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Analysis of a Prey-Predator System with Modified Transmission Function

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ABSTRACT - *In this paper, a predator–prey model with a non-homogeneous transmission functional response is studied. It is interesting to note that the system is persistent. The purpose of this work is to offer some mathematical analysis of the dynamics of a two prey one predator system. Criteria for local stability and global stability of the non- negative equilibria are obtained. Using differential inequality, we obtain sufficient conditions that ensure the persistence of the system.*

KEYWORDS – *Transmission function, Prey-Predator interaction, Local stability and Global Stability*

I. INTRODUCTION

Mathematical modelling is frequently an evolving process. Systematic mathematical analysis can often lead to better understanding of bio-economic models. System of differential equations has, to a certain extent, successfully described the interactions between species. There exists a huge literature documenting ecological and mathematical result from the model. Heathcoat et al. [6] proposed some epidemiological model with nonlinear incidence. Kesh et al. [3] proposed and analyzed a mathematical model of two competing prey and one predator species where the prey species follow Lotka - Volterra dynamics and predator uptake functions are ratio dependent. Some works in context of source-sink dynamics are due to Newman et al. [10]. His results show that the presence of refuge can greatly stabilize a population that otherwise would exhibit chaotic dynamics. Dubey et al. [2] analyzed a dynamic model for a single species fishery which depends partially on a logistically growing resource in a two patch environment. Ruan et al. [9] studied the global dynamics of an epidemic model with vital dynamics and nonlinear incidence rate of saturated mass action. Kar [11] considered a prey- predator fishery model and discussed the selective harvesting of fishes age or size by incorporating a time delay in the harvesting terms. Feng [14] considered a differential equation system with diffusion and time delay which models the dynamics of predator prey interactions within three biological species. Kar et al. [13] describe a prey predator model with Holling type II functional response where harvesting of each species is taken into consideration. Braza [8] considered a two predator; one prey model in which one predator interferes significantly with the other predator is analyzed. Kar and Chakraborty [12] considered a prey predator fishery model with prey dispersal in a two patch environment, one of which is a free fishing zone and other is protected zone. Sisodia et al. [1] proposed a generalized mathematical model to study the depletion of resources by two kinds of populations, one is weaker and others stronger. The dynamics of resources is governed by generalized logistic equation whereas the population of interacting species follows the logistic law. We have formulated and analyzed two species prey-predator model in which the prey dispersal in a two patch environment. Mehta et al. [4] considered prey predator model with reserved and unreserved area having modified transmission function. A model of predator-prey in homogeneous environment with Holling type-II functionl response is introduced to Alebraheen et al. [7]. Recently Mehta et al. [5] describe the epidemic model with an asymptotically homogeneous transmission function.

In this paper biological equilibria of the system are obtained and criteria for local stability and global stability of the system derived. We have investigate the model persistence with an asymptotically transmission function.

II. MATHEMATICAL MODEL

Mathematical Model considered is based on the predator –prey system WITH MODIFIED change transmission rate:

$$\frac{dx_1}{dt} = rx_1(1-\frac{x_1}{K_1}) - px_1x_2 - \omega_1 x_1 y,$$
$$\frac{dx_2}{dt} = sx_2(1-\frac{x_2}{K_2}) - qx_1 y - \frac{\omega_2 x_2 y}{A+Bx_2+Cy}, \qquad (1)$$
$$\frac{dy}{dt} = b_1\omega_1 x_1 y + b_2\omega_2 \frac{x_2 y}{A+Bx_2+Cy} - ky,$$

Where X_1, X_2 denote population densities of prey and y denote population density of the predator. In model (1) r and s are the intrinsic growth rate of two prey species, K_1 and K_2 are their carrying capacities, k is the mortality rate coefficient of the predator, p, q are inter species interference coefficient of two prey species. b_1 and b_2 are the conversion factors denoting the number of newly born predators for each captured of first and second prey respectively, ω_1 is the first prey specie's searching efficiency and ω_2 is the second type prey specie's searching efficiency of the predator.

