

I- Continuous Functions in Ideal Bitopological Spaces

Mandira Kar, S. S. Thakur, S. S. Rana, J. K. Maitra

Department of Mathematics St. Aloysius College, Jabalpur (M.P.) 482001 India

Department of Applied Mathematics Government Engineering College, Jabalpur (M.P.) 482011 India

Department of Mathematics & Computer Science Rani Durgawati Vishwavidyalaya Jabalpur (M.P.) 482011 India

Department of Mathematics & Computer Science Rani Durgawati Vishwavidyalaya Jabalpur (M.P.) 482011 India

Abstract: - In this paper we introduce and study the concepts of (i,j) - I - continuous, (i,j) - I - open and (i,j) - I - closed functions in Ideal Bitopological Spaces.

AMS Mathematics Subject Classification (2000): 54A10, 54A05, 54E55

Key words and phrases: - Ideal Bitopological spaces, (i,j) - I - open and (i,j) - I - closed sets, (i,j) - I - continuous and (i,j) - precontinuous functions.

I. INTRODUCTION

In 1961 Kelly introduced the concept of bitopological spaces as an extension of topological spaces. A bitopological space (X, τ_1, τ_2) is a nonempty set X equipped with two topologies τ_1 and τ_2 [4]. The notion of ideal in topological spaces was studied by Kuratowski [5] and Vaidyanathaswamy [10]. An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the conditions (i) $A \in I$ and $B \subset A$ then $B \in I$ and (ii) $A \in I$ and $B \in I$ then $A \cup B \in I$. An ideal topological space is a topological space (X, τ) with an ideal I on X , and is denoted by (X, τ, I) . Given an ideal topological space (X, τ, I) if $\mathcal{P}(X)$ is the set of all subsets of X , a set operator, $(.)^*: \mathcal{P}(X) \rightarrow \mathcal{P}(X)$ is called the local function $A^*(\tau, I)$ (in short A^*) [10] of A with respect to the topology τ and ideal I defined as $A^* = \{x \in X \mid \cup \cap A \notin I, \forall U \in \tau, \text{ where } x \in U\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(\tau, I)$, called the $*$ -topology, finer than τ , is defined by $Cl^*(A) = A \cup A^*$ [10]. The collection $\{V \mid V \in \tau \text{ and } J \in I\}$ is a basis for τ^* [9]. A subset A of X is called I -open if $A \subseteq \text{int}(A^*)$ and I -closed if its complement is open. A subset A of X is called $*$ -dense in itself, (resp. τ^* -closed, $*$ -perfect) if $A \subseteq A^*$ (resp. $A^* \subseteq A, A = A^*$) [2]. A subset A of X is called preopen [8] if $A \subseteq \text{int}(Cl(A))$. The complement of a preopen set is called preclosed. Every I -open set is preopen, but the converse may not be true.

II. PRELIMINARIES

Definition 2.1: [6] A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be precontinuous if the inverse image of every open set in Y is preopen in X .

Definition 2.2: [7] A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be I -continuous if for every $V \in \sigma, f^{-1}(V)$ is I -open in X .

Definition 2.3: [1] A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be pairwise continuous if inverse image of every σ_i -open (resp. σ_j -open) set in Y is τ_i -open (resp. τ_j -open) in X .

Definition 2.4: [3] An ideal bitopological space is a quadruple (X, τ_1, τ_2, I) where I is an ideal defined on a bitopological space (X, τ_1, τ_2) .

