The Efficiency Level in the Estimation of the Nigerian Population: A Comparison of One-Stage and Two-Stage Sampling Technique (A Case Study of the 2006 Census of Nigerians)

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ABSTRACT: This research work compares the one-stage sampling technique (Simple Random Sampling) and two-stage sampling technique for estimating the population total of Nigerians using the 2006 census result of Nigerians. A sample size of twenty (20) states was selected out of a population of thirty six (36) states at the Primary Sampling Unit (PSU) and one-third of each state selected at the PSU was sample at the Secondary Sampling Unit (SSU) and analyzed. The result shows that, with the same sample size at the PSU, one-stage sampling technique (Simple Random Sampling) is more efficient than two-stage sampling technique and hence, recommended.

KEYWORDS: Population, Simple Random Sampling, Cluster with Sampling, Mean Square Error.

I. INTRODUCTION

Our knowledge, our attitudes, and our actions are based to a very large extent on samples. This is equally true in everyday life and in scientific research. This work contains an account of the body of theory that has been built up to provide a background for good sampling methods. For most application for which this theory was constructed, the aggregate about which information is desired is finite and delimited. In some cases, it may seem feasible to obtain the information by taking a complete enumeration or census of the aggregate. The purpose of sampling survey theory is to make sampling more efficient. It attempts to develop methods of sample selection and of estimation that provide, at the lowest possible cost, estimates that are precise enough for our purpose. This principle of specified precision at minimum cost recurs repeatedly in the presentation of theory. In order to apply this principle, we must be able to predict, for any sampling procedure that is under consideration, the precision to be expected. So far as precision is concerned, we cannot foretell exactly how large an error will be present in an estimate of any specific situation, for this would require knowledge of the true value for the population. Instead, the precision of a sampling procedure is judged by examining the frequency distribution generated for the estimate if the procedure is applied again and again to the same population. This is, of course, the standard technique by which precision is judged in statistical theory.

The aim and objective of this research work is to determine the most efficient sampling technique in the comparison between one-stage and two-stage sampling technique with the same sample size and to estimate the population total of the Nigerian.

Definition of Terms used

- BIAS: In general terms, it is the deviation of results or inferences from the truth, or process leading to such deviation. In estimation, it is usually measured by the difference between a parameter estimate \( \theta \) and its expected value.
- EFFICIENCY: it is a term applied in the context of comparing different methods of estimating the same parameter; the estimate with the lower variance being regarded as the most efficient.
- ESTIMATION: the process of providing a numerical value for a population parameter on the basis of information collected from a sample. If a single figure is calculated for the unknown parameter, the process is called point estimation. If an interval is calculated which is likely to contain the parameter, then the procedure is called interval estimation.
- **ESTIMATOR**: A statistic used to provide an estimate for a parameter. The sample mean for example, is an unbiased estimator of the population mean.

- **FRAME**: a list element of the population from which a sample could be drawn. Its properties are completeness, up-to-date, non-duplicate, adequacy and clear identifiability.

- **INFERENC**E: the process of drawing conclusion about a population on the basis of measurements or observations made on a sample of individuals from the population.

- **PRECISION**: it is the measure of how close an estimate is to the average value over all possible samples. An estimate is said to be precise if it has a smaller variance.

- **SAMPLE**: it is the subset of a population obtained with the objective of investigating the population.

- **SAMPLING ERROR**: the difference between the sample result and the population characteristic being estimated. In practice, the sampling error can rarely be determined because the population characteristic is not usually unknown. With appropriate sampling procedures, however, it can be kept small and the investigator can determine its probable limits of magnitude.

**Symbolic Notification**

In course of data analysis, the following statistical notations were used:

- $N$: the total number of states in Nigeria.
- $n$: the number of state selected.
- $M_i$: the total number of elements (LGAs) in state $i$.
- $m_i$: the number of element (LGAs) selected in an SRS from state $i$.
- $M = \sum_i M_i$: the total number of element (LGAs) in Nigeria.
- $X_{ij}$: the $j$th element (LGA) total in the sample from the $i$th state.
- $\bar{x}_i$: the sample mean for the $i$th state.
- $f_1$: The population correction factor (p.c.f.).
- $f_{2,i}$: The sample correction factor or the second stage correction factor.
- $S_p^2$: The variance between the sample states or variance in the first stage.
- $S_i^2$: The variance within each unit (state) sampled.