III. EQUILIBRIUM ANALYSIS

The system (2) has seven equilibrium points, $E_0(0,0,0), E_1(K_1,0,0), E_2(0,K_2,0), E_3(x_1^*,x_2^*,0)$, $E_4(0,\bar{x}_2,\bar{y}), E_5(\tilde{x}_1,0,\tilde{y}), E_6(\hat{x}_1,\hat{x}_2,\hat{y})$ where three of them, namely $E_0(0,0,0), E_1(K_1,0,0)$, $E_2(0,K_2,0)$ always exist. We show the existence of other equilibria as follows:

Existence of $E_3(x_1^*, x_2^*, 0)$

Here x_1^*, x_2^* are the positive solutions of the following algebraic equations.

$$r(1-\frac{x_1}{K_1}) - px_2 = 0 \qquad (3)$$

$$s(1-\frac{x_2}{K_2}) - qx_1 = 0 \qquad (4)$$

Solving (3) and (4) we get

$$x_1^* = \frac{sK_1(r-pK_2)}{rs-pqK_1K_2}, x_2^* = \frac{rK_2(s-qK_1)}{rs-pqK_1K_2} \qquad (5)$$

Thus the equilibrium $E_3(x_1^*,x_2^*,0)$ exists

if $(r-pK_2)$ and $(s-qK_1)$ are of same sign, that is either

$$r > pK_2 \text{ and } s > qK_1 \qquad (6)$$
$$r < pK_2 \text{ and } s < qK_1 \qquad (7)$$

Existence of $E_4(0,\bar{x}_2,\bar{y})$

Here $E_4(0,\bar{x}_2,\bar{y})$ is the positive solution of the following algebraic equations.

$$s(1-\frac{x_2}{K_2}) - \frac{\omega_2 x_2 y}{(A+Bx_2+Cy)} = 0 \qquad (8)$$

$$b_2\omega_2 \frac{x_2^2}{A+Bx_2+Cy} - k = 0 \qquad (9)$$

Solving (8) and (9) we get

$$\bar{x}_2 = \frac{kA+Cky}{b_2\omega_2 - kB} \text{ and } \bar{y} = \frac{s(A+Bx_2)(K_2-Bx_2)}{\omega_2 K_2^2 - Cs(K_2-sx_2)} \qquad (10)$$

It can be seen that $E_4(0,\bar{x}_2,\bar{y})$ exists if

$(b_2\omega_2 - kB) > k(A+Cx_2)$.

Existence of $E_5(\tilde{x}_1, 0, \tilde{y})$

Here $\tilde{x}_1, \tilde{y}$ are the positive solutions of

$$r(1-\frac{x_1}{K_1})-\omega_1 y=0 \quad 0$$

(11)

$$b_1\omega_1 x_1 - k = 0 \quad (12)$$

Solving (11) and (12) we get

$$\tilde{x}_1 = \frac{k}{b_1\omega_1}, \tilde{y} = \frac{r}{\omega_1}(1-\frac{k}{K_1 b_1\omega_1}) \quad (13)$$

It can be seen that $E_5(\tilde{x}_1, 0, \tilde{y})$ exists if

$$Kb_1\omega_1 > k \quad (14)$$

Existence of $E_6(\hat{x}_1, \hat{x}_2, \hat{y})$

Here $E_6(\hat{x}_1, \hat{x}_2, \hat{y})$ is the positive solution of the system of algebraic equations given below:

$$rx_1(1-\frac{x_1}{K_1})-px_1x_2-\omega_1 x_1 y=0 \quad (15)$$

$$sx_2(1-\frac{x_2}{K_2})-qx_1 y-\frac{\omega_2 x_2 y}{A+Bx_2+Cy}=0 \quad (16)$$

$$b_1\omega_1 x_1 y + b_2\omega_2\frac{x_2 y}{A+Bx_2+Cy}-ky=0 \quad (17)$$

Solving (15) and (16) and eliminate x_1 we get

$$(qr-pqx_2-\omega_1 yq)K_1K_2(A+Bx_2+Cy)-sr(K_2-x_2)(A+Bx_2+Cy)+r\omega_2K_2y=0 \quad (18)$$

when $y \to 0$, then $x_2 \to x_{2a}$

where

$$x_{2a} = \frac{rK_2(s-qK_1)}{rs-pqK_1K_2}$$

We note that $x_{2a} > 0$, if the inequalities

$r > pK_2$ and $s > qK_1$ hold.

Also from the equation (18), we have

$$\frac{dx_2}{dy} = \frac{F_1}{F_2},$$

$F_1 = qKL(r-px_2)-\omega_1 qK_1K_2(A+Bx_2+2Cz)+\omega_2 rL$ and

$F_2 = qBK_1K_2(r-\omega_1 y)+(sr-pqK_1K_2)(A+2Bx_2+Cy)-srBK_2$

It is clear that $\frac{dy}{dz} > 0$, if either

$F_1 > 0$ and $F_2 > 0$, or

$F_1 < 0$ and $F_2 < 0$, hold.