Throughout this paper, A^{τ_i} {resp. A^{τ_j} } denote the local function of a subset A of X with respect topology τ_i {resp. τ_j } and $\tau_i - Cl(A)$ (resp. $\tau_j - Cl(A)$) and $\tau_i - int(A)$ {resp. $\tau_j - int(A)$ } denote the closure and interior of A with respect to topology τ_i (resp. τ_j)

Definition 2.5: [3] A subset A of an ideal bitopological space (X, τ_1, τ_2, I) is called (i,j)- preopen if $A \subseteq \tau_i - int(\tau_j - Cl(A))$ where ; $i, j=1, 2, i \neq j$

Definition 2.6: [3] A subset A of an ideal bitopological space (X, τ_1, τ_2, I) is called (i,j) I - open if $A \subseteq \tau_i - int(A^{\tau_j})$ where; $i, j=1, 2, i \neq j$

III. (I,J) -I-CONTINUOUS FUNCTIONS

Definition 3.1: A function $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-I - continuous if $f^{-1}(V)$ is (i,j)-I - open in X for every σ_i -open set V in Y; $i, j=1, 2, i \neq j$

Definition 3.2: A function $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be (i,j)- precontinuous if $f^{-1}(V)$ is (i,j)- preopen in X for every σ_i -open V in Y; $i, j=1, 2, i \neq j$.

Remark 3.1: Every (i,j)-I - continuous function is (i,j)- precontinuous but the converse is not true. For,

Example 3.1: Let $X = \{a, b, c, d\}$ with topologies $\tau_1 = \{X, \phi, \{a, c\}, \{a, b, c\}, \{b\}\}$; $\tau_2 = \{X, \phi, \{a, b\}, \{a, b, d\}, \{d\}\}$ and $I = \{\phi, \{a\}\}$ be an ideal on X. Let $Y = \{p, q, r, s\}$ with topologies $\sigma_1 = \{Y, \phi, \{p, r\}, \{p, q, r\}, \{q\}\}$; $\sigma_2 = \{Y, \phi, \{p, q\}, \{p, q, s\}, \{s\}\}$. Then $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ defined by $f(a) = p, f(b) = q, f(c) = r, f(d) = s$ is (1,2)- precontinuous but not (1,2)-I- continuous because (p, r) is open and $f^{-1}(p, r)$ is (1,2)- preopen but not (1,2)-I- open.

Definition 3.3: A function $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be pairwise I- continuous if $f^{-1}(V)$ is τ_i -I- open (resp. τ_j -I- open) in X for every σ_i - open (resp. σ_j - open) set V in Y

Remark 3.2: The concepts of pairwise I- continuity and (i,j)-I- continuity are independent.

Example 3.2: Let $X = \{a, b, c, d\}$ with topologies $\tau_1 = \{X, \phi, \{a, c\}, \{a, b, c\}, \{b\}\}$; $\tau_2 = \{X, \phi, \{a, b\}, \{a, b, d\}, \{d\}\}$ and $I = \{\phi, \{a\}\}$ be an ideal on X. Let $Y = \{p, q, r, s\}$ with topologies $\sigma_1 = \{Y, \phi, \{p, r\}, \{p, q, r\}, \{q\}\}$ $\sigma_2 = \{Y, \phi, \{p, q\}, \{p, q, s\}, \{s\}\}$. Then $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ defined by $f(a) = p, f(b) = q, f(c) = r, f(d) = s$ is pairwise I- continuous but not (1,2)-I- continuous, because (p, r) is open, $f^{-1}(p, r)$ is τ_1 -I- open, but not (1,2)-I- open in X.

Example 3.3: Let $X = \{a, b, c, d\}$ with topologies $\tau_1 = \{X, \phi, \{a, c\}, \{a, b, c\}, \{b\}\}$; $\tau_2 = \{X, \phi, \{a, b\}, \{a, b, d\}, \{d\}\}$ and $I = \{\phi, \{a\}\}$ be an ideal on X. Let $Y = \{p, q, r, s\}$ with topologies $\sigma_1 = \{Y, \phi, \{p, r\}, \{p, q, r\}, \{q\}\}$ $\sigma_2 = \{Y, \phi, \{p, q\}, \{p, q, s\}, \{s\}\}$. Then $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ defined by $f(a) = p, f(b) = q, f(c) = r, f(d) = s$ is (1,2)-I- continuous but not pairwise I- continuous because (p, q, r) is open in Y, $f^{-1}(p, q, r)$ is (1,2)-I- open but not τ_i -I- open in X.