**Sampling Survey**: Sampling theory deals with the study of the relationship between population and samples drawn from the population. It is important in most application. For instant, it is used in the estimation of parameters from knowledge of the corresponding sample statistics. Also, survey sampling, or population sampling, deals with methods for selecting and observing a part (sample) of the population in order to make inferences about the whole population. Survey sample theory deals with the process of sample data collection of the population characteristics using the sample data collection and determine the accuracy of the estimates.

**Survey Design and Sample Design**: Designing a survey is the most important stage of a survey since design deficiencies cannot always be compensated for when editing and analyzing the data. Kish, (1965) state that sample design has two aspects: a selection process, the rules and operations by which some members of the population are included in the sample; and an estimate of the population values.

For a sample survey to be conducted, its objectives must be clearly spelt out along with the manner in which the results are going to be used. It is visualize as comprehensively as possible the nature of the statistical data required to satisfy the current and the future needs of the users. Raj, (1972) stated that the surveyor (a person or an establishment in charge of collecting or recording data) or researcher initial task is to formulate a rational justification for the use of sampling in his research. If sampling is found appropriate for a research, the researcher then;

1. Identifies the target population as precisely as possible, and in a way that make sense in terms of the purpose of the study.
2. Put together the list of the target population from which the sampling will be selected. This list is termed as a frame (more appropriately list frame) by many statisticians.
3. Selects the samples and decide on a sampling technique and
4. Makes an inference about the population.

All these steps are inter-related and cannot be considered as isolated from one another.

**PURPOSE OF SAMPLE SURVEY**: There are a wide variety of reasons why sampling is important. In most situations, a study of an entire population is impossible; hence sampling may represent the only possible or practice able method to obtain the desired information.
Mahalanobis,(1965) summarize the advantages of sampling surveys: “large scale sample surveys, when conducted in the proper way within a satisfactory survey design, can supply with great speed and low cost information of sufficient accuracy for practical purposes and with the possibility of ascertainment of the satisfactory survey design”. Cochran, (1977); Ali, (2010); Okafor,(2002) and many other scholars summarize the purpose of sample survey in terms of its advantages compared to complete enumeration (census) as:

1. **It reduces cost**: if data are secured from only a small fraction of the aggregate, expenditures are smaller than if a complete census is attempted. With large populations, result accurate enough to be useful can be obtained from the samples that represent only a small fraction of the population.

2. **Greater Speed**: Data can be collected and summarized more quickly with a sample than with a complete count. This is a vital consideration when the information is urgently needed.

3. **It has greater scope**: in certain types of inquiry, highly trained personnel or specialized equipment, limited in availability, must be used to obtain the data. A complete census is impracticable; the choice lies between obtaining the information by sampling or not at all. Thus, surveys that rely on sampling have more scope and flexibility regarding the types of information that can be obtained.

4. **It has greater accuracy**: because personnel of higher quality can be employed and given intensive training, and because more careful supervision of the field work and processing of results becomes feasible when the volume of work is reduced, a sample may produce more accurate results than the kind of complete enumeration that can be taken.

5. **Sometimes, it is the only alternative as in destructive sampling of a finite population e.g. in the test of the life-span of light bulb, it is highly advantageous to test few of the bulb light than testing the complete population of the bulb life.

**Types of Sampling:** Basically, we have two types of sampling namely; Non-probability sampling and Probability sampling techniques. Non-probability sampling is the type of sampling in which the probability of choosing a sample unit of the population is not known and may not be easily determined. Therefore, measurement of sample error or bias becomes almost impossible. Examples of Non-probability sampling are:

1. Purposive sampling.
2. Haphazard sampling.
3. Quota sampling.
5. Convenience sampling.
6. Judgmental sampling.

Probability sampling (Random sampling):- under this, each element, unit or member of the population has a known probability of being included in the sample. Here, sampling bias is eliminated as no particular unit favored. It is then possible to assess the risk of erroneous conclusion and incorrect decision by the use of probability theory. Examples of probability sample are:

1. Simple random sampling.
2. Multi-stage sampling.
3. Cluster sampling.
4. Systematic sampling.
5. Stratified sampling.

Salant et al.,(1994) stated that complex sampling techniques are used only in the process of large experimental data sets, when efficiency is required, and, while making precise estimates about relatively small groups within large population.