Again solving (14) and (15) and eliminate again x_1 then we get

$$(rb_1\omega_1K_1 - a_1b_1\omega_1 yK_1 - \omega_1^2 b_1 yK_1 - kr)(A+Bx_2+Cy)+rb_1\omega_1x_2=0 \quad (19)$$

when $y \to 0$, then $x_2 \to x_{2b}$

where

$$x_{2b} = \frac{-G_2 + \sqrt{G_2^2 - 4G_1G_2}}{2G_1}$$

In which

$$G_1 = -pb_1\omega_1K_1B$$

$$G_2 = b_1\omega_1(rK_1B - pK_1A - r)$$

$$G_3 = rA(b_1\omega_1K_1 - k)$$

Clearly $G_1 < 0$ and $G_3 > 0$ if the inequalities (14) is satisfied.

We also get from the equation $\frac{dx_2}{dy} < -\frac{D_1}{D_2}$,

Where

$$D_1 = -pb_1\omega_1K_1 + rb_1\omega_1\frac{(A+Cy)}{(A+Bx_2+Cy)^2}$$

$$D_2 = -\omega_1^2b_1K_1 + rb_1\omega_1\frac{(A+Bx_2)}{(A+Bx_2+Cy)^2}$$

It is clear that $\frac{dy}{dz} < 0$, if either

$D_1 > 0$ and $D_2 > 0$, or

$D_1 < 0$ and $D_2 < 0$, hold.

From above conditions we note that $x_{2a} < x_{2b}$ holds. Knowing the value of $\hat{x}_2, \hat{y}$, the value of $\hat{x}_1$ can be calculated from

$$\hat{x}_1 = \frac{k(A+C\hat{y}) - \hat{x}_2(b_2\omega_2 - kB)}{(A+B\hat{x}_2+C\hat{y})b_1\omega_1}$$

We can see that $E_6(\hat{x}, \hat{y}, \hat{z})$ exists if x^* to be positive,

if $k(A+C\hat{y}) > \hat{x}_2(b_2\omega_2 - kB)$ condition is hold.

III. STABILITY ANALYSIS

Now we check the stability of model (2). For that matrix is

$$J = \begin{pmatrix} r - \frac{2x_1r}{K_1} - px_2 - \omega_1 y & -px_1 & -\omega_1x_1 \\ -qx_2 & s - \frac{2sx_2}{K_2} - qx - \frac{\omega_2 y(A+Cy)}{(A+Bx_2+Cy)^2} & -\frac{\omega_2x_2(A+Bx_2)}{(A+Bx_2+Cy)^2} \\ b_1\omega_1 y & -\frac{b_2\omega_2 y(A+Cy)}{(A+Bx_2+Cy)^2} & b_1\omega_1x_1 - \frac{b_2\omega_2x_2(A+Bx_2)}{(A+Bx_2+Cy)^2} - k \end{pmatrix}$$

(a) The viariational matrix at equilibrium point $E_0(0,0,0)$

$$J_0 = \begin{pmatrix} r-\lambda & 0 & 0 \\ 0 & s-\lambda & 0 \\ 0 & 0 & -c-\lambda \end{pmatrix}$$

Thus E_0 is a saddle point which is stable in y direction and unstable manifold in the x_1-x_2 plane.

(b) The viariational matrix at equilibrium point $E_1(K_1,0,0)$

$$J_1=\begin{pmatrix} -r-\lambda & -pK_1 & -\omega_1K_1 \\ 0 & s-qK_1-\lambda & 0 \\ 0 & 0 & b_1\omega_1K_1-k-\lambda \end{pmatrix}$$

E_1 is a saddle point with locally stable manifold in x_1 direction and with locally unstable manifold in x_2-y plane, if $s-qK_1>0$ and $b_1\omega_1K_1-k>0$ hold, but if $s-qK_1<0$ and $b_1\omega_1K_1-k<0$, then E_1 is locally asymptotically stable in x_1-x_2- y plane.

(c) The viariational matrix at equilibrium point $E_2(0,K_2,0)$

$$E_2=\begin{pmatrix} r-pK_2-\lambda & 0 & 0 \\ -qK_2 & -s-\lambda & -\dfrac{\omega_2K_2(A+BK2)}{(A+BK_2)^2} \\ 0 & 0 & -\dfrac{b_2\omega_2K_2(A+BK_2)}{(A+BK_2)^2}-k-\lambda \end{pmatrix}$$

E_2 is a saddle point with locally stable manifold in x_2 direction and with locally unstable manifold in x_1-y plane if $r-pK_2>0$ holds, but if $r-pK_2<0$, then E_2 is locally asymptotically stable in x_1–x_2-y plane.

