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Design of Discrete Optimal Multirate-Output Controllers Applied to a Hydrogenerator Power System

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Abstract: - In the present work an H^{∞} -control technique is presented and applied to the design of optimal multirate-output controllers. The technique is based on multirate-output controllers (MOCs) having a multirate sampling mechanism with different sampling period in each measured output of the system. The proposed technique relies on multirate-output controllers. Its main feature consists in reducing the original problem, to an associate discrete H^{∞}-control problem for which a fictitious static state feedback controller is to be designed. The proposed H^{∞}-control technique is applied to the discrete linear open-loop system model which represents a 117 MVA hydrogenerator unit supplying power through a step-up transformer and a transmission line to a infinite grid and give good assurance that the controllers designed by the H^{∞} -control technique may be implementable.

Keywords: *- Digital multirate control, Disturbance attenuation,* H^{∞}*-control, hydrogenerator system*

I. INTRODUCTION

The H^{∞}-control problem for discrete-time and sampled data singlerate and multirate systems has successfully been treated in the past [1-9,11,12]. Generally speaking, when the state vector is not available for feedback, the H^{∞}-control problem is usually solved in both the continuous and the discrete-time cases, by the use of dynamic measurement feedback.

This technique is based on multirate-output controllers (MOCs). MOCs contain a multirate sampling mechanism with different sampling period to each system measured output. The technique proposed[10], relies mainly on the reduction, under appropriate conditions, of the original H^{∞} -disturbance attenuation problem, to an associated discrete H^{∞}-control problem for which a fictitious static state feedback controllers is to be designed, even though state variables are not available for feedback. This fact has beneficial impact on the theoretical and the numerical complexity of the problem since using the technique reported in [10,11], only one algebraic Riccati equation is to be solved, as compared to two algebraic Riccati equations needed by other well known H^{∞} -control techniques.

In the present paper the ultimately investigated discrete linear open-loop power system model was obtained through a systematic procedure using a linearized continuous, with impulse disturbances, 6^{th}-order SIMO open-loop model representing a practical power system, which consists of a 117 MVA hydrogenerator with a single stage excitations system supplying power to an infinite gird via a step-up transformer and a double-circuit transmission line [14]. The digital controller, which will lead to the associated designed discrete closed-loop power system model displaying enhanced dynamic stability characteristics, is accomplished by applying properly the presented MOCs technique.

II. OVERVIEW OF RELEVANT MATHEMATICAL CONSIDERATIONS

The general description of the controllable and observable continuous, linear, time-invariant, multivariable mimo dynamical open-loop system expressed in state-space form is:

$$\dot{\mathbf{x}}(\mathrm{t}) = \mathbf{A}\mathbf{x}(\mathrm{t}) + \mathbf{B}\mathbf{u}(\mathrm{t}) \qquad (1)$$
$$\mathbf{y}(\mathrm{t}) = \mathbf{C}\mathbf{x}(\mathrm{t})$$

where: $\mathbf{x}(t) \in R^n$, $\mathbf{u}(t) \in R^m$, $\mathbf{y}(t) \in R^p$ are state, input and output vectors respectively; and **A**, **B** and **C** are real constant system matrices with proper dimensions.

The associated general discrete description of the system of equation 1 is as follows

$$\mathbf{x}(\mathrm{k}+1) = \mathbf{A}\mathbf{x}(\mathrm{k}) + \mathbf{B}\mathbf{u}(\mathrm{k}) \qquad (2)$$
$$\mathbf{y}(\mathrm{k}) = \mathbf{C}\mathbf{x}(\mathrm{k})$$

where: $\mathbf{x}(k) \in \mathrm{R}^n$, $\mathbf{u}(k) \in \mathrm{R}^m$, $\mathbf{y}(k) \in \mathrm{R}^p$ are state, input and output vectors respectively; and **A**, **B** and **C** are real constant system matrices with proper dimensions.

III. OVERVIEW OF H$^\infty$- CONTROL TECHNIQUE USING MOC [5,8]

Consider the controllable and observable continuous linear state-space system model of the general form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{q}(t), \ \mathbf{x}(0) = \mathbf{0} \qquad (3a)$$

$$\mathbf{y}_m(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{J}_1\mathbf{u}(t), \ \mathbf{y}_c(t) = \mathbf{E}\mathbf{x}(t) + \mathbf{J}_2\mathbf{u}(t) \qquad (3b)$$

where: $\mathbf{x}(t) \in R^n$, $\mathbf{u}(t) \in R^m$, $\mathbf{q}(t) \in \mathbf{L}_2^d$, $\mathbf{y}_m(t) \in R^{p_1}$, $\mathbf{y}_c(t) \in R^{p_2}$ are the state, input, external disturbance, measured output and controlled output vectors, respectively. In equation 3, all matrices have real elements and appropriate dimensions. Now follows a useful definition.

Definition. For an observable matrix pair $(\mathbf{A}, \mathbf{C})$, with $\mathbf{C}^T = \begin{bmatrix}\mathbf{c}_1^T & \mathbf{c}_2^T & \cdots & \mathbf{c}_{p_1}^T\end{bmatrix}$ and $\mathbf{c}_i$ with i=1, …,p$_1$, the *i*th row of the matrix **C**, a collection of p_1 integers $\{n_1, n_2, \cdots, n_{p_1}\}$ is called an *observability index vector* of the pair $(\mathbf{A}, \mathbf{C})$, if the following relationships simultaneously hold

$$\sum_{i=1}^{p_1} n_i = n \ , \ rank\begin{bmatrix}\mathbf{c}_1^T & \cdots & (\mathbf{A}^T)^{n_1-1}\mathbf{c}_1^T & \cdots & \mathbf{c}_{p_1}^T & \cdots & (\mathbf{A}^T)^{n_{p_1}-1}\mathbf{c}_{p_1}^T\end{bmatrix} = n$$

Next the multirate sampling mechanism [6,8,10], is applied to system 3.

