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Concept of Hydrodynamic Load Calculation on Fixed Jacket Offshore Structures — An Overview of Vertically Mounted Cylinder

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Abstract: **-** This paper focuses on the analysis of hydrodynamic loads on fixed offshore structures (vertical cylinder) that are operating in shallow water and are often subjected to huge wave loading. For the purpose of this study, linear (Airy) wave theory was adopted together with the application of (21) in the load computation. The loads for six different sea states were computed using spread sheet for the following values of time interval t = 0, T/4, T/2.

Keywords: **–** *Airy wave theory, Fixed Jacket Offshore Structure, Morrison equation, Vertically Mounted Cylinder*

I. INTRODUCTION

Hydrodynamic wave loading on fixed offshore structures has been an issue of concern to the offshore oil and gas industry. The analysis, design and construction of offshore structures are arguably one of the most demanding sets of tasks faced by the engineering profession. Over and above the usual conditions and situations met by land-based structures, offshore structures have the added complication of being placed in an ocean environment where hydrodynamic interaction effects and dynamic response become major considerations in their design. In general, wave and current can be found together in different forms in the ocean. The existence of waves and currents and their interaction play a significant role in most ocean dynamic processes and are important for ocean engineers.

In addition, the range of possible design solutions, such as: Tension Leg Platform (TLP) deep water designs; the more traditional jacket and jack-up oil rigs; and the large number of sized gravity-style offshore platforms themselves, pose their own peculiar demands in terms of hydrodynamic loading effects, foundation support conditions and character of the dynamic response of not only the structure itself but also of the riser systems for oil extraction adopted by them. Invariably, non-linearity in the description of hydrodynamic loading characteristics of the structure-fluid interaction and in the associated structural response can assume importance and need be addressed. Access to specialist modelling software is often required to be able to do so [1].

1.1 Basics of Offshore Engineering

A basic understanding of a number of key subject areas is essential to an engineer likely to be involved in the design of offshore structures, [2], [3], [4] and [5].
These subject areas, though not mutually exclusive, would include;

- Hydrodynamics
- Structural dynamics
- Advanced structural analysis techniques
- Statistics of extreme among others.

1.2 Hydrodynamics

Hydrodynamics is concerned with the study of water in motion. In the context of an offshore environment, the water of concern is the ocean. Its motion, (the kinematics of the water particles) stems from a number of sources including slowly varying currents from the effects of the tides and from local thermal influences and oscillatory motion from wave activity that is normally wind-generated [1].

The characteristics of currents and waves, themselves would be very much site dependent, with extreme values of principal interest to the LFRD approach used for offshore structure design, associated with the statistics of the climatic condition of the site interest [6].

The topology of the ocean bottom also has influence on the water particle kinematics as the water depth changes from deeper to shallower conditions, [7]. This influence is referred to as the "shoaling effect", which assumes significant importance to the field of coastal engineering. For so called deep water conditions (where the depth of water exceeds half the wavelength of the longest waves of interest), the influence of the water bottom topology on the water particle kinematics is considered negligible, removing an otherwise potential complication to the description of the hydrodynamics of offshore structures in such deep water environment.

II. METHODOLOGY AND MATERIALS

The jacket structure used for this study (Fig. 5) is a HD accommodation platform to be operated in shallow water and is similar to all fixed jacket offshore structures. The part of the structure under water was discretized in to (264) beam elements. The water depth for the HD field is approximately 25.3m. The loads were computed using spread sheet. See TABLE I for the most probable wave heights and time periods for different sea states.

3.1 Wave Theories

All wave theories obey some form of wave equation in which the dependent variable depends on physical phenomena and boundary conditions [8]. In general, the wave equation and the boundary conditions may be linear and non linear.

3.1.1 Airy Wave Theory

The surface elevation of an Airy wave amplitude ζ_a, at any instance of time *t* and horizontal position *x* in the direction of travel of the wave, is denoted by η(x,t) and is given by:

$$\eta(x,t) = \zeta_a \cos(kx - \omega t) \qquad \text{"equation 1"}$$

where wave number k = 2π/L in which *L* represents the wavelength (see fig. 1) and circular frequency ω = 2π/T in which *T* represents the period of the wave. The celerity, or speed, of the wave *C* is given by L/T or ω/k, and the crest to trough wave height, *H* is given by $2\zeta_a$. The along wave u(x, t) and vertical v(x, t) water particle velocities in an Airy wave at position *z* measured from the Mean Water Level (MWL) in depth water *h* are given by:

$$u(x,t) = \frac{\omega\zeta_a \cosh[k(z+h)]}{\sinh(kh)} \cos(kx - \omega t) \qquad \text{"equation 2"}$$

$$v(x,t) = \frac{\omega\zeta_a \text{sinsh}[k(z+h)]}{\sinh(kh)} \sin(kx - \omega t) \qquad \text{"equation 3"}$$