Theorem 3.1: For a function $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ the following conditions are equivalent:

- (a) f is (i,j)-I- continuous.
- (b) For each $x \in X$ and each $V \in \sigma_i$ containing $f(x)$, there exists an (i,j)-I- open set W in X such that $x \in W$ and $f(W) \subset V$.
- (c) For each $x \in X$ and each $V \in \sigma_i$ containing $f(x)$, $(f^{-1}(V))^{\tau_j}$ is a τ_i - neighborhood of x

Proof:

(a) \Rightarrow (b) $V \in \sigma_i$ containing $f(x)$. Hence by (a), $f^{-1}(V)$ is (i,j)- I- open set in X containing x. Put $W = f^{-1}(V)$ then $x \in W$ and $f(W) \subset V$.

(b) \Rightarrow (c) Since $V \in \sigma_i$ containing $f(x)$, then by (b), there exists an (i,j)- I- open set W in X containing x s.t. $f(W) \subset V$. So, $x \in W \subseteq (\tau_i - int(W))^{\tau_j} \subseteq (\tau_i - int(f^{-1}(V)))^{\tau_j} \subseteq (f^{-1}(V))^{\tau_j}$. Hence $(f^{-1}(V))^{\tau_j}$ is a τ_i - neighborhood of x.

(c) \Rightarrow (a) Obvious.

Theorem 3.2: For a function $f: (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ the following conditions are equivalent:

- (a) f is (i,j)- I- continuous

- (b) The inverse image of each σ_j - closed set in Y is (i,j) - \mathbf{I} - closed in X .
- (c) $\tau_i - \text{int } f^{-1}(M) \tau_j^* \subseteq \tau_i - (f^{-1}(M^{\sigma_i})) \tau_j^*$, for each σ_i^* -dense-in-itself subset M of Y .
- (d) $\tau_i - f(\text{int}(U)) \tau_j^* \subseteq \tau_i - f(U) \tau_j^*$, for each $U \subseteq X$, and for each σ_i^* - perfect subset of Y .

Proof:

(a) \Rightarrow (b) Let $M \subseteq Y$ be σ_i - closed in Y , then $Y \setminus M$ is σ_i open in Y , then by (a), $f^{-1}(Y \setminus M) = X \setminus f^{-1}(M)$ is (i,j) - \mathbf{I} - open in X . Thus, $f^{-1}(M)$ is (i,j) - \mathbf{I} - closed in X .

(b) \Rightarrow (c) Let $M \subseteq Y$ be σ_i - closed in Y , Since M^{σ_i} is also σ_i - closed in Y , then by (b) $f^{-1}(M^{\sigma_i})$ is (i,j) - \mathbf{I} - closed in X . Next, by using Theorem 2.4 [6], $\tau_i - \text{int } f^{-1}(M^{\sigma_i}) \tau_j^* \subseteq f^{-1}(M^{\sigma_i})$ and since M^{σ_i} is σ_i^* - dense in itself, $\tau_i - \text{int } f^{-1}(M) \tau_j^* \subseteq \tau_i - \text{int } f^{-1}(M^{\sigma_i}) \tau_j^* \subseteq f^{-1}(M^{\sigma_i})$.

(c) \Rightarrow (d) Let $U \subseteq X$ and $W = f(U)$, then by (c), $\tau_i - \text{int}(U) \tau_j^* \subseteq \tau_i - \text{int } f^{-1}(W) \tau_j^* \subseteq \tau_i - \text{int } f^{-1}(W^{\sigma_i}) \tau_j^* \subseteq f^{-1}(W) \tau_j^*$ (because W^{σ_i} is perfect). Hence, $\tau_i - f(\text{int}(U)) \tau_j^* \subseteq \tau_i - (W) \tau_j^* = \tau_i - f(U) \tau_j^*$.