Assuming that all samplers start simultaneously at t = 0, a sampler and a zero-order hold with period T_0 is connected to each plant input $\mathrm{u}_i(\mathrm{t})$, i=1,2,…,m, such that

$$\mathbf{u}(\mathrm{t}) = \mathbf{u}(kT_0), \ t \in [kT_0, (\mathrm{k}+1)\mathrm{T}_0) \qquad (4)$$

while the ith disturbance $q_i(t)$, i=1,…,d, and the ith controlled output $y_{c,i}(t)$, i=1,…, p_2, are detected at time kT_0, such that for $t \in [kT_0, (\mathrm{k}+1)\mathrm{T}_0)$

$$\mathbf{q}(\mathrm{t}) = \mathbf{q}(kT_0), \ \mathbf{y}_c(kT_0) = \mathbf{E}\mathbf{x}(kT_0) + \mathbf{J}_2(kT_0) \qquad (5)$$

The ith measured output $\mathrm{y}_{m,i}(\mathrm{t})$, i=1,…, p_1, is detected at every T_i period, such that for $\mu = 0, \ldots, N_i - 1$

$$y_{m,i}(kT_0 + \mu T_i) = \mathbf{c}_i\mathbf{x}(kT_0 + \mu T_i) + (\mathbf{J}_1)_i\,\mathbf{u}(kT_0) \qquad (6)$$

where $(\mathbf{J}_2)_i$ is the ith row of the matrix $\mathbf{J}_2$. Here $\mathrm{N}_i \in Z^+$ are the output multiplicities of the sampling and $\mathrm{T}_i \in R^+$ are the output sampling periods having rational ratio, i.e. $T_i = T_0 / N_i$ with i=1,…, p_1.

The sampled values of the plant measured outputs obtained over $[kT_0, (k+1)T_0)$ are stored in the N^*-dimensional column vector given by

$$\hat{\gamma}(kT_0) = \begin{bmatrix} y_{m,1}(kT_0) & \cdots & y_{m,1}(kT_0 + (N_1-1)T_1) & \cdots & \mathrm{y}_{m,p_1}(\mathrm{kT}_0) & \cdots & \mathrm{y}_{m,p_1}[\mathrm{kT}_0 + (\mathrm{N}_{p_1} - 1)\mathrm{T}_{p_1}]\end{bmatrix}^{\mathrm{T}} \qquad (7)$$

(where $N^* = \sum_{i=1}^{p_1} N_i$), that is used in the MROC of the form

$$\mathbf{u}[(k+1)T_0] = \mathbf{L}_\mathbf{u}\mathbf{u}(kT_0) - \mathbf{L}_\gamma\hat{\gamma}(kT_0) \qquad (8)$$

where $L_\mathbf{u} \in R^{\mathrm{mxm}}$, $L_\gamma \in R^{\mathrm{mxN}^*}$.

The H^{∞}-disturbance attenuation problem treated in this paper, is as follows: Find a MOC of the form (2), which when applied to system (1), asymptotically stabilizes the closed-loop system and simultaneously achieves the following design requirement

$$\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_{\infty} \leq \gamma \tag{9}$$

for a given $\gamma \in \mathbf{R}^{+}$, where $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_{\infty}$ is the H$^{\infty}$-norm of the proper stable discrete transfer function $\mathbf{T}_{\mathbf{qy}_c}(z)$, from sampled-data external disturbances $\mathbf{q}(kT_0) \in \ell_2^d$ to sampled-data controlled outputs $\mathbf{T}_{\mathbf{qy}_c}(z)$, defined by

$$\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_{\infty} = \sup_{\mathbf{q}(kT_0)\in l_2} \frac{\left\|\mathbf{y}_c(kT_0)\right\|_2}{\left\|\mathbf{q}(kT_0)\right\|_2} = \sup_{\theta\in[0,2\pi]} \sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}\left(e^{j\theta}\right)\right] = \sup_{|z|=1} \sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right]$$

where, $\sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right]$ is the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$, and where use was made of the standard definition of the ℓ_2-norm of a discrete signal $\mathbf{s}(kT_0)$

$$\left\|\mathbf{s}(kT_0)\right\|_2^2 = \sum_{k=0}^{\infty} \mathbf{s}^T(kT_0)\mathbf{s}(kT_0)$$

Our attention will now be focused on the solution of the above H^{∞}-control problem. To this end, the following assumptions on system (1) are made:

Assumptions:

a) The matrix triplets $(\mathbf{A},\mathbf{B},\mathbf{C})$ and $(\mathbf{A},\mathbf{D},\mathbf{E})$ are stabilizable and detectable.

b) $\operatorname{rank}\begin{bmatrix} A & D \\ C & 0_{\mathrm{p_1xd}} \end{bmatrix} = \mathrm{n+d}, \quad \operatorname{rank}\begin{bmatrix} A & B & D \\ C & 0_{\mathrm{p_1xm}} & 0_{\mathrm{p_1xd}} \end{bmatrix} = \mathrm{n+m+d}$

c) $\mathbf{J}_2^T[\mathbf{E} \quad \mathbf{J}_2] = [\mathbf{0}_{m\times n} \quad \mathbf{I}_{m\times m}]$

d) There is a sampling period T_0, such that the open-loop discrete-time system model in general form becomes

$$\begin{aligned} \mathbf{x}[(k+1)T_0] &= \Phi\mathbf{x}(kT_0) + \hat{\mathbf{B}}\mathbf{u}(kT_0) + \hat{\mathbf{D}}\mathbf{q}(kT_0) \\ \mathbf{y}_c(kT_0) &= \mathbf{E}\mathbf{x}(kT_0) + \mathbf{J}_2\mathbf{u}(kT_0) \end{aligned} \tag{10}$$

where $\Phi = \mathbf{exp}(A\mathrm{T}_0)$, $\left(\hat{B},\hat{D}\right) = \int_0^{\mathrm{T}_0} \mathbf{exp}(A\lambda)(B,D)\mathrm{d}\lambda$

is stabilizable and observable and does not have invariant zeros on the unit circle.

From the above it fellows that the procedure for H^{∞}-disturbance attenuation using MOCs essentially consists in finding for the control law a fictitious state matrix **F**, which equivalently solves the problem and then, either determining the MOC pair $(\mathbf{L}_{\gamma},\mathbf{L}_u)$ or choosing a desired $\mathbf{L}_{\mathbf{u}}$ and determining the $\mathbf{L}_{\gamma}$. As it has been shown in [3], matrix **F** takes the form

$$\mathbf{F} = \left(\mathbf{I} + \hat{\mathbf{B}}^T\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}^T\mathbf{P}\Phi \tag{11}$$

where **P** is an appropriate solution of the following Riccati equation

$$\mathbf{P} = \mathbf{E}^T\mathbf{E} + \Phi^T\mathbf{P}\Phi - \Phi^T\mathbf{P}\hat{\mathbf{B}}\left(\mathbf{I} + \hat{\mathbf{B}}^T\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}\mathbf{P}\Phi + \mathbf{P}\hat{\mathbf{D}}_{\gamma}\left(\mathbf{I} + \hat{\mathbf{D}}_{\gamma}^T\mathbf{P}\hat{\mathbf{D}}_{\gamma}\right)\hat{\mathbf{D}}_{\gamma}^T\mathbf{P} \ , \ \hat{\mathbf{D}}_{\gamma} = \gamma^{-1}\hat{\mathbf{D}} \tag{12}$$

It is to be noted that $\gamma \in R^{+}$, such that $\left\|T_{qy_c}(\mathrm{z})\right\| \geq \gamma$ where $\left\|T_{qy_c}(\mathrm{z})\right\|_{\infty}$ is the H$^{\infty}$-norm of the proper stable discrete transfer function $T_{qy_c}(\mathrm{z})$, from sampled-data external disturbances $q(\mathrm{kT}_o) \in \ell_2^d$ to sampled-data controlled output $y_c(\mathrm{kT_o})$.

