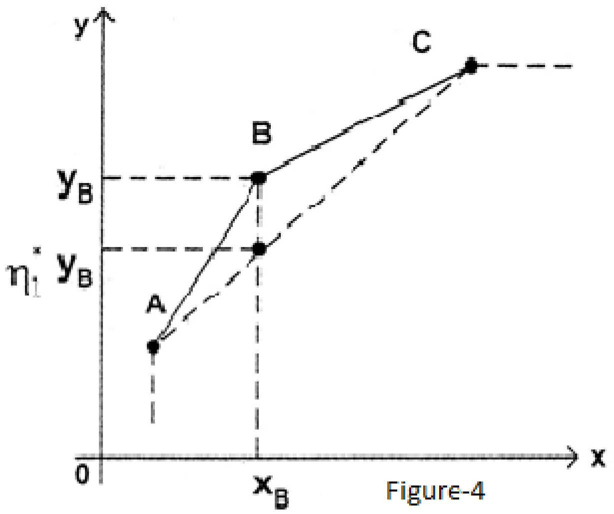

Figure-4

Consider the following output super –efficiency problem

$$\gamma_1^* = \text{Max} \quad \gamma_1$$

Subject to

$$\left.\begin{aligned} &\sum_{j\neq 0} \theta_j x_{ij} \le x_{i0}, \qquad i = 1, 2, \ldots\ldots, m \\ &\sum_{j\neq 0}^{n} \theta_j y_{rj} \le \gamma_1 y_{r0}, \qquad r = 1, 2, \ldots\ldots, s \\ &\sum_{j\neq 0}^{n} \theta_j = 1, \qquad \theta_j \ge 0, \forall j \neq 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots[4]$$

The super-efficiency score of DMU_0 = DMU B in the above diagram is $\gamma_1^* < 1$. With the inputs of DMU_0 the other DMUs or a convex combination of them shall be able to produce lesser output than what DMU_0 actually produces. The output super-efficient DMU enjoys output gains. The output gain of output super-efficient DMU is $(1 - \gamma_1^*) y_0$.

DMU B which is both input and output super-efficient has its constraints feasible in input and output orientations.

A study of super-efficiency problem has at least three applications

[1] If an input oriented or output oriented DEA problem is solved, it is likely that there exists more than one technical efficient DMU. In such a situation these efficient DMUs have to be ranked. If all super-efficiency problems are feasible, the extremely efficient DMUs can be ranked according to their super-efficiency. Larger super-efficiency implies better the DMU is ranked.

[2] An input oriented super efficiency problem can be solved to estimate input gains of an extreme efficient DMU. An output oriented super-efficiency problem may be solved to estimate output gains of extreme point DMU.

[3] In sensitivity analysis of an extreme efficient DMU, the super-efficient Data Envelopment Analysis (DEA) is used.

III. CONCLUSION:

Characterized the input and output oriented targets in decision making analysis. The concept of 'Super-efficiency' is evaluated if (i) the Decision Making Unit (DMU) is extremely efficient and (ii) the DMU under evaluation is removed from the reference set.

BIBLIOGRAPHY:

[1] **Bankar, R.D., Charnes, A., Cooper, W.W. (1984):-** "Some models for the estimation of technical and scale inefficiency in Data Envelopment Analysis", Management Science, 30, 1078-1092.

[2] **Banker, R.D. and Morey, R. (1986):-** "Efficiency Analysis for exogenously fixed inputs and outputs in operations research", 34, 89-98.

[3] **Banker, R.D., and Thrall, R.M. (1992):-** "Estimation of Returns to Scale using Data Envelopment Analysis", European Journal of Operations Research, 62, pp.74-84.

[4] **Charnes A., Cooper, W.W. and Rhodes, E. (1978):-** "Measuring the efficiency of decision making units", European Journal of Operations Research, No.2: 429-444.

[5] **Sciford and Zhu (1999):-** "Infeasibility of Super Efficiency DEA models", Infor 33, pp. 174-187.

[6] **Subramanyam, G., Swamy, S.B., (1994):-** "Production Efficiency Difference between Large and Small Banks", Artha Vijnana, September, Vol. 36, No. 3, pp. 183-193.

[7] **Yue, Piyu, (1992):-** " Data envelopment analysis and commercial bank performance: A primer with applications to Missouri banks", Federal Reserve Bank of St. Louis Economic Review 74(1), pp. 31-45.

[8] **Zenios, Christiana V., Stavros, A., Zenios, Kostas Agathocleous and Andreas C. Soteriou, (1999):-** "Benchmarks of the efficiency of bank branches", Interfaces 29(3), pp. 37-51.A. and Revanker, N.B., (1969), "Generalized production functions, Review of Economic Studies, 36, 241-250.