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Random Supply, Costant Lead Times and Quadratic Backorder Costs. For Inventory Model (M, T)

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Abstract: - This paper considers the inventory costs for the (M.T) model in which the backorder costs is quadratic, supply is continuous and lead time is constant.
Use is made of series 1: Inventory Model (M.T) with quadratic costs and continuous lead times.
The inventory cost for fixed M (maximum re-order level) and constant lead times is averaged over the states of M. Supply is assumed to follow a gamma distribution. The inventory costs for the model, random supply, constants lead time and quadratic costs are thereby derived.

Keywords: *- Gamma distribution, normal distribution, maximum reorder level, quadratic backorder costs and constant lead times.*

I. INTRODUCTION

In inventory costs for fixed M and constant lead times obtained in series 1 is averaged over the states of M. M is assured to be a gamma variate. Demand during the lead time is normally distributed.
The inventory costs for fixed M (maximum re-order level) when the cost of a backorder was quadratic function of the length of time excluding the cost of stockouts dependent only on the number of stockouts equation 14 of series 1.

II. LITERATURE REVIEW

Zipkin (2006) threats both fixed and random lead times and examines both stationary and limiting distributions under different assumptions.
Bartsimas (1999) in his paper "probabilistic service level guarantee in make-to-stock, considered both linear and quadratic inventory costs and backorder costs".
Pritibhushan (2008) in his paper. 'A note in Bernoulli demand inventory model presents a simple-item, continuous monitoring inventory model with probabilistic demand for the item and probabilistic lead time of order replacement'.

Hadley and Within (1972) extensively developed the inventory model (M.T) for constant lead times and linear backorder costs.

Random Supply Constant lead time and quadratic backorder costs. Series 2

Since the supply is a random variable the maximum re-order cover M would vary. Similarly M follows a gamma distribution. T becomes the only control parameter. The probability density function of M, u(M)

$$u(\mathrm{M}) = \frac{\mathrm{esp}\,(\mu\mathrm{M})\mathrm{M}^{v-1}\mu^{v}}{\Gamma(v)} \qquad v > 0 \quad M > 0 \qquad (1)$$

$$C = \frac{Rc + S}{T} + hc\left(M - DL - \frac{DT}{2}\right) + \frac{b_1}{T}(G_5(M, T+L) - (G_5(M, L)) + \left(b_2 + \frac{hc}{T}\right)$$

$$(G_6(M, T+L) - (G_6(M, L) + \frac{b_3}{T}(G_7(M, T+L) - G_7(M, L))$$

Where $G_5(M,T) = \sqrt{\sigma^2 T}\left(g\left(M - \frac{DT}{\sqrt{\sigma^2 T}}\right) - (M - DT)F\left(\frac{M - DT}{\sqrt{\sigma^2 T}}\right)\right)$

Multiplying by U(M)

$$U(M)G_5(M,T)\frac{\sqrt{\sigma^2 T}\, esp - \mu M\, M^{v-1}\mu^v}{\lceil(v)}\, g\left(\frac{M - DT}{\sqrt{\sigma^2 T}}\right) - \frac{\sqrt{\sigma^2 T}\, esp^{-\mu M}\mu^v}{\lceil(v)}\, F\left(\frac{M - DT}{\sqrt{\sigma^2 T}}\right)$$

Let $G_{15}(T) = \int_U^\infty u(M)\, G_5(M,T)dM$ (2)

Substituting for U (M)G_5(M, T)integrating and simplifying we have

$$G_{15}(T) = \frac{\sqrt{\sigma^2 T}}{\lceil(v)}\mu^v esp\left(\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)\sum_{i=0}^{v-1}\binom{v-1-1}{i}(D - \mu\sigma^2 T)^{v-1-i}\left(\frac{\sigma^2 T}{2}\right)^i$$

$$+\frac{\mu^v}{\lceil(v)} esp\left(\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)\left[DT\sum_{z=1}^{v}\binom{v-1}{i}\sum_{i=0}^{\frac{v-z}{2}}\binom{v-z-i}{i}(DT - \mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i\right]$$

$$-\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{v-2}{2}-z}\left(\frac{v!}{\mu^2(v+1-z)!}\right)\binom{v+1-2+z}{i}\left[(DT - \mu\sigma^2 T)^{v+1-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i\right]$$

$$G_6(M,T) = \left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2} + \frac{\sigma^2 M}{2D^2} - TMV + \frac{M^2}{2D}\right)$$

$$F\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sqrt{\sigma^2 T}}{2}\left(T - \frac{\sigma^2}{D^2}\right) - \frac{M}{D}\, g\left(M - \frac{D}{\sqrt{\sigma^2 T}}\right) - \frac{\sigma^4}{4D^3}\, esp\left(\frac{2DM}{\sigma^2}\right)F\left(\frac{M+DT}{\sqrt{\sigma^2 T}}\right) \quad (3)$$