Once matrix **F** is obtained the MROC matrices $\mathbf{L}_{\gamma}$ and $\mathbf{L}_{\mathbf{u}}$ (in the case where $\mathbf{L}_{\mathbf{u}}$ is free), can be computed according to the following mathematical expressions

$$\begin{aligned} \mathbf{L}_{\gamma} &= [\mathbf{F} \quad \mathbf{0}_{m\times d}]\tilde{\mathbf{H}} + \Lambda\left(\mathbf{I}_{N^*\times N^*} - [\mathbf{H} \quad \Theta_{\mathbf{q}}]\tilde{\mathbf{H}}\right) \\ \mathbf{L}_{\mathbf{u}} &= \left\{[\mathbf{F} \quad \mathbf{0}_{m\times d}]\tilde{\mathbf{H}} + \Lambda\left(\mathbf{I}_{N^*\times N^*} - [\mathbf{H} \quad \Theta_{\mathbf{q}}]\tilde{\mathbf{H}}\right)\right\}\Theta_{\mathbf{u}} \end{aligned} \tag{13}$$

where $\tilde{\mathbf{H}}\begin{bmatrix}\mathbf{H} & \Theta_{\mathbf{q}}\end{bmatrix} = \mathbf{I}$ and $\mathbf{\Lambda} \in \mathbf{R}^{\mathbf{m x N^*}}$ is an arbitrary specified matrix. In the case where $\mathbf{L}_u = \mathbf{L}_{u,sp}$, we have

$$\mathbf{L}_\gamma = \begin{bmatrix}\mathbf{F} & \mathbf{L}_{\mathbf{u},sp} & \mathbf{0}_{m\times d}\end{bmatrix}\hat{\mathbf{H}} + \Sigma\left(\mathbf{I}_{N^*\times N^*} - \begin{bmatrix}\mathbf{H} & \Theta_{\mathbf{u}} & \Theta_{\mathbf{q}}\end{bmatrix}\hat{\mathbf{H}}\right)$$

where $\hat{\mathbf{H}}\begin{bmatrix}\mathbf{H} & \Theta_{\mathbf{u}} & \Theta_{\mathbf{q}}\end{bmatrix} = \mathbf{I}$ and $\mathbf{\Sigma} \in \mathbf{R}^{\mathbf{m x N^*}}$ is arbitrary.

The resulting closed-loop system matrix $(\mathbf{A}_{\mathbf{cl/d}})$ takes the following general form

$$\mathbf{A}_{\mathbf{cl/d}} = \mathbf{A}_{\mathbf{ol/d}} - \mathbf{B}_{\mathbf{ol/d}}\mathbf{F} \tag{14}$$

where **cl** = closed-loop, **ol** = open-loop and **d** = discrete.

IV. DESIGN AND SIMULATIONS OF OPEN- AND CLOSED-LOOP MODELS OF THE POWER SYSTEM

In the present work, the aforementioned optimal control strategy is used to design a desirable excitation controller of a hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The hydrogenerator system studied [14], is an 117 MVA hydrogenerator unit of the Greek Electric Utility Power System, which supplies power through a step-transformer and a transmission line to an infinite grid.

The numerical values of the parameters, which define the total system as well as its operating point, come from [14] and are given in Appendix A.

Based on the state variables Fig. 1 and the values of the parameters and the operating point (see Appendix A), the system of Fig. 1 may be described in state-space form, in the form of system 3, where

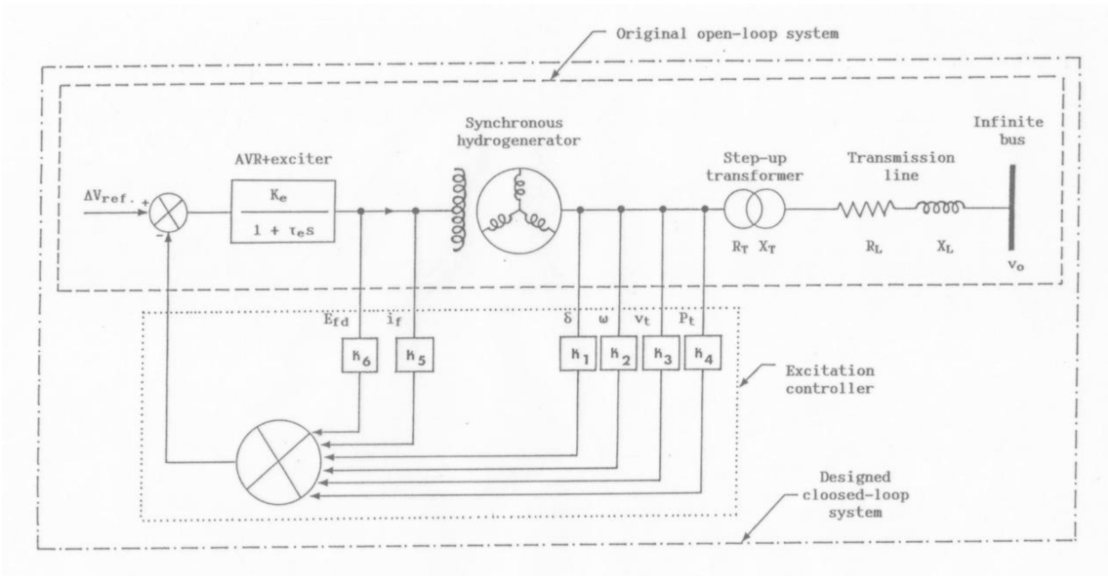


Fig. 1. Simplified representation of hydrogenerator system supplying power by an infinite grid.

$$\mathbf{x} = \begin{bmatrix}\Delta\delta & \Delta\omega & \Delta v_t & \Delta P_t & \Delta i_f & \Delta E_{fd}\end{bmatrix}^T,$$

$$\text{u}=\Delta\text{V}_{\text{ref.}},\quad \mathbf{y} = \mathbf{x},\quad \text{q=u},\quad \mathbf{y_m} = \mathbf{x},\quad \mathbf{y_c} = \mathbf{x},$$

$$\mathbf{E} = \mathbf{I}_{6x6},\quad \mathbf{J}_1 = \mathbf{0}_{6x1},\quad \mathbf{J}_2 = \mathbf{0}_{6x1}.$$

The matrices **A**, **B**, **C** and **D** are given in Appendix B.