The dispersion relationship relates wave number *k* to circular frequency *ω* (as these are not independent), via:

$$\omega^2 = gk\tanh(kh) \qquad \text{"equation 4"}$$

where *g* is the acceleration due to gravity (9.8 m/s^2). The along wave acceleration $\dot{u}$(x, t) is given by the time derivative of (2) as:

$$\dot{u}(x,t) = \frac{\omega^2\zeta_a \cosh[k(z+h)]}{\sinh(kh)} \sin(kx - \omega t) \qquad \text{"equation5a"}$$

while the vertical velocity $\dot{v}$(x, t) is given by the time derivative of (3) as:

$$\dot{v}(x,t) = -\frac{\omega^2\zeta_a \sinh[k(z+h)]}{\sinh(kh)} \cos(kx - \omega t) \qquad \text{"equation 5b"}$$

It should be noted here that wave amplitude, $a = \zeta_a$, is considered small (in fact negligible) in comparison to water depth *h* in the derivation of Airy wave theory.

For deep water conditions, kh > π, (2) to (5) can be approximated to:

$$u(x,t) = \omega\zeta_a e^{kz} \cos(kx - \omega t) \qquad \text{"equation 6"}$$

$$v(x,t) = \omega\zeta_a e^{kz} \sin(kx - \omega t) \qquad \text{"equation 7"}$$

$$\omega^2 = gk \qquad \text{"equation 8"}$$

$$\dot{v}(x,t) = \omega^2\zeta_a e^{kz} \sin(kx - \omega t) \qquad \text{"equation 9"}$$

This would imply that the elliptical orbits of the water particles associated with the general Airy wave description in (2) and (3), would reduce to circular orbits in deep water conditions as implied by (6) and (7).

3.1.2 Stoke's Second Order Wave Theory

Stokes employed perturbation techniques to solve the wave boundary value problem and developed a theory for finite amplitude wave that he carried to the second order. In this theory, all the wave characteristics (velocity potential, celerity, surface profile, particle kinetics….e.tc) are formulated in terms of a power series in successively higher orders of the wave steepness (H/L).

A condition of this theory is that (H/d) should be small so that the theory is applicable only in deep water and a portion of the immediate depth range.

For engineering applications, the second-order and possibly the fifth-order theories are the most commonly used [9].

Stoke's wave expansion method is formally valid under the conditions [10]:

H/d << (kd)2 for kd < 1 and H/L << 1.

Stoke's wave theory is considered most nearly valid in water where the relative depth (D/L) is greater than about (1/10) [11]. Stoke's theory would be adequate for describing water waves at any depth of water. In shallow water, the connective terms become relatively large, the series convergence is slow and erratic and a large number of terms are required to achieve a uniform accuracy [12].

The fluid particle velocities are then given by;

$$v_x = \frac{\pi H}{T}\frac{\cosh[k(z+h)]}{\sinh(kh)}\cos(k(kx-\omega t)) + \frac{3(\pi H)^2}{4TL}\frac{\cosh[2k(z+h)]}{\sinh^4(kh)}\cos 2(kx-\omega t) \qquad \text{“equation 9a”}$$

$$v_z = \frac{\pi H}{T}\frac{\sinh[k(z+h)]}{\sinh(kh)}\sin(k(kx-\omega t)) + \frac{3(\pi H)^2}{4TL}\frac{\sinh[2k(z+h)]}{\sinh^4(kh)}\sin 2(kx-\omega t) \qquad \text{“equation 9b”}$$

The fluid particle accelerations are then given by;

$$a_x = 2\frac{\pi^2 H}{r^2}\frac{\cosh[k(z+h)]}{\sinh(kh)}\sin(kx-\omega t) + \frac{3\pi^3 H^2}{T^2 L}\frac{\cosh[2k(z+h)]}{\sinh^4(kh)}\sin 2(kx-\omega t) \qquad \text{“equation 10a”}$$

$$a_z = 2\frac{\pi^2 H}{r^2}\frac{\sinh[k(z+h)]}{\sinh(kh)}\cos(kx-\omega t) + \frac{3\pi^3 H^2}{T^2 L}\frac{\sinh[2k(z+h)]}{\sinh^4(kh)}\cos 2(kx-\omega t) \qquad \text{“equation 10b”}$$

These velocities and accelerations in (9) and (10) are used in Morison's equation to calculate load vectors of hydrodynamic loading by using Stoke's wave theory after being transformed from global coordinates for each member of the offshore structure.

3.2 Morison's Equation

The along wave or in-line force per unit length acting on the submerged section of a rigid vertical surface-piercing cylinder, *F* (z, t), from the interaction of the wave kinematics at position *z* from the MWL, (see Fig. 2), is given by Morison's equation. This equation is originally developed to compute hydrodynamic forces acting on a cylinder at a right angle to the steady flow, and is given by:

$$F(z,t) = \rho\,\pi\frac{D^2}{4}C_m\,a(z,t) + \frac{1}{2}\rho D C_d\,V|V|(z,t) \qquad \text{“equation 11”}$$

in this (11), it is assumed that the wave force is acting on the vertical distance (z, t) of the cylinder due to the velocity (v) and acceleration (a) of the water particles, where (ρ) is the density of water, (D) is the cylinder diameter, (C_m) and (C_d) are inertia and drag coefficients, F_D and F_I are drag force and inertia force [13].