**One-stage Sampling Technique (Simple Random Sampling)** : One-stage sampling technique is a technique that involves sampling in only one stage. Examples one one-stage sampling technique may be Simple Random Sampling, Cluster sampling, stratified etc.In this research work, the simple random sampling technique was adopted as the one-stage sampling technique. Simple random sampling is a probability selection scheme where each unit in the population is given equal probability of selection, and thus every possible sample of a given size has the same probability of being selected. Simple random sampling can involve the units being selected either with or without replacement. With replacement sampling allows the units to be selected multiple times whilst without replacement, only allows a unit to be selected once. Without replacement, sampling is by far the more commonly used method and hence, it is been adopted in this work. The advantage of simple random sampling lies in its simplicity and ease of use, especially when only a small sample is taken.
Simple random sampling does, however, require a complete list of all population units as each unit needs to have a unique number associated with it to enable random selection. This sampling scheme also becomes unwieldy for large sample sizes and can be expensive if the sample is spread over a wide geographic area. In practice, simple random sampling is rarely used because there is almost always a more efficient method of designing the sample (in terms of producing accurate results for a given cost). Nevertheless, simple random sampling forms the basis of a number of the more complex methods of sample design, and is used as a benchmark to which other designs are compared.

**Two-stage Sampling**: Two-stage sampling is a type of multi-stage sampling which involves sampling in two stages namely first stage sampling (primary sampling unit) and the second stage sampling (second sampling unit). It involves two stage clustering with sampling be made at each stage. Cochran, (1977), Raj, (1972) and Neyman, (1938), stated that the procedure of first selecting clusters and then choosing specified number of elements from each selected cluster is known as two-stage sampling or double sampling or sub sampling. Mahalanobie, (1944) called this design two-stage sampling because the sampling is taken in two stages. The first is to select a sample of units, often called the primary units and the second stage is called the secondary sampling units (SSU). Kish, (1967), Hansen et al, (1975); Fink, (2002); and Tate et al, (2007) stated that the criteria for selecting a unit at a given stage typically depend on attributes observed in the previous stage.

Cochran, (1977); Kalton, (1983); and Okafor, (2002) stated that in designing a study, it can be advantageous to sample units in more than one stage.

The above view show that two-stage sampling is highly advantageous when compared with some single stage sampling like simple random sampling, systematic sampling and cluster sampling. Alfredo et al, (2006) pointed out some of these advantages of two-stage sampling. Some of these advantages are:

- It provides good coverage.
- It is simple to implement.
- It allows for control of field-work quality.

Adams et al, (2003) opines that if it cost little to determine the attribute that are necessary to classify the units, it can be cost efficient to sample in stage one and then in stage two to subsample the cluster at different stages.

**Efficiency of an Estimator**: Efficiency is a term applied in the context of comparing different method of estimating the same parameter; the estimate with the lower variance being regarded as the most efficient. It is also used when comparing competing experimental designs; with one design being more efficient than another if it can achieve the same precision with fewer resources. If we have more than one consistency estimators of a parameter Y, then efficiency is the criterion which enables us to choose between them by considering the variances of the sampling distribution of the estimators. Thus, if \( \hat{Y}_1, \hat{Y}_2, \ldots, \hat{Y}_n \) are consistency estimator of parameter Y such that \( \text{Var}(\hat{Y}_1) < \text{Var}(\hat{Y}_2) < \text{Var}(\hat{Y}_3) < \ldots < \text{Var}(\hat{Y}_n) \); for all n, then \( \hat{Y}_1 \) is said to be more efficient than \( \hat{Y}_2, \hat{Y}_3, \ldots, \hat{Y}_n \). In other words, an estimator with lesser variability is said to be more efficient and consequently more reliable than the others.