The eigenvalues of the original continuous open-loop power system models and the simulated responses of the output variables $\left(\Delta\delta, \Delta\omega, \Delta v_t, \Delta P_t, \Delta i_f, \Delta E_{fd}\right)$, are shown in Table 1 and Fig. 2, rispectivly.

Table 1. Eigenvalues of original open-loop power system models.

Original open-loop power system model	λ	-25.6139 0.0931+7.7898i 0.0931-7.7898i -8.1191+6.2036i -8.1191-6.2036i -6.4021

As it can be easily checked the above linear state space model is unstable, since matrix **A** has two unstable complex eigenvalues at $\lambda_{1,2}$=0.0931±j7.7898.

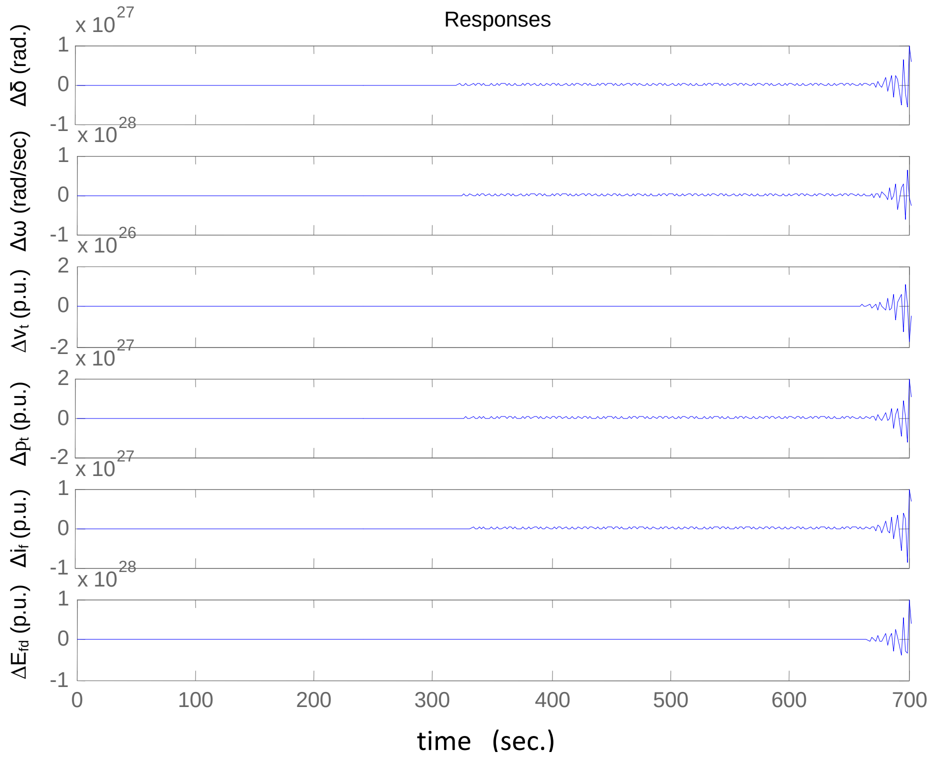


Fig. 2. Responses of the output variables of the original continuous open-loop power system models to step input change: $\Delta V_{ref.}$=0.05 p.u.

The computed discrete linear open-loop power system model, based on the associated linearized continuous open-loop system model described in Appendix B, is given below in terms of its matrices with sampling period T_0 = 1.0 sec.

$$\mathbf{A}_{ol/d} = \begin{bmatrix} -0.5757 & 0.1325 & -1.6853 & 0.2743 & -0.0556 & -0.0005 \\ -13.9929 & -0.0004 & -15.0928 & 3.2806 & -1.6663 & -0.0703 \\ 0.2546 & -0.0197 & 0.4331 & -0.0805 & 0.0284 & 0.0009 \\ -0.1185 & 0.2090 & -1.8064 & 0.2483 & 0.0065 & 0.0032 \\ 0.3721 & 0.1108 & -0.4875 & 0.0289 & 0.0553 & 0.0039 \\ -7.4355 & 1.0654 & -16.5864 & 2.8682 & -0.7774 & -0.0176 \end{bmatrix}$$

$$\mathbf{B}_{ol/d} = \begin{bmatrix} -0.4208 & -0.4652 & 0.7994 & 0.6004 & 2.6836 & 10.9119 \end{bmatrix}^T$$

$$\mathbf{C}_{ol/d} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D}_{ol/d}=\mathbf{B}_{ol/d}$$

The H^∞-control using MOCs, the computed discrete linear open-loop model of the power system under study, and the two discrete closed-loop power system models were designed considering two distinct cases:

a) with γ =10.5, the $\mathbf{L_u}$ and $\mathbf{L_\gamma}$ feedback gain matrices were computed as

$$\mathbf{L_u} = 0.00000256$$
$$\mathbf{L}_\gamma = \begin{bmatrix} -0.1583 & 0.0897 & -0.8912 & 0.1314 & -0.0100 & 0.0009 \end{bmatrix}$$

and

b) with γ=5.5, the associated $\mathbf{L_u}$ and $\mathbf{L}_\gamma$ feedback gain matrices were computed as