Multiplying by U (M)

$$U(M)G_6(M,T) = \left[\left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2}\right) + \left(\frac{\sigma^2}{2D^2} - T\right) + M^v + \frac{M^{v+1}}{2D}\right]\frac{(esp - \mu^v)\mu^v}{\lceil(v)}$$

$$\left(\frac{M - DT}{\sqrt{\sigma^2 T}}\right) + \frac{\sqrt{\sigma^2 T}}{2}\left(\left(T - \frac{\sigma^2}{2D}\right)M^{v-1} - \frac{M^v}{D}\right)\frac{esp - \mu M\, \mu^v}{\lceil(v)}\, g\left(\frac{M - DT}{\sqrt{\sigma^2 T}}\right)$$

$$-\frac{esp\,(-\mu M)\,\mu^v M^{v-1}}{4D^3\lceil(v)}\, esp\left(\frac{2DM}{\sigma^2}\right)F\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right) \quad (4)$$

Let $G_{16}(T) = \int_0^\infty u(M)\, G_5(M,T)dM$ (5)

Substituting for U(M) G_2 (M, T) integrating and simplifying we have

$$G_{16}(T) = \frac{\mu^v esp\,(\mu^2\sigma^2 T - D\mu T)}{\lceil(v)}\left[\left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2}\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-z}{2}}\frac{(v-1)!}{\mu^2(v-2)!}\binom{v-z-i}{2}\right)$$

$$\sum_{z=1}^{\frac{v+2}{2}}\frac{(v+1)!}{\mu^2(v+2-z)!}\sum_{z=0}^{\frac{v+2-z}{2}}\binom{v+2+z+i}{i}(DT - \mu\sigma^2 T)^{v+2-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i + \left(\frac{\sigma^2 T}{2}\right)^i$$

$$\sum_{z=1}^{v+1}\sum_{i=0}^{(v+1-z)/2}\frac{v!}{\mu^2(v+1-z)}\binom{v+1-z-i}{i}(DT - \mu\sigma^2 T)^{v+1-z-2i}\left(\frac{\sigma^2}{2D^2} - T\right)^i$$

$$+\frac{\sigma^2 T}{2}\frac{\mu^v esp\left(\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)}{\lceil(v)}\left(T - \frac{\sigma^2}{D^2}\right)\sum_{Z=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}(DT - \mu\sigma^2 T)^{v-1-2i}\left(\frac{\sigma^2 T}{2}\right)^{2i} - \frac{1}{D}$$

$$\sum_{i=0}^{\frac{v}{2}}\binom{v-2}{i}(DT - \mu\sigma^2 T)^{v-2i}\left(\frac{\sigma^2 T}{2}\right)^i - \frac{\mu^v esp\left(\frac{\mu^2\sigma^2 T}{2} - D\mu T\right)}{\lceil(v)\, 4D^3}$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\left(\mu - \frac{2D}{\sigma^2}\right)^2(v-2)}(DT - \mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i \quad (6)$$

Substituting T for L

$$\frac{G_7(M,D)}{D} = -\left(-\frac{M^3}{3D^3} - \frac{\sigma^2 M^2}{2D^4} + \frac{\sigma^4 M^2}{2D^5} + \frac{\sigma^6}{4D^6} - \frac{\sigma^2 T^2}{2D^2} + \frac{T^2 M}{D} - \frac{T^3}{3} - \frac{TM^2}{D^2}\right)F\left(\frac{M - DT}{\sqrt{\sigma^2 T}}\right)$$

$$+\frac{1}{\sqrt{\sigma^2 T}} g\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right)\left(-\frac{2}{3}\frac{\sigma^2 MT^2}{D^2}-\frac{\sigma^2 T^3}{3D}+\frac{\sigma^2 MT}{3D^3}+\frac{\sigma^4 MT}{2D^4}+\frac{\sigma^4 T^2}{6D^3}+\frac{8\sigma^6 T}{D^5}\right)$$
$$+\frac{\sigma^6}{4D^6} esp\left(\frac{2DM}{\sigma^2}\right) F\left(\frac{M+DT}{\sqrt{\sigma^2 T}}\right) \tag{7}$$