Gupta, (2004) write that, if there exist more than two consistent estimators for a parameter Y, then, considering the class of all such possible estimators, we can choose the one whose sampling variance is minimum. Such an estimator is known as the most efficient estimator and provides a measure of the efficiency of the other estimators.

**Comparison of One-stage Sampling Technique (Simple Random Sampling) with Two-stage Sampling Technique in Terms of their efficiency**: Cochran, (1977); Kalton, (1983); and Okafor, (2002) stated that in designing a study, it can be advantageous to sample units in more than one stage. From the above, the advantage explained by those scholars is majorly in terms of the efficiency of the stages of a multi-stage sampling technique. From this, we can infer that as the stage increases, the precision of that estimate also increases. Typically, a multi-stage sample gives less precise estimates than a simple random sample of the same size. However, this view was opposed by Nafiu, (2012) who said that multi-stage sample is often more precise than a simple random sample of the same cost. From the above argument, it is seen that under the same sample size, one-stage sample (simple random sampling) is more precise than two-stage sampling technique. But in opposition, if the cost levels are the same, two-stage sampling technique will be more efficient than the sampling random sampling. It is for this reason that this method is been investigated in this work.
**Population Census**: Census is the total process of collecting, compiling, analyzing, evaluating, publishing and disseminating demographic, economic, social and housing data pertaining at a specified time to all persons and all buildings in a country or in a well delineated part. Censuses are the most elaborate and detail and are supposed to be conducted every 10 years to provide up-to-date data in population characteristics and dynamics. UNECE, (2006) defined population census as “the operation that produces at regular intervals the official counting (or benchmark) of the population in the territory of a country and in its smallest geographical sub-territories together with information on a selected number of demographic and social characteristics of the total population”. The essential features of a census, as specified by the commission (UNECE), include Universality, Simultaneity of information, and individual enumeration.

Yates, (1960) pointed out the special advantages of complete census. These advantages are:-
(1). Data for small units can be obtained.
(2). Public acceptance is easier to secure for complete data.
(3). Public compliance and response may be better secured.
(4). Bias of coverage may be easier to check and reduce.
(5). Sampling Statisticians are not required.

A population census is exposed to different types of errors, including Coverage, Content, and Operational errors.

### II. METHODOLOGY AND DATA PRESENTATION

The area of study of this research work is the population of Nigerian. The study covers the population of peoples in the 774 Local Government Areas (LGAs) across the 36 States in Nigeria.

**Sample Selection procedure**: The country, Nigeria consists of \(N=36\) number of states out of which a simple random sample of \(n=20\) number of state is selected using “table of random number”. Each state consists of a total of \(M_i\) number of Local Government Area (LGA) out of which a sample size of \(m_i\) number of LGA is selected from each sampled state at the PSUs using the Loitering method. \(m_i\) is one-third of each sampled state at the PSUs.

**Population Total**
- For one-stage,
  \[ \hat{X} = N \bar{x} , \]
  where the sample mean \( \bar{x} = \frac{\sum x_i}{n} \)
- For two-stage sampling technique, \( \hat{x}_i = M_i \bar{x}_i \)
  The total population in the ith state, \( \hat{X}_i = \sum_{i=1}^{n} \hat{x}_i \)
  The estimated population total is
  \[ \hat{X} = \sum_{i=1}^{n} \hat{X}_i = \sum_{i=1}^{n} M_i \hat{x}_i \]

**Variance of the Population Total**
- For one-stage sampling technique,
  \[ \text{Var}(\hat{X}) = (1-f) \frac{S^2}{n} \]
  Therefore, the population variance, \( \text{Var}(\hat{X}) = N^2 \text{Var}(\hat{X}) \).
- For the two-stage sampling technique,
  \[ \text{Var}(\hat{X}) = N(1-f)^2 S^2 + \sum_{i=1}^{n} M_i \left( \frac{(1-f) \hat{X}}{f^2 \hat{X}_i} \right) S_i^2 \]