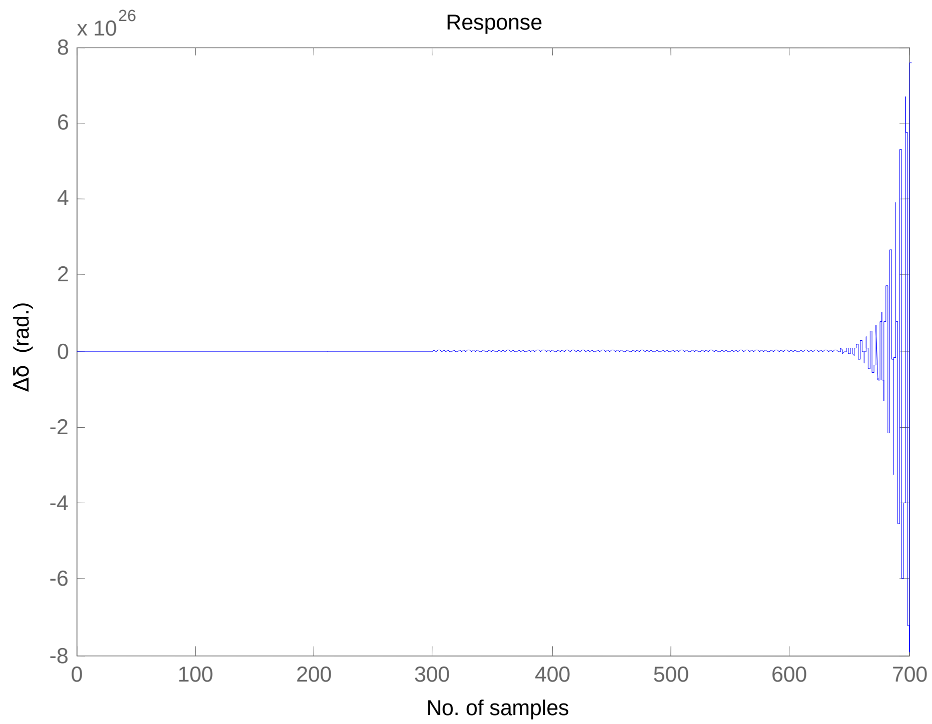
$$\mathbf{L_u} = 0.00000029$$
$$\mathbf{L}_\gamma = \begin{bmatrix} -0.1248 & 0.0928 & -0.8803 & 0.1269 & -0.0056 & 0.0011 \end{bmatrix}$$

The numerical values of the matrices referring to the discrete closed-loop power system models of the above two cases are not included here due to space limitations.
The magnitude of the eigenvalues of the discrete original open-loop and designed closed-loop power system models are shown in Table 2.

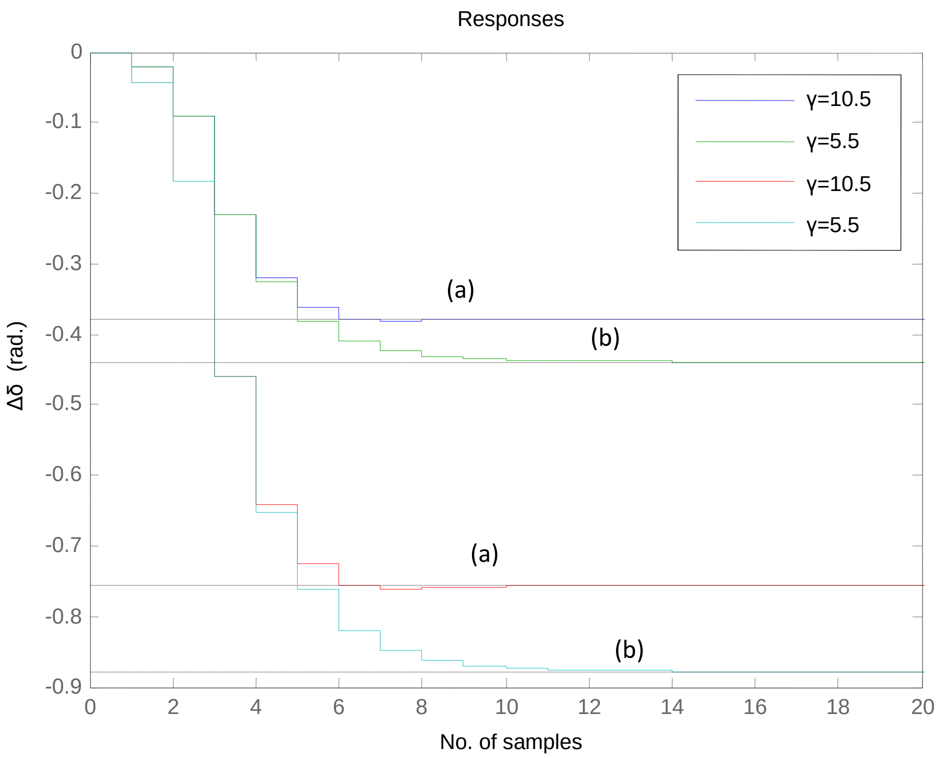
Table 2. Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.

Original open-loop power system model		$\lvert\lambda\rvert$	1.0976 1.0976 0.0017 0.0 0.0003 0.0003
Designed closed-loop power system model	with γ=10.5	$\lvert\hat{\lambda}\rvert$	0.4426 0.4426 0.0053 0.0 0.0003 0.0003
	with γ=5.5	$\lvert\hat{\lambda}\rvert$	0.4933 0.2626 0.0081 0.0 0.0003 0.0003

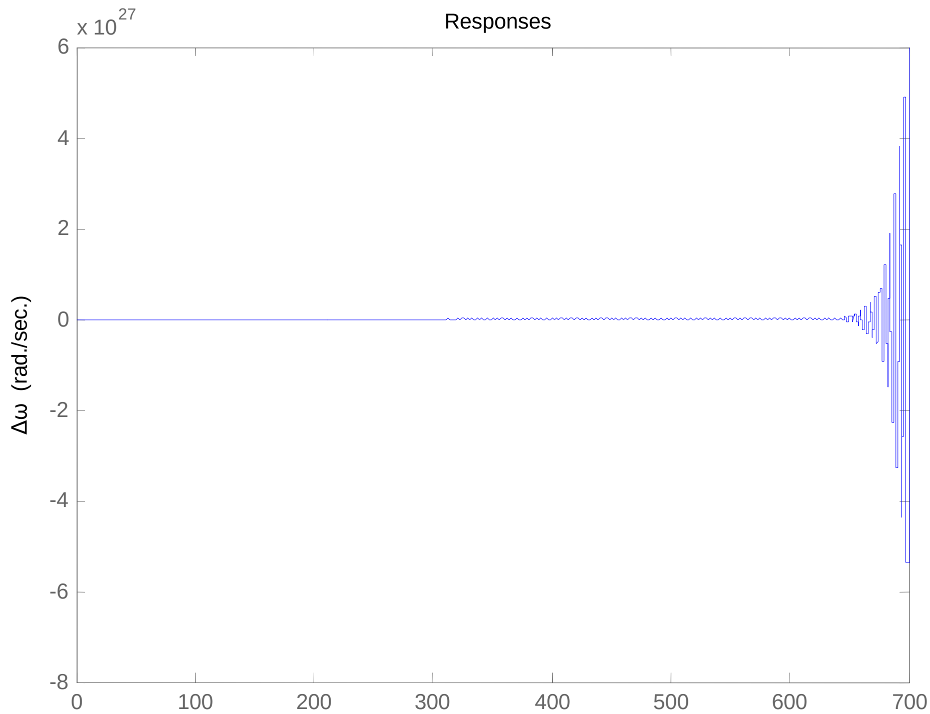
By comparing the eigenvalues of the designed closed-loop power system models to those of the original open-loop power system model the resulting enhancement in dynamic system stability is judged as being remarkable.
The responses of the output variables ($\Delta\delta$, $\Delta\omega$, Δv_t, ΔP_t, Δi_f, ΔE_{fd}) of the original open-loop and designed closed-loop power system models for zero initial conditions and unit step input disturbance are shown in Figs. 3, respectively.