(d) \Rightarrow (a) Let $V \in \sigma_i, W = Y \setminus V$, and $U \subseteq f^{-1}(W)$, then $f(U) \subseteq W$ and by (d), $\tau_i - f(\text{int}(U)) \tau_j^* \subseteq \tau_i - f(U) \tau_j^* \subseteq \tau_i - (W) \tau_j^* = W$ (because W is σ_i^* - perfect). Thus, $\tau_i - (\text{int}(f^{-1}(W))) \tau_j^* = \tau_i - (\text{int}(U)) \tau_j^* \subseteq f^{-1}(W)$, and therefore, $f^{-1}(Y \setminus V)$ is (i,j) - \mathbf{I} - closed. Hence, $f^{-1}(V)$ is (i,j) - \mathbf{I} - open in X and f is (i,j) - \mathbf{I} - continuous.

Theorem 3.4: Let $f: (X, \tau_1, \tau_2, \mathbf{I}) \rightarrow (Y, \sigma_1, \sigma_2)$ is (i,j) - \mathbf{I} - continuous and $U \in \tau_1 \cap \tau_2$. Then the restriction $f|U$ is (i,j) - \mathbf{I} - continuous.

Proof:

Let $V \in \sigma_i$. Then, $\tau_i - f^{-1}(V) \tau_j^* \subseteq \tau_i - \text{int}(f^{-1}(V)) \tau_j^*$ and so $U \cap \tau_i - f^{-1}(V) \tau_j^* \subseteq U \cap \tau_i - \text{int}(f^{-1}(V)) \tau_j^*$. Thus $(f|U)^{-1}(V) \subseteq U \cap \tau_i - \text{int}(f^{-1}(V)) \tau_j^*$. Since $U \in \tau_i$, we get $(f|U)^{-1}(V) = \tau_i - \text{int}(U \cap (f^{-1}(V)) \tau_j^* [5] \subseteq \tau_i - \text{int}(U \cap f^{-1}(V)) \tau_j^* = \tau_i - \text{int}((f|U)^{-1}(V)) \tau_j^*$. Hence $(f|U)^{-1}(V)$ is (i,j) - \mathbf{I} - open and $f|U$ is (i,j) - \mathbf{I} - continuous.

Theorem 3.5: For a function $f: (X, \tau_1, \tau_2, \mathbf{I}) \rightarrow (Y, \sigma_1, \sigma_2)$ and $\{U_\alpha: \alpha \in \Delta\}$ be a biopen cover of X . If the restriction function $f|U_\alpha$ is (i,j) - \mathbf{I} - continuous, for each $\alpha \in \Delta$, then f is (i,j) - \mathbf{I} - continuous.

Proof:

Similar to Theorem 1.4

Theorems that follow are immediate and their obvious proofs have been omitted

Theorem 3.6: Let $f: (X, \tau_1, \tau_2, \mathbf{I}) \rightarrow (Y, \sigma_1, \sigma_2)$ is (i,j) - \mathbf{I} - continuous and a biopen function, then the inverse image of each open set in Y , which is (i,j) - \mathbf{I} - open set in X is also (i,j) - preopen in X .

Theorem 3.7: Let $f: (X, \tau_1, \tau_2, \mathbf{I}) \rightarrow (Y, \sigma_1, \sigma_2)$ is (i,j) - \mathbf{I} - continuous and $\tau_i - f^{-1}(V) \tau_j^* \subseteq \tau_i - (f^{-1}(V)) \tau_j^*$, for each $V \in \sigma_i \subseteq Y$. Then the inverse image of each (i,j) - \mathbf{I} - open set is (i,j) - \mathbf{I} - open.

Remark 3.3: Composition of two (i,j) - \mathbf{I} - continuous functions need not be (i,j) - \mathbf{I} - continuous, in general, as shown by the following example.