**Standard Error of the Population Total**
\[ S.E.(\hat{X}) = \sqrt{\text{Var}(\hat{X})} \]

**Coefficient of Variation of the Population Total**
\[ C.V.(\hat{X}) = \frac{S.E.(\hat{X})}{\hat{X}} \times 100\% \]

**Confidence Interval of the Population Total**
\[ \hat{X} \pm Z_{\alpha/2} S.E.(\hat{X}) \]
Bias of the Population Estimate

\[ \text{Bias (}\hat{Y}\text{)} = |\hat{Y} - \mu| \]

Mean square Error of the Population Estimate

\[ \text{M.S.E.}(\hat{Y}) = \hat{V}(\hat{Y}) + (\text{Bias})^2 \]

III. DATA ANALYSIS

3.1 One stage technique (Simple Random Sampling)

\[ \sum X_i = 85,810,417 \quad \text{and} \quad \sum X_i^2 = 4.412541867 \times 10^{14} \]

\[ \text{Sampling fraction, } f = \frac{n}{N} = \frac{20}{36} = 0.556 \]

\[ \text{Sample mean, } \hat{X} = \frac{\sum X_i}{n} = \frac{85,810,417}{20} = 4,290,520.85 \]

\[ \text{Variance, } V(\hat{X}) = (1 - f) \frac{S^2}{n} \]

\[ \text{where } S^2 = \frac{1}{n-1} \left[ \sum X_i^2 - \left( \sum X \right)^2 \right] \]

\[ = 3.846463321 \times 10^{12} \]

\[ V(\hat{X}) = 8.546841497 \times 10^{10} \]

\[ \text{Population total, } \hat{X} = N\hat{X} = 36 \times 4,290,520.85 \]

\[ = 154,458,750 \]

\[ \text{Population variance} = N^2V(\hat{X}) = 36^2 \times 8.546841497 \times 10^{10} \]

\[ = 1.10767065 \times 10^{14} \]

\[ \text{Standard error of the population estimate} \]

\[ \text{S.E.}(\hat{X}) = \sqrt{V(\hat{X})} = \sqrt{1.10767065 \times 10^{14}} \]

\[ = 10,524,593.37 \]

\[ \text{Coefficient of variation of the population estimate} \]

\[ C.V.(\hat{X}) = \frac{\text{S.E.}(\hat{X})}{\hat{X}} \times 100\% \]

\[ = \frac{10,524,593.37}{154,458,750} \times 100\% \]

\[ = 6.81\% \]

\[ \text{Confidence interval of the population estimate} \]

\[ C.I. (\hat{X}) = \hat{X} \pm Z_{\frac{\alpha}{2}} \text{S.E.}(\hat{X}) \]

\[ = 154,458,750 \pm 1.96 \times (10,524,593.37) \]

\[ = 154,458,750 \pm 20,628,203.08 \]

\[ = (133,830,546.7; 175,086,953.08) \]
Bias of the population estimate

\[ \text{Bias} \left( \hat{X} \right) = \left| \left( \hat{X} - \mu \right) \right| \]
\[ = \left| (154,458,750 - 140,003,542) \right| \]
\[ = 14,455,208 \]

Mean square error of the population estimate

\[ \text{MSE} \left( \hat{X} \right) = V \left( \hat{X} \right) + \left( \text{Bias} \right)^2 \]
\[ = (1.10767065 \times 10^{14}) + (14,455,208) \]
\[ = 3.197201031 \times 10^{14} \]

3.2 Two stage Technique

Sample mean, \( \bar{X} = \frac{\sum \hat{X}_i}{n} \)
\[ = \frac{84,802,984.34}{20} \]
\[ = 4,240,149.217 \]

Population total, \( \hat{X} = N\bar{X} = 36 \times 4,240,149.217 \)
\[ = 152,645,371.8 \]