$$F_D = \frac{1}{2}\rho D C_d\,V|V|(z,t) \qquad \text{“equation 12”}$$

$$F_I = \rho\,\pi\frac{D^2}{4}C_m\,a(z,t) \qquad \text{“equation 13”}$$

These coefficients are found to be dependent upon Reynold's number, *Re*, Keulegan Carpenter number, KC, and the β parameter, Viz;

$$KC = \frac{u_m}{D}T;\ \beta = \frac{Re}{KC} \qquad \text{“equation 14”}$$

Where u_m = the maximum along wave water particle velocity. It is found that for KC < 10, inertia forces progressively dominate; for 10 < KC < 20 both inertia and drag force components are significant and for KC > 20, drag force progressively dominates [1].

Various methods exist for the calculation of the hydrodynamic loads on an arbitrary oriented cylinder by using Morison's equation. The method adopted here assumes that only the components of water particles and accelerations normal to the member produce [14].

To formulate the hydrodynamic load vector F_w, consider the single, bottom mounted cylindrical member (as shown in Fig. 2). The forces are found by the well known semi-empirical Morison's formula (11). It also represents the load exerted on a vertical cylinder, assuming that the total force on an object in the wave is the sum of drag and inertia force components. This assumption (introduced by Morison) takes the drag term as a function of velocity and the inertia force as a function of acceleration [15], [16] and [17], so that:

$$F_n = \rho\,\pi \frac{D^2}{4}\, C_m\, v_n' - (C_m - 1)\,\rho\,\pi \frac{D^2}{4}\, u_n'' + \frac{1}{2}\rho\, D\, C_d\, (v_n - u_n')\,|(v_n - u_n')| \quad \text{“equation 15a”}$$

This can be simplified to:

$$F_n = \rho\,\pi \frac{D^2}{4}\, C_m\, v_n' + \frac{1}{2}\rho\, D\, C_d\, (v_n)\,.\,|(v_n)| \quad \text{“equation15b”}$$

Where:

F_n = nodal hydrodynamic force normal to the cylinder, D = Outer diameter of cylinder, ρ = Sea water density. C_d = Drag coefficient (= 1.05). v_n' = water particle acceleration. C_m = Inertia coefficient (= 1.2). v_n = water particle velocity. u_n' = Structural velocity. u_n'' = Structural acceleration.

(15b) neglects the non-linear terms of drag coefficient [2] and [18] water particle velocity and acceleration can be evaluated by potential velocity computed from wave theories; the absolute value of velocity is needed to preserve the sign variation of the force.

3.2.1 Global and Local System

The kinematics of cross-flow with resultant velocity (see Fig. 3) is;

$$[U_L] = [t][U_g] \quad \text{“equation 16”}$$

$$W_n = \sqrt{u' + w'} \quad \text{“equation 17”}$$

is determined using wave theory applied in the global system and then transferred to the local system using transformation matrix (see Fig. 4).

Application of Morison's equation leads to:

$$\mathrm{F_L} = \rho\,\pi \frac{\mathrm{D}^2}{4}\, \mathrm{C_m}\, \frac{\mathrm{du}}{\mathrm{dt}} + \frac{1}{2}\rho\, \mathrm{D}\, \mathrm{C_d}\, (\mathrm{w_n})\,.\,|(\mathrm{w_n})| \quad \text{“equation 18”}$$

The components of the forces in the local axis system then become;

$$\begin{bmatrix} f_y' \\ f_z' \end{bmatrix} = \frac{1}{2}\rho\, \mathrm{D}\, \mathrm{C_d}\, (\mathrm{w_n})\,|(\mathrm{w_n})| \begin{bmatrix} u' \\ w' \end{bmatrix} + \rho\,\pi \frac{\mathrm{D}^2}{4}\, \mathrm{C_m} \begin{bmatrix} u'' \\ w'' \end{bmatrix} \quad \text{“equation 19”}$$

To get the local forces, we need to get the matrices as follows;

$$\mathrm{T} = \begin{array}{c|ccc} & X & Y & Z \\ \hline x' & 0 & 0 & 1 \\ y' & 0 & 1 & 0 \\ z' & -1 & 0 & 0 \end{array} \quad ; \quad \mathrm{T}^{-1} = \begin{array}{c|ccc} & X & Y & Z \\ \hline x' & 0 & 0 & -1 \\ y' & 0 & 1 & 0 \\ z' & 1 & 0 & 0 \end{array}$$

Therefore, the local forces are given as:-

$$\begin{bmatrix} f_x' \\ f_y' \\ f_z' \end{bmatrix} = \frac{1}{2}\rho\, \mathrm{D}\, \mathrm{C_d}\, (\mathrm{w_n})\,|(\mathrm{w_n})|[\mathrm{T}] \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} + \rho\,\pi \frac{\mathrm{D}^2}{4}\, \mathrm{C_m} \begin{bmatrix} u'' \\ v'' \\ w'' \end{bmatrix} \quad \text{“equation 20”}$$

The local forces are then transferred in to global forces by the transpose matrix.

$$\mathrm{F_w} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = [\mathrm{T}]^{-1} \begin{bmatrix} 0 \\ f_y' \\ f_z' \end{bmatrix} \quad \text{“equation 21”}$$

III. RESULTS

The loads for six different sea states were computed using spread sheet for the following values of time intervals, t = 0, T/4, T/2. The magnitudes of these forces are presented graphically in Figures 6, 7, 8, 9, 10 and 11when x= 0 and x = 16m.