Example 3.4: Let $X = \{a, b, c\}$ with topologies $\tau_1 = \{X, \phi, \{a\}\}$, $\tau_2 = \{X, \phi\}$, and $\mathbf{I} = \{\phi, \{c\}\}$ be an ideal on X ; $Y = \{a, b, c, d\}$ with topologies $\sigma_1 = \{Y, \phi, \{a, c\}\}$ $\sigma_2 = \{Y, \phi\}$ and $\mathbf{J} = \{\phi, \{a\}\}$ be an ideal on Y ; $Z = \{a, b, c\}$ $\eta_1 = \{Z, \phi, \{c\}, \{b, c\}\}$ $\eta_2 = \{Z, \phi\}$. Let $f: (X, \tau_1, \tau_2, \mathbf{I}) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ be the identity function and let $g: (Y, \sigma_1, \sigma_2, \mathbf{J}) \rightarrow (Z, \eta_1, \eta_2)$ be defined as $g(a) = a$, $g(b) = g(d) = b$, $g(c) = c$. It is clear that both f and g are (i,j) - \mathbf{I} - continuous but the composition function $g \circ f$ is not (i,j) - \mathbf{I} - continuous, because $\{c\}$ is open, but $(g \circ f)^{-1}\{c\} = \{c\}$ is not (i,j) - \mathbf{I} - open.

Theorem 3.8: For the functions $f: (X, \tau_1, \tau_2, \mathbf{I}) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g: (Y, \sigma_1, \sigma_2, \mathbf{J}) \rightarrow (Z, \eta_1, \eta_2)$ if f is (i,j) - \mathbf{I} - continuous and g is pairwise continuous then $g \circ f$ is (i,j) - \mathbf{I} - continuous.

Proof: Obvious.

IV. (\mathbf{I}, \mathbf{J}) \mathbf{I} - OPEN AND (\mathbf{I}, \mathbf{J}) \mathbf{I} - CLOSED FUNCTION

Definition 4.1: A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$; $i, j = 1, 2, i \neq j$ is called (i,j) - \mathbf{I} - open function (resp. (i,j) - \mathbf{I} - closed function) if for each $U \in \tau_i$ (resp. $U \in \tau_i^c$), $f(U)$ is an (i,j) - \mathbf{I} - open set in Y (resp. (i,j) - \mathbf{I} - closed set in Y)

Remark 4.1: (i,j)-I- open (resp. (i,j)-I- closed) function \Rightarrow preopen (resp. preclosed) function but the converse is not true.

Example 4.1: Let $X = Y = \{a, b, c\}$ with topologies $\tau_1 = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$ $\tau_2 = \{X, \phi\}$ the discrete topology; $\sigma_1 = \{Y, \phi, \{a\}, \{a, b\}\}$; $\sigma_2 = \{Y, \phi\}$ the discrete topology and $\mathbf{J} = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ an ideal on Y. Then the identity function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ is preopen but not (1,2)-I- open, because $\{b\}$ is open, but $f(b)$ is a (2,1)-I- preopen set but not a (2,1)-I- open set.

Remark 4.2: The concepts of (i,j)-I- open functions and pairwise open functions are independent concepts

Example 4.2: Let $X = Y = \{a, b, c, d\}$ with topologies $\tau_1 = \{X, \phi, \{a, b\}, \{a, b, d\}\}$ τ_2 the discrete topology; $\sigma_1 = \{Y, \phi, \{a, b\}, \{a, b, c\}\}$; σ_2 the discrete topology and $\mathbf{J} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$ an ideal on Y. Then the identity function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ is a (1,2)-I- open function but not pairwise open function.

Example 4.3: Let $X = Y = \{a, b, c\}$; $\tau_1 = \{X, \phi, \{a\}\}$; $\sigma_1 = \{Y, \phi, \{a\}, \{a, b\}\}$; τ_2 and σ_2 the respective discrete topologies on X and Y and $\mathbf{J} = \{\phi, \{a\}\}$ an ideal on Y. Then the identity function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ is an open function but not a (1,2)-I- open function because, $\{a\}$ is open, but $f(a) = a$ is σ_2 open but not a (1,2)-I- open.