(A)

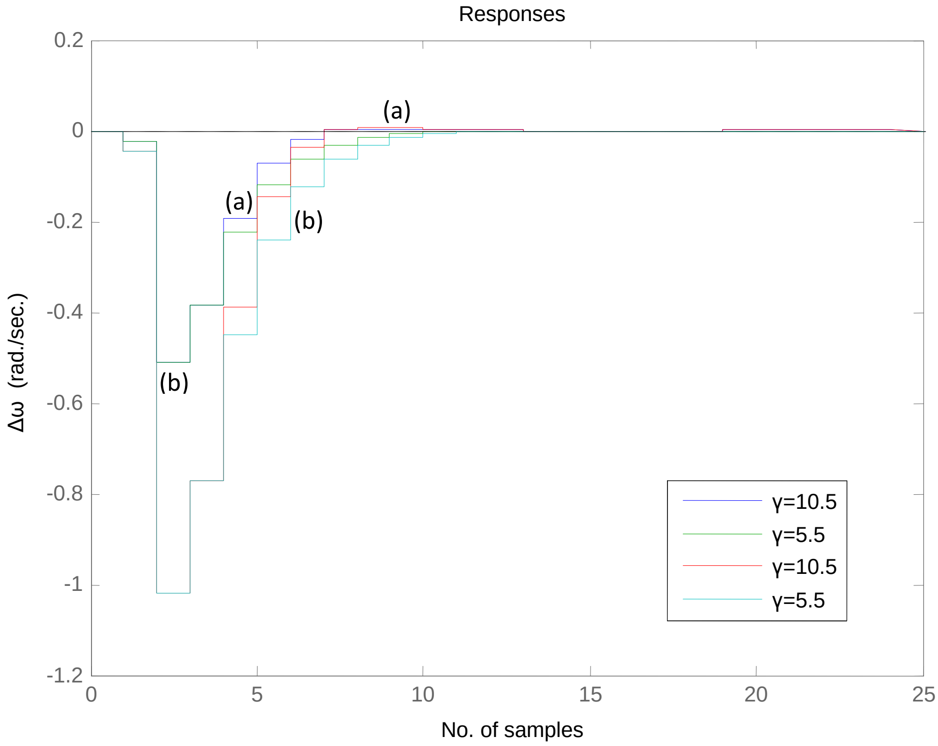


(B)

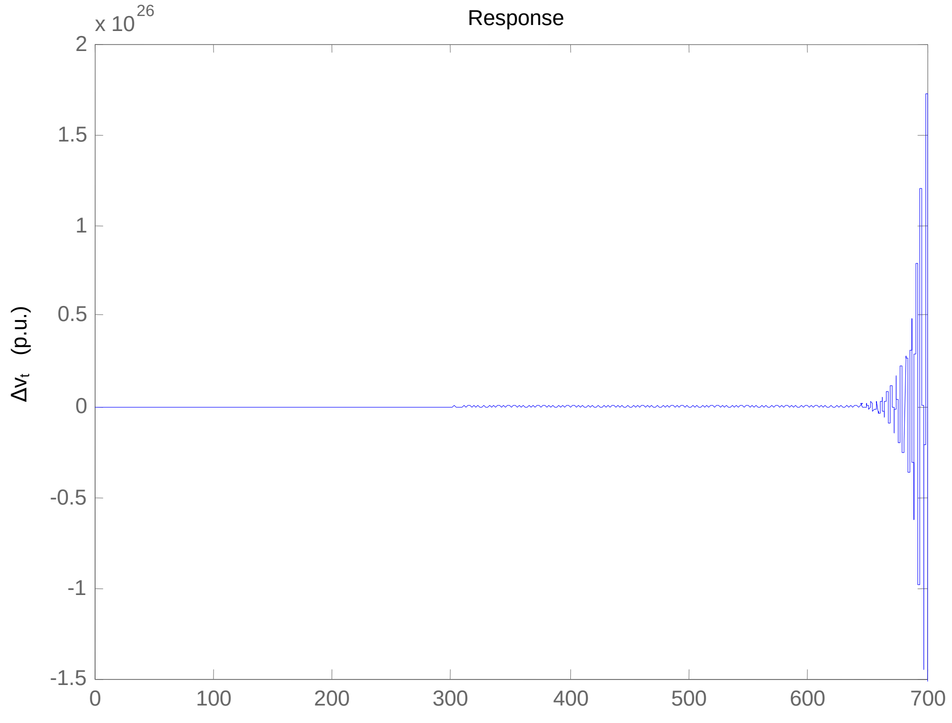


No. of samples

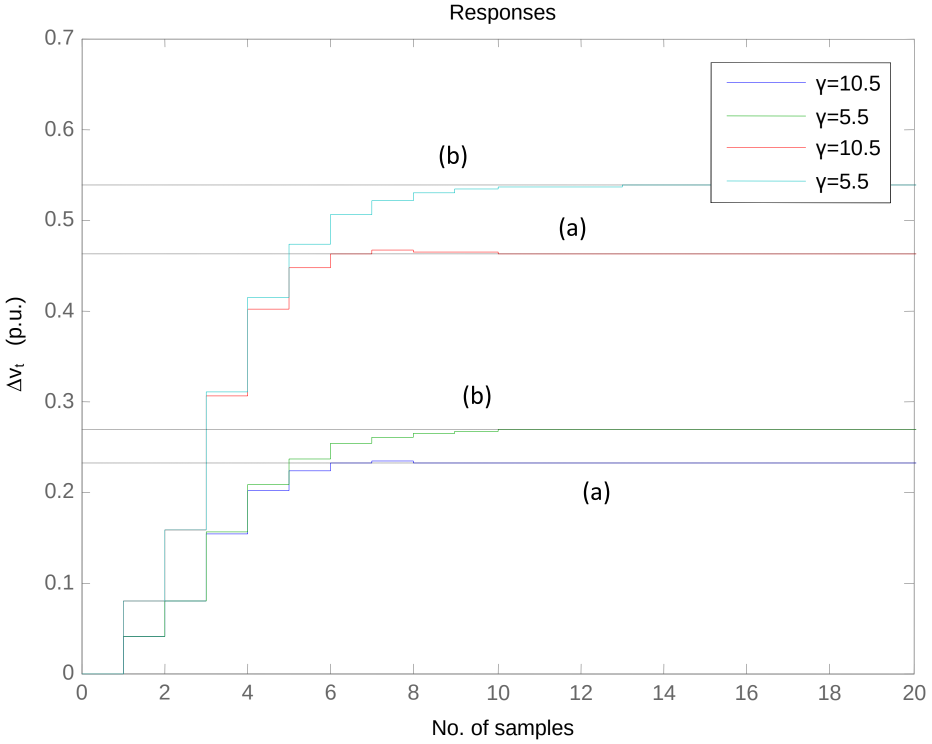
(A)