IV. CONCLUSION

The results for this work are thus;

1. The hydrodynamic force is directly proportional to the depth *z* and is minimum at *z = 0.*
2. For all the sea state, all the hydrodynamic forces follow the same directions as the direction of the wave propagation (as forces pushing the member) at time *t = T/2* and *t = 0* when distance *x = 0* and 16m respectively. (see Figures 8 and 9)
3. Also, all the hydrodynamic forces at time *t = 0, t = T/4* and *t = T/2* when distance *x = 0, = 0* and *x =* 16m respectively, are in direction (as forces pulling the member) similar to the direction of the wave propagation for all the sea states. (see Figures 6, 7 and 11)
4. In Figure 10, the hydrodynamic forces follow the same directions of the wave propagation (as both pushing and pulling forces) due to changes in sea states.
5. At constant time *t*, distance *x* and depth *z*, all the hydrodynamic forces are different for different sea state.

V. ACKNOWLEDGEMENT

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REFERENCES

[1] Haritos, N, University of Melbourne, Australia, *Introduction to the Analysis and Design of Offshore Structures-An Overview, 2007.*

[2] Sarpakava, T and Isaacson, M, John Wiley and Sons, *Mechanics of Wave Forces on Offshore Structures*, 1981.

[3] Chakrabarti, S. K, Southampton, Computational Mechanics Publication, *Fluid Structure Interaction in Offshore Engineering*, (ed) 1987.

[4] Graff, W. J, Houston, Gulf Publishing Company, I*ntroduction to Offshore Structures – Design, fabrication, Installation*, 1981.

[5] DNV-OS-C101, General (LFRD Method), Det Norske Veritas, Norway, *Design of Offshore Steel Structures*, 2004.

[6] Nigam, N.C and Narayanan, S, New York, Springer-Verlag, *Applications of Random Vibrations*, 1994.

[7] Dean, R. G., and Dalrymple, R. A, New Jersey, world scientific, *Water waves Mechanics for engineers and Scientist*s, 1991.

[8] Al-Salehy, S. M. S, *Dynamic Analysis of Offshore Structure Using Finite Element Method*, MSc Thesis, Basrah University, Iraq, 2002.

[9] Sorensen, R.M, Springer Science and Business Media, Inc, *Basic Coastal Engineering*, 2006.

[10] Iraninejid, B, *Dynamic Analysis of Fixed Offshore Platforms Subjected to Non-linear hydrodynamic Loading*, PhD Thesis, Illinois Institute of Technology, 1988.

[11] Patal, M, Buttler and Tanner Co., England, *Dynamics of Offshore Structures*, 1989.

[12] Muga, B. J, Edited by James F. Wilson, John Wily and Sons, *Deterministic Description of Offshore Waves-Dynamic of Offshore Structure*, 2003.

[13] Madhujit, M. and Sinha, S.K, Journal of the Ocean Engineering, *Modeling of Fixed Offshore Towers in Dynamic Analysis*, 15(6), 1988.

[14] Qian, J. and Wang, X, International Journal of computers and Structures, *Three –Dimensional Stochastic Response of Offshore Towers to Random Sea waves*, 43(2), 1992.

[15] Zienkiewicz, O.C., Lewis, R.W and Stagg, K.G, John Wiley and Sons, *Numerical Methods In Offshore Engineering*, 1978.

[16] Dean, R. G., and Dalrymple, R. A, Prentice – Hall, Inc., *Water waves Mechanics for Engineers and Scientist*s, 1984.

[17] McCormic, M. E, John Wiley and Sons: New York, *Ocean Engineering Wave Mechanics*, 1973.

[18] Al-Jasim, S. A. J, *Dynamic Analysis of Offshore Template Structures with Soil Structure Interaction*, PhD Thesis: University of Basrah, Iraq, March, 2000.

Figures and Table

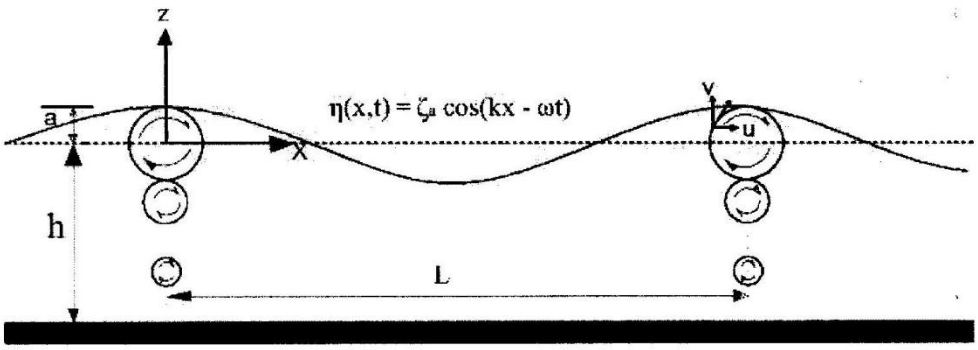

Figure 1: definition diagram for an airy wave [1]