Multiplying by U(M) and simplifying

$$U(M)=\frac{G_7(M,T)}{D}=\frac{esp(-\mu M)\mu^v}{\lceil(v)}\left[\frac{M^{2+v}}{3D^3}+M^{1+v}\left(\frac{\sigma^2}{2D^4}-\frac{T}{D^2}\right)+M^v\left(\frac{\sigma^4}{2D^5}+\frac{T^2}{D}\right)\right.$$
$$+M^{v+1}\left(\frac{\sigma^6}{4D^6}-\frac{\sigma^2 T^2}{6D^3}+\frac{8\sigma^6 T}{D^5}\right)M^{v-1}+\frac{\sigma^6}{4D^6} esp\left(\frac{2DM}{\sigma^2}\right)\frac{esp(-\mu M)M^{v-1}\mu^v}{\lceil(v)} F\left(\frac{M+DT}{\sqrt{\sigma^2 T}}\right) \tag{8}$$

Let $G_{17}(T)=U(M)G_7(M,T)dM$

Then substituting for U(M) G_7 (M,L+T)

Integrating and simplifying

$$G_{17}(T)=\frac{\mu^v}{\lceil(v)} esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)\left[\frac{1}{3D^3}\sum_{z=1}^{v+3}\sum_{i=0}^{(v-z+3)/2}\frac{(v+z)!}{\mu^z(v+3-z)}\binom{v+3-z-i}{i}\right.$$
$$(DT-\mu\sigma^2 T)^{v+2-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i+\left(\frac{\sigma^4}{2D^5}+\frac{T^2}{D}\right)\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{v+1-2}{2}}\frac{v!}{\mu^z(v+1-z)}\binom{v+1-z-i}{i}$$
$$(DT-\mu\sigma^2 T)^{v+1-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i+\left(\frac{\sigma^6}{4D^6}+\frac{\sigma^2 T^2}{2D^2}+\frac{T^3}{3}\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\mu^z(v-z)}\binom{v-z-i}{i}$$
$$\left.(DT-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i\right]+\frac{\mu^v}{\lceil(v)} esp\ (\mu^2\sigma^2 T-D\mu T)\left(-\frac{2}{3}\frac{\sigma^2 T^2}{D^2}+\frac{\sigma^2 T}{3D^3}+\frac{\sigma^4 T}{2D^4}\right)$$
$$\sum_{i=0}^{v/2}\binom{v-1}{i}(DT-\mu\sigma^2 T)^{v-2i}\left(\frac{\sigma^2 T}{2}\right)^i+\left(\frac{\sigma^2 T^3}{3D}+\frac{\sigma^4 T^2}{6D^3}+\frac{8\sigma^6 T}{D^5}\right)\sum_{i=0}^{\frac{v-1}{2}}\binom{v-i-1}{i}$$
$$\left.(DT-\sigma^2 T)^{v-2i}\left(\frac{\sigma^2 T}{2}\right)^i\right]+\frac{\sigma^6}{4D^6} esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^z(v-z)!}\binom{v-z-i}{i} \tag{9}$$
$$\binom{v-z-i}{i}(DT-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^{2i}$$

Hence averaging the inventory costs for fixed M over the states of M over the states of M we obtain the inventory cost for variable M

$$C=\frac{Rc+S}{T}+\frac{hcv}{\mu}-hc\left(DL+\frac{DT}{2}\right)+\frac{b_1}{T}\left(G_{15}(T+L)-G_{15}(L)\right)+\frac{b_2+hc}{T}-G_{16}(L)+\frac{b_3}{T}\left(G_{17}(T+L)-G_{17}(L)\right)$$
$$\left(G_{16}(T+L)-G_{16}(L)\right) \tag{10}$$

III. IMPACT OF THE STUDY

The study will enable industries or organizations having thousands of items in their warehouses located in various locations of the world and items supplied by different manufacturers with accurately use realistic lead times in arriving at their inventory cost.

In many of such cases lead times are not constant.

Expressing lead time as continuous gives a more realistic estimate of inventory costs.

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