(d) The viariational matrix at equilibrium point $E_3(x_1^*,x_2^*,0)$

$$J_3=\begin{pmatrix} r-\dfrac{2x_1^*r}{K_1}-px_2^*-\lambda & px_1^* & \omega_1x_1^* \\ -qx_2^* & s-\dfrac{2sx_2^*}{K_2}-qx_1^*-\lambda & 0 \\ 0 & 0 & b_1\omega_1x_1^*-\dfrac{b_2\omega_2x_2^*(A+Bx_2^*)}{(A+Bx_2^*)^2}-k-\lambda \end{pmatrix}$$

Put $x_1^*=\dfrac{sK_1(r-pK_2)}{rs-pqK_1K_2}, x_2^*=\dfrac{rK_2(s-qK_1)}{rs-pqK_1K_2}$, then j_3

$$J_3=\begin{pmatrix} \dfrac{sr(pK_2-r)}{rs-pqK_1K_2}-\lambda & -\dfrac{psK_1(r-pK_1)}{rs-pqK_1K_2} & -\dfrac{\omega_1sK_1(r-pK_1)}{rs-pqK_1K_2} \\ -\dfrac{qrK_2(s-qK_1)}{rs-pqK_1K_2} & \dfrac{rs(qK_1-s)}{rs-pqK_1K_2}-\lambda & 0 \\ 0 & 0 & \dfrac{b_1\omega_1sK(r-a_1L)}{rs-pqK_1K_2}-\dfrac{b_2\omega_2qrK_2(s-qK_1)}{A(rs-pqK_1K_2)+BrK_2(s-qK_1)}-k-\lambda \end{pmatrix}$$

Here sum of two eigen values are

$$\frac{sr(pK_2 - r)}{rs - pqK_1K_2} + \frac{rs(qK_1 - s)}{rs - pqK_1K_2}$$

and product of two eigen values are

$$\frac{sr(pK_2 - r)(qK_1 - s)}{rs - pqK_1K_2}$$

If $r > pK_2$ and $s > qK_1$ holds, then the sum of two eigen values is negative and product is positive. So that we can say that $E_3(x_1^*, x_2^*, 0)$ exists and is asymptotically stable in $x_1 - x_2$ plane, but if $Kr < pK_2$ and $s < qK_1$ holds, then the product of two eigenvalues is negative. Then $E_3(x_1^*, x_2^*, 0)$ exists and in that case it will be unstable in $x_1 - x_2$ plane. Moreover, it will be stable in x_1- x_2-y plane if for the other eigen value of the system is

$$\frac{b_1\omega_1 sK(r - a_1L)}{rs - pqK_1K_2} < \frac{b_2\omega_2 qrK_2(s - qK_1)}{A(rs - pqK_1K_2) + BrK_2(s - qK_1)} + k\,.$$

(e) The variational matrix at equilibrium point $E_4(0, \bar{x}_2, \bar{y})$

$$J_4 = \begin{pmatrix} r - p\bar{x}_2 - \omega_1\bar{y} - \lambda & 0 & 0 \\ -q\bar{x}_2 & s - \dfrac{2s\bar{x}_2}{K_2} - \dfrac{\omega_2\bar{y}(A + C\bar{y})}{(A + B\bar{x}_2 + C\bar{y})^2} - \lambda & -\dfrac{\omega_2\bar{x}_2(A + B\bar{x}_2)}{(A + B\bar{x}_2 + C\bar{y})^2} \\ b_1\omega_1\bar{y} & -\dfrac{b_2\omega_2\bar{y}(A + C\bar{y})}{(A + B\bar{x}_2 + C\bar{y})^2} & -\dfrac{b_2\omega_2\bar{x}_2(A + B\bar{x}_2)}{(A + B\bar{x}_2 + C\bar{y})^2} - k - \lambda \end{pmatrix}$$

Where $\bar{x}_2 = \dfrac{kA + Ck\bar{y}}{b_2\omega_2 - kB}$ and $\bar{y} = \dfrac{s(A + B\bar{x}_2)(K_2 - B\bar{x}_2)}{\omega_2 K_2^2 - Cs(K_2 - s\bar{x}_2)}$

$E_4(0, \bar{x}_2, \bar{y})$ exists and is asymptotically stable in x_2-y plane if the inequality $r - p\bar{x}_2 - \omega_1\bar{y} < 0$, and $\dfrac{2s\bar{x}_2}{K_2} + \dfrac{\omega_2\bar{y}(A + C\bar{y})}{(A + B\bar{x}_2 + C\bar{y})^2} < s$ holds , then it will be asymptotically stable in x-y-z plane.