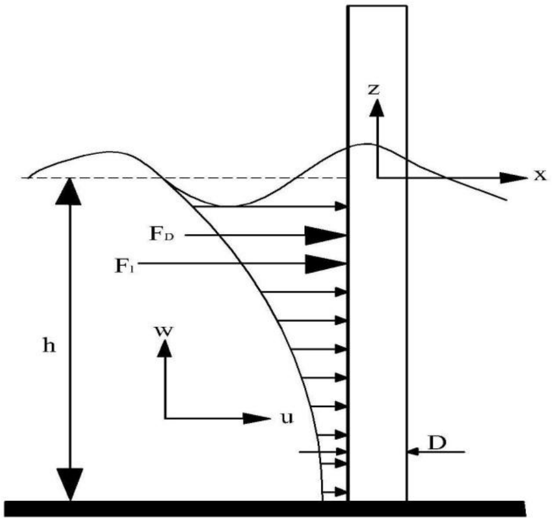

Figure 2: wave loading on a surface-piercing bottom mounted cylinder [1]

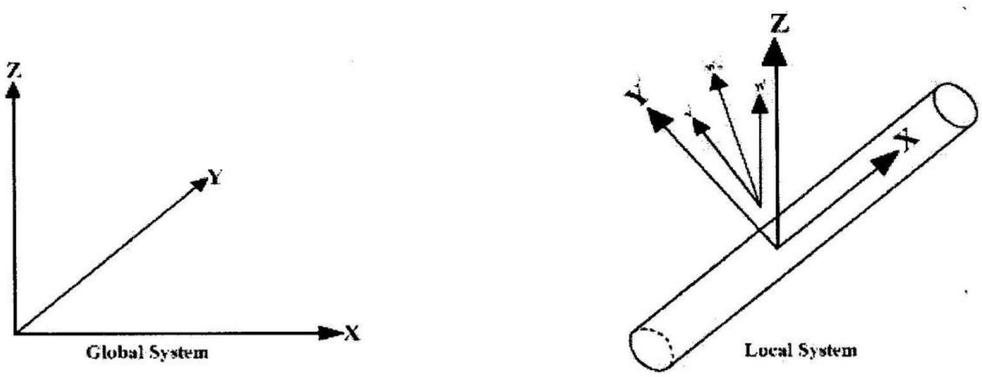

Figure 3: global and local system

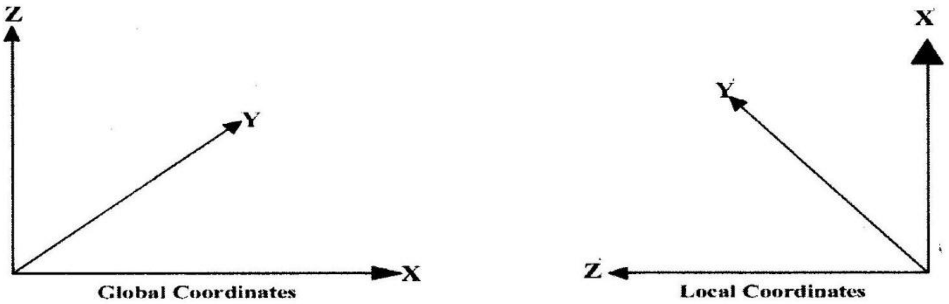