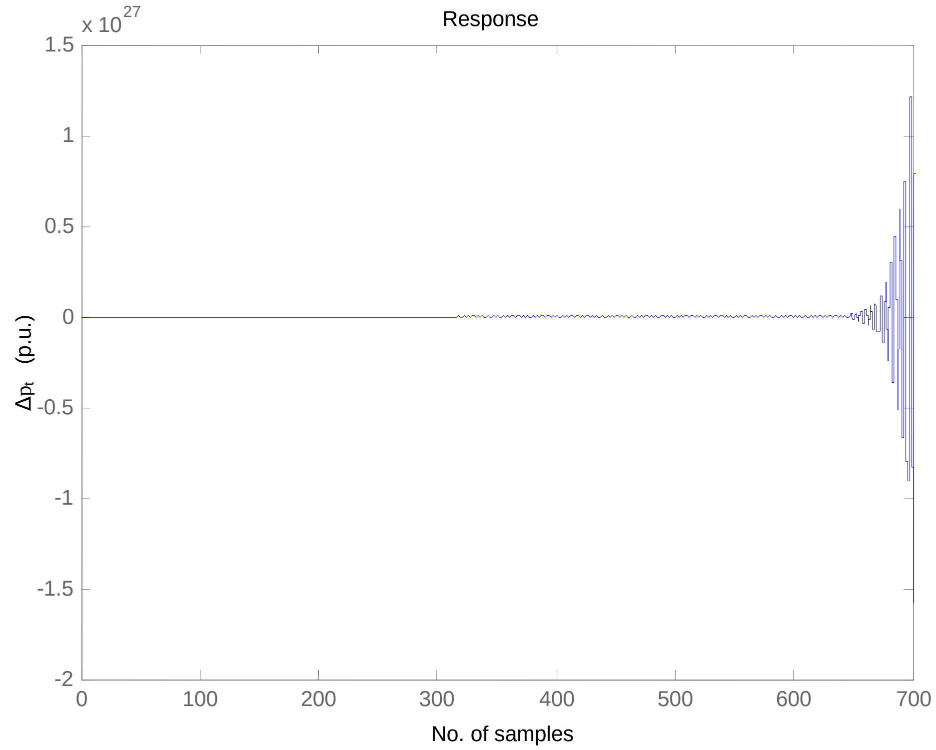

Responses
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
5
10
15
20
25
(a)
(a)
(b)
(b)
Δω (rad./sec.)
γ=10.5
γ=5.5
γ=10.5
γ=5.5
No. of samples

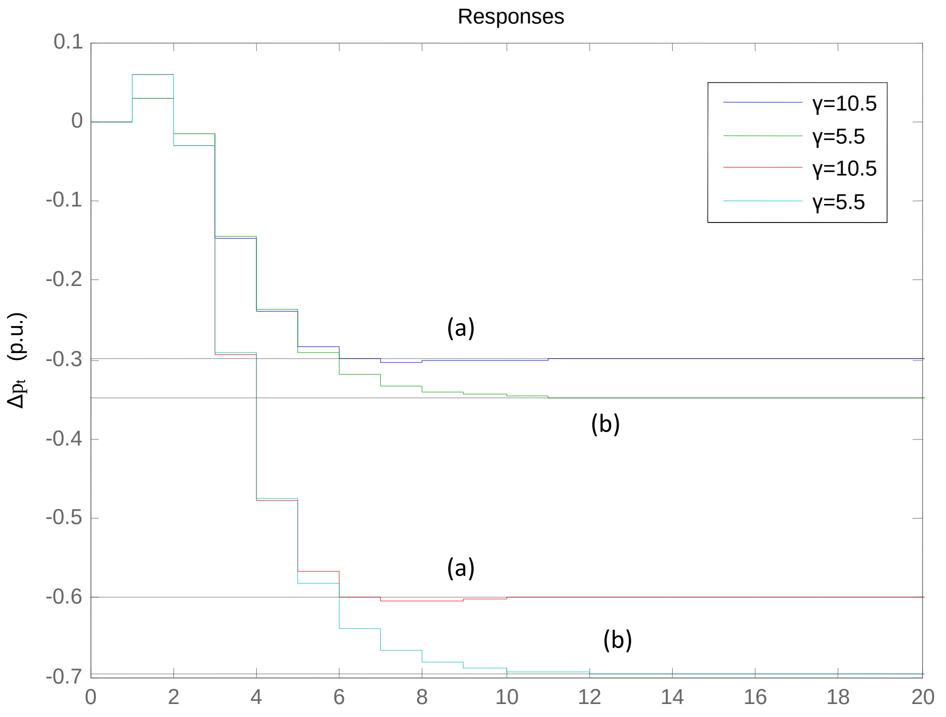
(B)


Response
x 10^26
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
100
200
300
400
500
600
700
Δvt (p.u.)
No. of samples