(f) The variational matrix at equilibrium point $E_5(\tilde{x}_1, 0, \tilde{y})$

$$J_5 = \begin{pmatrix} -\dfrac{rk}{K_1b_1\omega_1} - \lambda & -\dfrac{pk}{b_1\omega_1} & -\dfrac{k}{b_1} \\ 0 & s - \dfrac{qk}{b_1\omega_1} - \dfrac{r\omega_2}{\omega_1} / (\dfrac{AK_1b_1\omega_1}{K_1b_1\omega_1 - k} + \dfrac{Cr}{\omega_1}) - \lambda & 0 \\ rb_1(1 - \dfrac{k}{K_1b_1\omega_1}) & \dfrac{b_2\omega_2 r}{\omega_1} / (\dfrac{AK_1b_1\omega_1}{K_1b_1\omega_1 - k} + \dfrac{Cr}{\omega_1}) & -\lambda \end{pmatrix}$$

$E_5(\tilde{x}, 0, \tilde{z})$ exists and is asymptotically stable in x_1-x_2-y plane, if the inequality

$$\frac{qk}{b_1\omega_1} - \frac{r\omega_2}{\omega_1} / (\frac{AK_1b_1\omega_1}{K_1b_1\omega_1 - k} + \frac{Cr}{\omega_1}) > s \text{ holds}.$$

(g) The viariational matrix at equilibrium point $E_6(\hat{x}_1,\hat{x}_2,\hat{y})$

$$J_6=\begin{pmatrix} r-\frac{2\hat{x}_1 r}{K_1}-p\hat{x}_2-\omega_1\hat{y}-\lambda & -p\hat{x}_1 & -\omega_1\hat{x}_1 \\ -q\hat{x}_2 & s-\frac{2s\hat{x}_2}{K_2}-q\hat{x}_1-\frac{\omega_2\hat{y}(A+C\hat{y})}{(A+B\hat{x}_2+C\hat{y})^2}-\lambda & -\frac{\omega_2 x_2(A+B\hat{x}_2)}{(A+B\hat{x}_2+C\hat{y})^2} \\ b_1\omega_1\hat{y} & -\frac{b_2\omega_2\hat{y}(A+C\hat{y})}{(A+B\hat{x}_2+C\hat{y})^2} & b_1\omega_1\hat{x}_1-\frac{b_2\omega_2\hat{x}_2(A+B\hat{x}_2)}{(A+B\hat{x}_2+C\hat{y})^2}-k-\lambda \end{pmatrix}$$

The stability of the point $E_6(\hat{x}_1,\hat{x}_2,\hat{y})$ depends on the determinant and trace of the above Jacobean J_6. The point is stable if the det $J_6>0$ and Trace $J_6<0$.

In the following theorem, we show that all equilibrium point is globally asymptotically stable.

Theorem 1- The interior equilibrium E_3 is globally asymptotically stable in the interior of the quadrant of the x_1-x_2 plane.

Proof -Let $\Delta(x_1,x_2)=\frac{1}{x_1x_2}$ .Clearly $\Delta(x_1,x_2)$ is positive in the interior of the positive quadrant of the x_1,x_2-plane.

$$h_1(x_1,x_2)=rx_1(1-\frac{x_1}{K_1})-px_1x_2$$

$$h_2(x_1,x_2)=sx_2\,(1-\frac{x_2}{K_2})-qx_1x_2.$$

Then

$$\Delta(x_1,x_2)=\frac{\delta}{\delta x_1}(h_1H)+\frac{\delta}{\delta x_2}(h_2H)$$

$$\Delta(x_1,x_2)=-\frac{r}{x_2K_1}-\frac{s}{x_1K_2}<0.$$

From the above equation, we note that $\Delta(x_1,x_2)$ does not change sign and is not identically zero in the interior of the positive quadrant of the x_1,x_2 plane. In the following theorem, we show that E_3 is globally asymptotically stable.

Theorem 2- The interior equilibrium E_4 is globally asymptotically stable in the interior of the quadrant of the x_2,y plane.