Figure 4: global and local coordinates

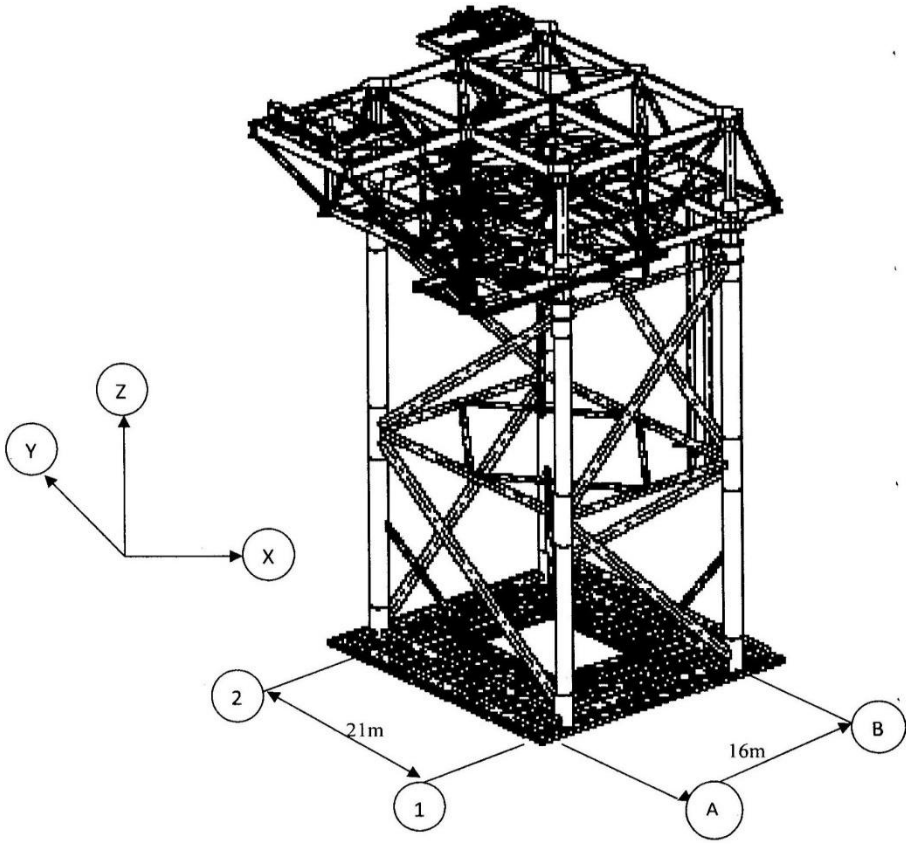

Figure 5: HD accommodation platform

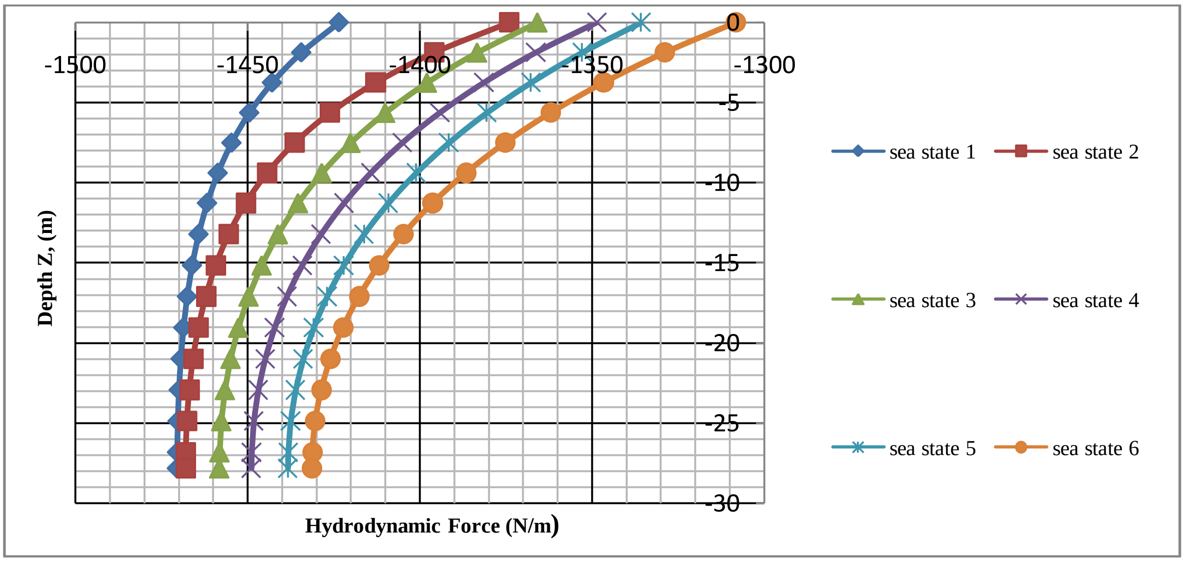

Figure 6: variation of hydrodynamic forces with depth (@ time t = 0 and distance x = 0)