Variance of the population total

\[ \text{variance,} \hat{V} \left( \hat{X} \right) = N \left( \frac{1 - f_i}{f_i} \right) S_i^2 + \sum M_i \left( \frac{1 - f_{z_i}}{f_i f_{z_i}} \right) S_i^2 \]

but \( N \left( \frac{1 - f_i}{f_i} \right) S_i^2 = 36 \left( \frac{1 - 0.556}{0.556} \right) \times 4.33172783 \times 10^{12} \)
\[ = 1.247381673 \times 10^{14} \]

and \( \sum M_i \left( \frac{1 - f_{z_i}}{f_i f_{z_i}} \right) S_i^2 = 1.1041288 \times 10^{13} \)

Therefore, \( \hat{V} \left( \hat{X} \right) = (1.247381673 \times 10^{14}) + (1.1041288 \times 10^{13}) \)
\[ = 1.357794553 \times 10^{14} \]

Standard error of the population total

\[ \text{S.E.} \left( \hat{X} \right) = \sqrt{\hat{V} \left( \hat{X} \right)} = \sqrt{(1.357794553 \times 10^{14})} \]
\[ = 11,652,444.17 \]
Coefficient of variation of the population total

\[ C.V.(\hat{X}) = \frac{S.E.(\hat{X})}{\hat{X}} \times 100\% \]

\[ = \frac{11,652,444.17}{152,645,371.8} \times 100\% \]

\[ = 7.63\% \]

Confident interval of the population total

\[ C.I.(\hat{X}) = \hat{X} \pm Z_{\alpha/2}S.E.(\hat{X}) \]

\[ = 152,645,371.8 \pm (1.96)(11,652,444.17) \]

\[ = 152,645,371.8 \pm 22,838,790.57 \]

\[ = (129,806,518.2;175,484,162.3) \]

Bias of the population estimate

\[ Bias(\hat{X}) = \left| \hat{X} - \mu \right| \]

\[ = \left| (152,645,371.8 - 140,003,542) \right| \]

\[ = 12,641,830 \]

Mean square error population estimate

\[ MSE(\hat{X}) = \hat{V}(\hat{X}) + (Bias)^2 \]

\[ = \left( 1.357794553 \times 10^{14} \right) + (12,641,830)^2 \]

\[ = 2.95595321 \times 10^{14} \]

Summary of the results

<table>
<thead>
<tr>
<th></th>
<th>Sampling Technique</th>
<th>One-stage</th>
<th>Two-stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Total</td>
<td>( \hat{X} )</td>
<td>154,458,750</td>
<td>152,645,372</td>
</tr>
<tr>
<td>Variance</td>
<td>( \hat{V}(\hat{X}) )</td>
<td>1.107670665 \times 10^{14}</td>
<td>1.357794553 \times 10^{14}</td>
</tr>
<tr>
<td>Standard error</td>
<td>S.E.(( \hat{X} ))</td>
<td>10,524,593</td>
<td>11,652,444</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>C.V.(( \hat{X} ))</td>
<td>6.81%</td>
<td>7.63%</td>
</tr>
<tr>
<td>Confident interval</td>
<td>C.I.(( \hat{X} ))</td>
<td>(133,380,547;175,086,953)</td>
<td>(129,806,581;175,484,162)</td>
</tr>
<tr>
<td>Bias</td>
<td>Bias(( \hat{X} ))</td>
<td>14,455,208</td>
<td>12,641,830</td>
</tr>
<tr>
<td>Mean square error</td>
<td>MSE(( \hat{X} ))</td>
<td>3.197201031 \times 10^{14}</td>
<td>2.95595321 \times 10^{14}</td>
</tr>
</tbody>
</table>

IV. DISCUSSION

From the above table, it was observed that the one-stage technique has the least for the variance, standard error and the coefficient of variation of the population except for bias and mean square error of the population total which two-stage has the least while confidence interval for both techniques shows that the population total fall within the computed interval as expected.
V. CONCLUSION

From the comparison of the results obtained so far, it shows that with the same sample size, one-stage sampling technique (simple random sampling) is more efficient than two-stage sampling technique and hence, recommended.

Recommendation: For efficient result in the estimation of population total, one-stage sampling technique (simple random sampling) gives a more precise result than two-stage sampling technique with the same sample size and hence, recommended.

REFERENCE