Proof -Let $H'(x_2,y)=\frac{1}{x_2y}$ .Clearly $H'(x_2,y)$ is positive in the interior of the positive quadrant of the x_2,y plane.

$$h'_1(x_2,y)=sx_2(1-\frac{x_2}{K_2})-\frac{\omega_2x_2y}{A+Bx_2+Cy},$$

$$h'_2(x_2,y)=\frac{b_2\omega_2x_2y}{A+Bx_2+Cy}-ky.$$

Then

$$\Delta(x_2,y)=\frac{\delta}{\delta x_2}(h'_1H')+\frac{\delta}{\delta y}(h'_2H')$$

$$\Delta(x_2,y)=\frac{\omega_2B}{(A+Bx_2+Cy)^2}-\frac{s}{yK_2}-\frac{b_2\omega_2C}{(A+Bx_2+Cy)^2}<0.$$

when $(A+Bx_2+Cy)^2-\frac{\omega_2ByK_2}{s}+\frac{b_2\omega_2yK_2C}{s}>0$

From the above equation, we note that $\Delta(x_2, y)$ does not change sign and is not identically zero in the interior of the positive quadrant of the x_2y - plane. In the following theorem, we show that E_4 is globally asymptotically stable.

Theorem 3- The interior equilibrium E_5 is globally asymptotically stable in the interior of the quadrant of the x_1y plane.

Proof - Let $H''(x_1y) = \dfrac{1}{x_1y}$ .Clearly H(x_1y) is positive in the interior of the positive quadrant of the x_1y plane.

$$h''_1(x_1y) = rx_1(1-\frac{x_1}{K_1}) - \omega_1 x_1 y,$$

$$h''_2(x_1, y) = b_1\omega_1 x_1 y - ky.$$

Then

$$\Delta(x_1, y) = \frac{\delta}{\delta x_1}(h''_1 H'') + \frac{\delta}{\delta y}(h''_2 H'')$$

$$\Delta(x_1, y) = -\frac{r}{yK_1} < 0.$$

From the above equation, we note that $\Delta(x_1, y)$ does not change sign and is not identically zero in the interior of the positive quadrant of the x_1, y plane. In the following theorem, we show that E_5 is globally asymptotically stable.

Theorem 4-The interior equilibrium E_6 is globally asymptotically stable with respect to (x_1, x_2, y) plane.

Proof-Consider the following positive definite function about E_6,

$$W(t) = \left(x_1 - x_1^* - x_1^* \ln\frac{x_1}{x_1^*}\right) + d_1\left(x_2 - x_2^* - x_2^* \ln\frac{x_2}{x_2^*}\right) + d_2\left(y - y^* - y^* \ln\frac{y}{y^*}\right)$$

Differentiating W with respect to time t along the solutions of model (1), we get

$$\frac{dW}{dt} = -\frac{r}{K_1}(x_1 - x_1^*)^2 - \frac{d_1 s}{K_2}(x_2 - x_2^*)^2 - \omega_1(y - y^*)$$
$$-\omega_2 d_1[\frac{A(y-y^*) + B(x_2^* y - x_2 y^*)}{(A+Bx_2+Cy)(A+Bx^*_2+C^*y)}] + \omega_2 d_2 b_2[\frac{A(x_2 - x_2^*) - C(x_2^* y - x_2 y^*)}{(A+Bx_2+Cy)(A+Bx^*_2+C^*y)}]$$

We choose $d_2 = \dfrac{d_1(y-y^*)}{b_2(x_2 - x^*_2)}$

So that,

$$\frac{dW}{dt} = -\frac{r}{K_1}(x_1 - x_1^*)^2 - \frac{d_1 s}{K_2}(x_2 - x_2^*)^2 - \omega_1(y - y^*)$$
$$-[\frac{\omega_2(x_2^* y - x_2 y^*)(B(x_2 - x_2^*) + C(y-y^*))}{(A+Bx_2+Cy)(A+Bx^*_2+C^*y)(y-y^*)}]$$

Hence W is a Lipunov function with respect to $E_6(x_1, x_2, y)$.

IV. CONCLUSION

In this paper, a mathematical model has been discussed with the transmission function, we have analyzed a prey-predator fishery model change the transmission function in a two patch environment, one is assumed to a free fishing zone and the other is a reserved zone where fishing and other extractive activities are prohibited. The population and the resource both the growing logistically. The existence of equilibrium point has been discussed and local stability and global stability analysis has been carried out by Variational Matrix and Liapunov function method. It has been observed that, whether in the absence or in the presence of predators, the fishing populations may be sustained at an appropriate equilibrium level.

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