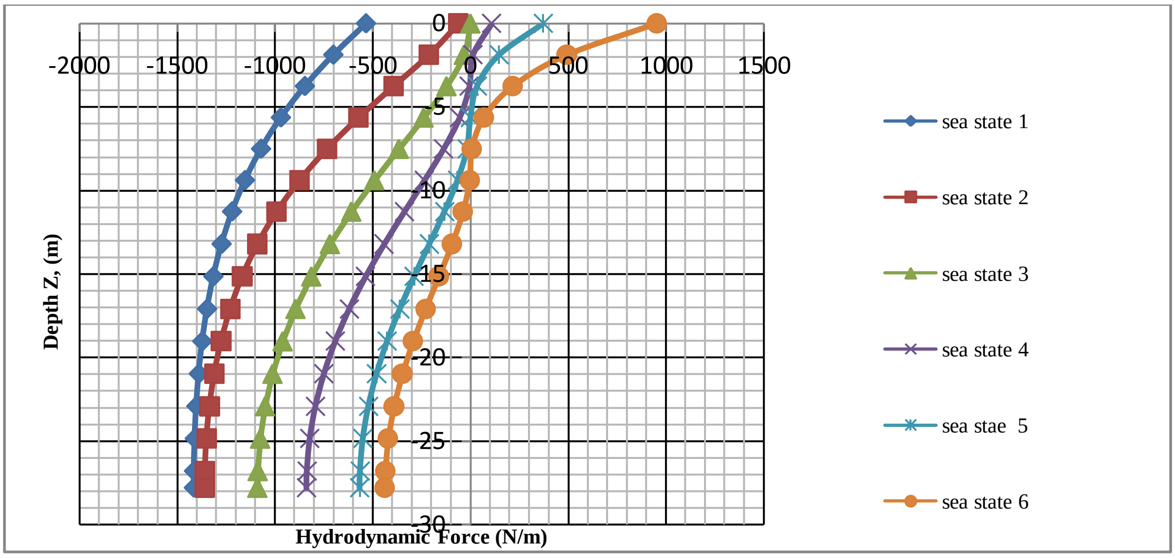

Figure 7: variation of hydrodynamic forces with depth (@ time t = T/4 and distance x = 0)

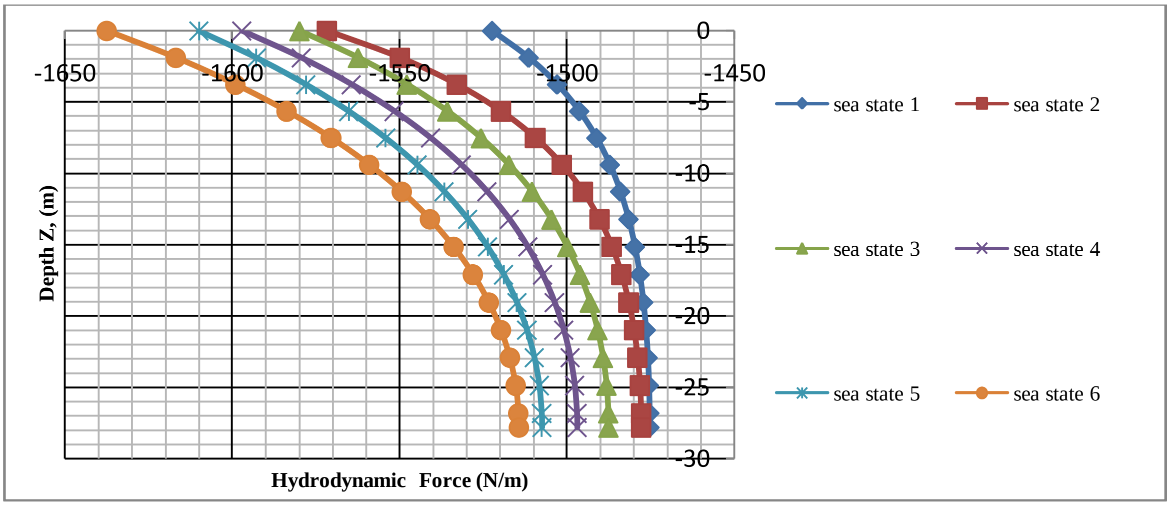

Figure 8: variation of hydrodynamic forces with depth (@ time t = T/2 and distance x = 0)

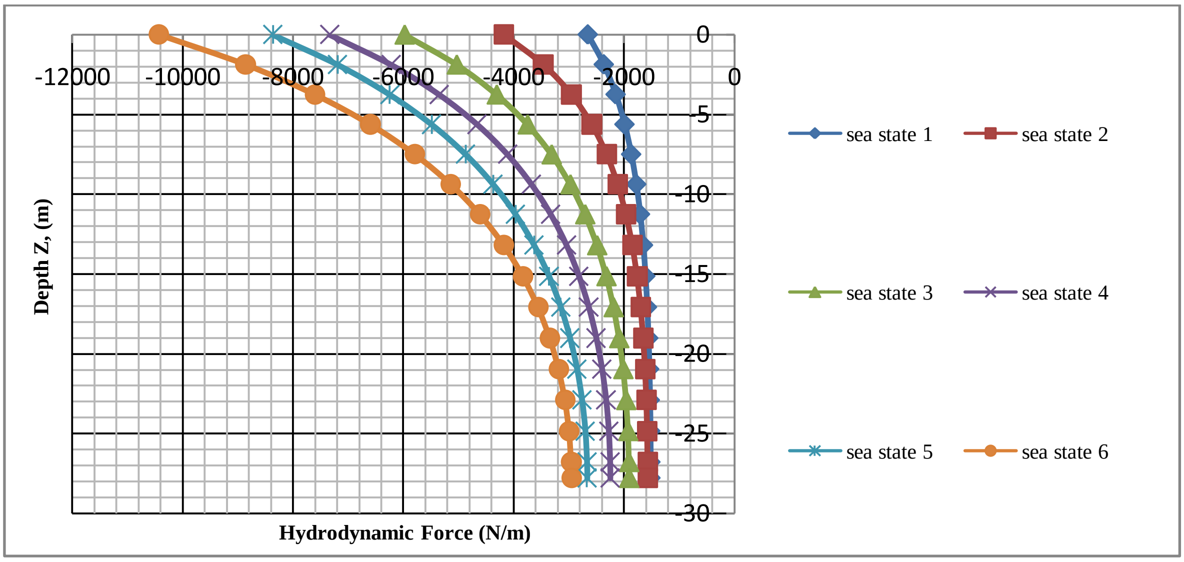

Figure 9: variation of hydrodynamic forces with depth (@ time t = 0 and distance x = 16m)