Theorems that follow are immediate and their proofs obvious from the definitions

Theorem 4.1: Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ be a function. Then the following are equivalent:

- f is a (i,j)-I- open function.
- For each $x \in X$ and each neighborhood U of x , there exists an (i,j)-I- open set $W \subset Y$ containing $f(x)$ such that $W \subset f(U)$

Theorem 4.2: Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ be an (i,j)-I- open function (resp. (i,j)-I- closed function) if $W \subset Y$, and $F \subset X$ is a closed (resp. open) set containing $f^{-1}(W)$, then there exists an (i,j)-I- closed (resp. (i,j)-I- open) set H containing W such that $f^{-1}(H) \subset F$

Theorem 4.3: If function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ is (i,j)-I- open, then $\tau_i \cdot f^{-1}(\text{int}(B))^{\sigma_i} \subset \tau_i \cdot (f^{-1}(B))^{\sigma_i}$ such that $f^{-1}(B)$ is σ_i^* dense-in-itself, for every $B \subset Y$

Theorem 4.4: For any one-one onto function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ the following are equivalent:

- $f^{-1}(Y, \sigma_1, \sigma_2, \mathbf{J}) \rightarrow (X, \tau_1, \tau_2)$ is (i,j)-I- continuous
- f is (i,j)-I- closed

Theorem 4.5: If function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ is (i,j)-I- open and for each $A \subset X$, $\sigma_i \cdot f(A)^{\tau_j} \subset \sigma_i \cdot [f(A)]^{\tau_j}$, then the image of each (i,j)-I- open set is (i,j)-I- open.

Theorem 4.6: Let function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2, \mathbf{J})$ and $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2, \mathbf{K})$ be two functions, where \mathbf{I}, \mathbf{J} and \mathbf{K} are ideals on X, Y and Z respectively, then

- If f is open and g is (i,j)-I- open then $g \circ f$ is (i,j)-I- open
- f is (i,j)-I- open if $g \circ f$ is open; g is one-one and (i,j)-I- continuous
- If f and g are (i,j)-I- open; f is surjective and $g(V)^{\sigma_i} \subset [g(V)]^{\sigma_i}$ for each $V \subset Y$, then $g \circ f$ is (i,j)-I- open

REFERENCES

- [1] Dontchev J., On pre-I -open sets and a decomposition of O- continuity, Banyan Math J; 2, (1996)
- [2] Hayashi E., Topologies defined by local properties, Math. Ann; 156, (1964), 205-215
- [3] Kar M. and Thakur S.S., Pair-wise open sets in Ideal Bitopological Spaces, Int. J of Math. Sc. and Appln; Vol. 2(2) (2012) 839-842
- [4] Kelly J.C., Bitopological Spaces, Proc. London Math. Soc.; 13, (1963), 71-89
- [5] Kuratowski K., Topology, Vol.1, Academic Press New York; (1966)
- [6] Mashhour A.S., Monsef M.E. Abd El and Deeb S.N. El., On precontinuous and weak precontinuous mappings, Proc. Math and Phys. Soc. Egypt; 53, (1982), 47-53

- [7] Monsef M.E. Abd El; Lashien E.F and Nasef A.A; On I- open sets and I- continuous functions, Kyungpook Math. J.; 32(1), (1992), 21-30
- [8] Noiri T; Hyperconnectedness and preopen sets, Rev. Roumania Math. Pure Appl.; 29 (1984), 329-334
- [9] Samuel P; A topology formed from a given topology and ideal, J. London Math. Soc.; 2(10), (1975), 409-416
- [10] Vaidyanathaswamy R; The localization theory in set topology, Proc. Indian Acad. Sci.; 20, (1945), 51-61