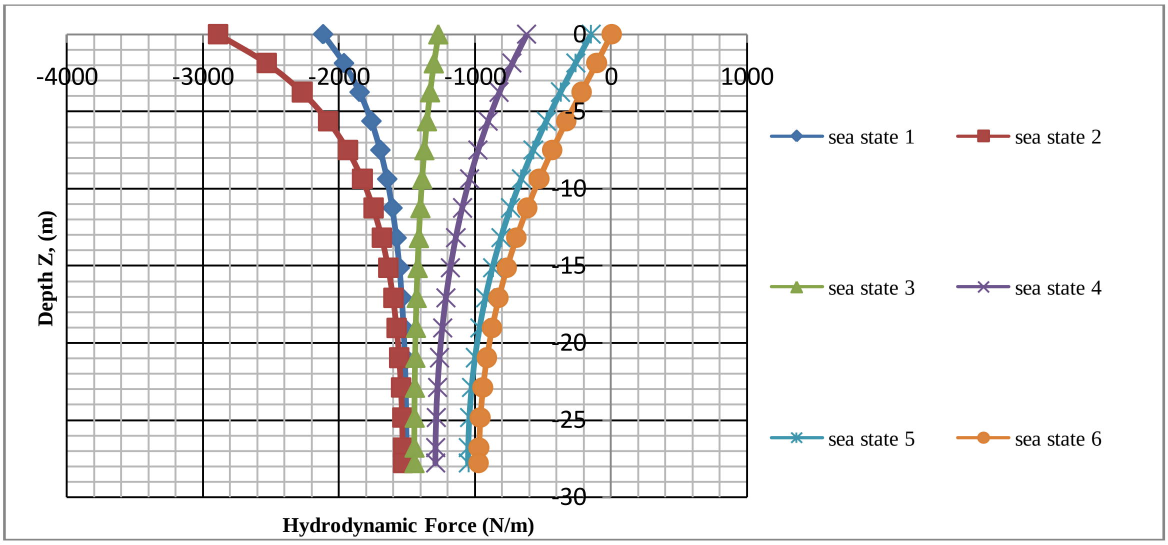

Figure 10: variation of hydrodynamic forces with depth (@ time t = T/4 and distance x = 16m)

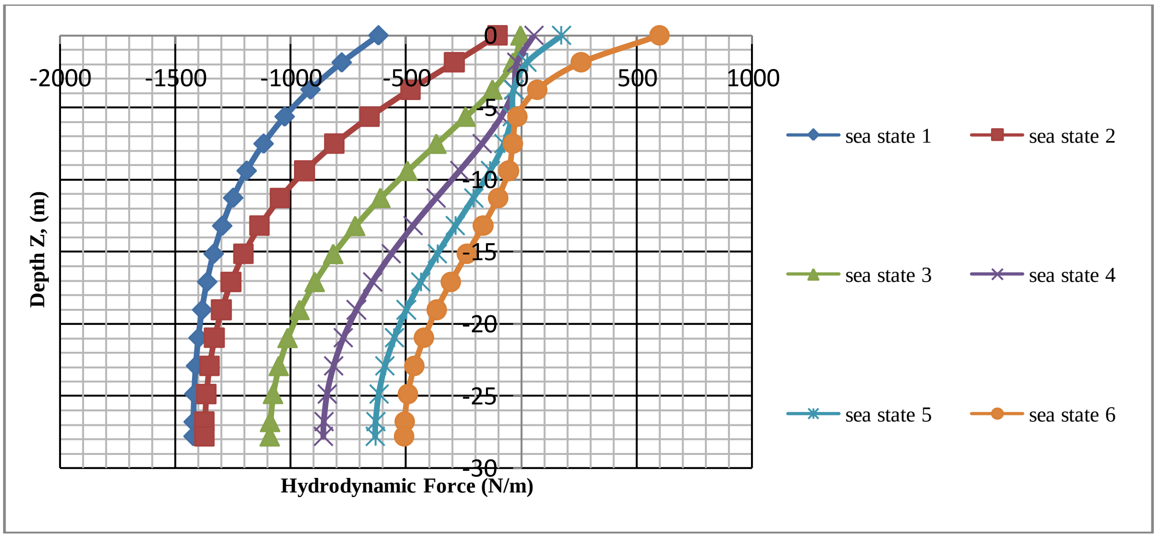

Figure 11: variation of hydrodynamic forces with depth (@ time t = T/2 and distance x = 16m)

Table I: Most Probable Wave Heights and Time Periods for Different Sea States (Area 59)

Sea State	Hs (m)	Tp (sec)	ζa (m)	t (sec)	k
1	1	5.5	0.5	4.125	0.133
2	2	5.5	1	4.125	0.133
3	3	6.5	1.5	4.875	0.0961
4	4	7	2	5.25	0.08371
5	5	7.5	2.5	5.625	0.0739
6	6	7.5	3	5.625	0.0739