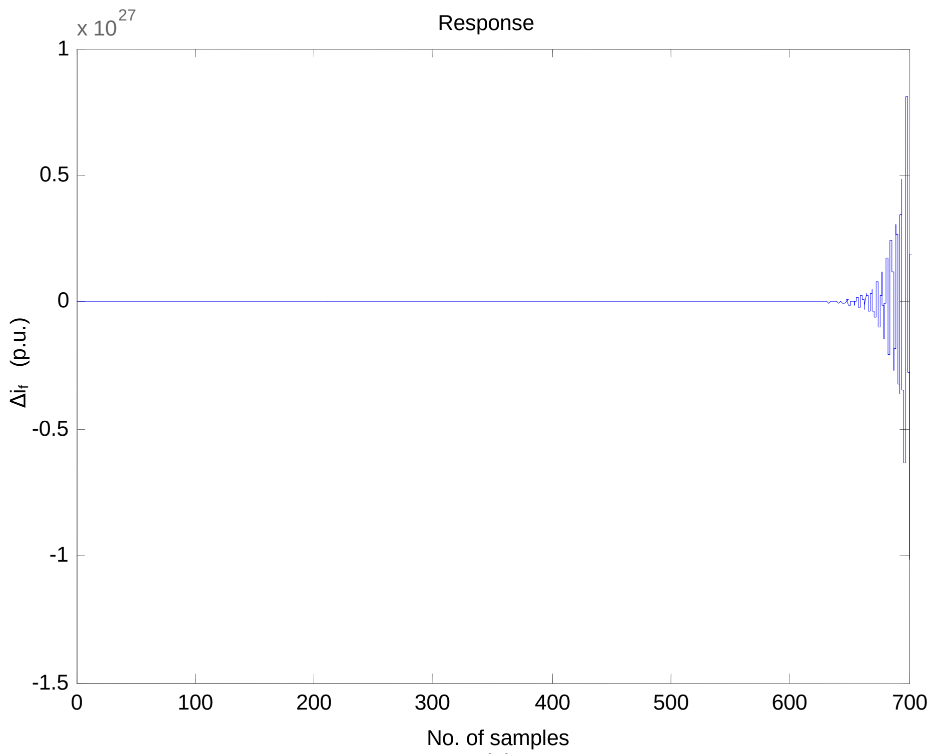
(A)


Responses
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Δvt (p.u.)
γ=10.5
γ=5.5
γ=10.5
γ=5.5
(b)
(a)
(b)
(a)
0 2 4 6 8 10 12 14 16 18 20
No. of samples
(B)

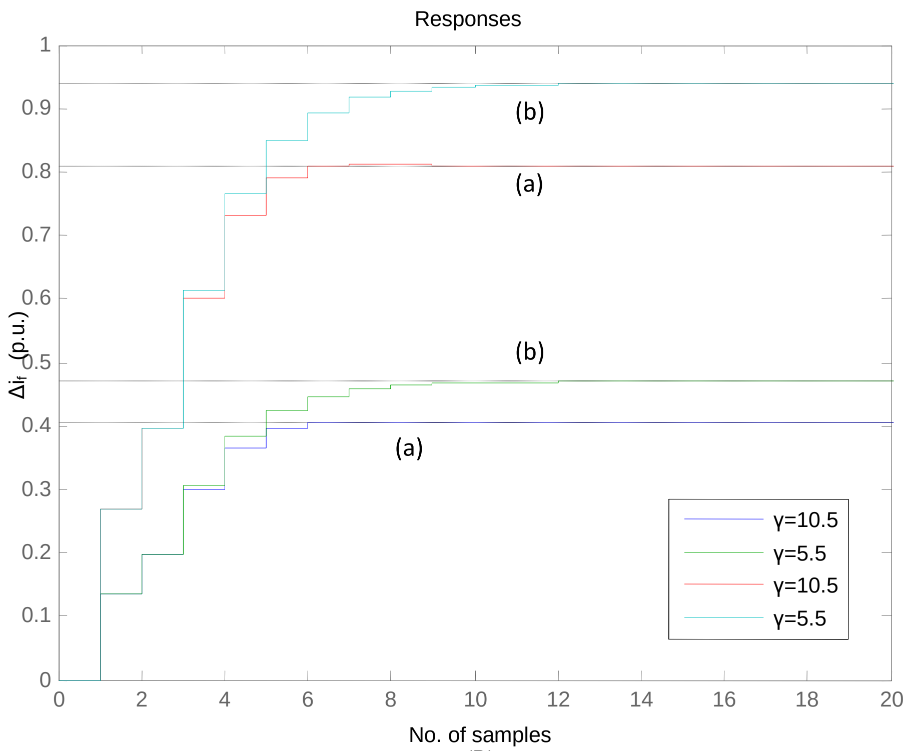

Response
x 10^27
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Δpt (p.u.)
0 100 200 300 400 500 600 700
No. of samples
(A)


Responses
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
Δpt (p.u.)
γ=10.5
γ=5.5
γ=10.5
γ=5.5
(a)
(b)
(a)
(b)
0 2 4 6 8 10 12 14 16 18 20
No. of samples

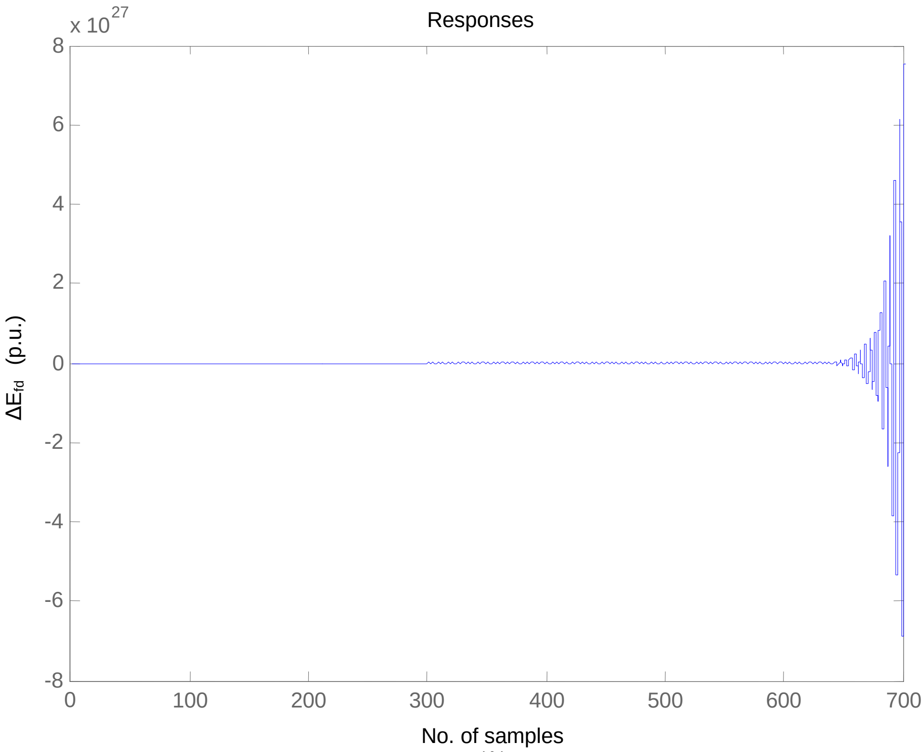
(B)


Response
x 10^27
1
0.5
0
-0.5
-1
-1.5
Δif (p.u.)
0 100 200 300 400 500 600 700
No. of samples

(A)


Responses
(b)
(a)
(b)
(a)
γ=10.5
γ=5.5
γ=10.5
γ=5.5
Δi_f (p.u.)
No. of samples

(B)


x 10^27
Responses
ΔE_fd (p.u.)
No. of samples

(A)

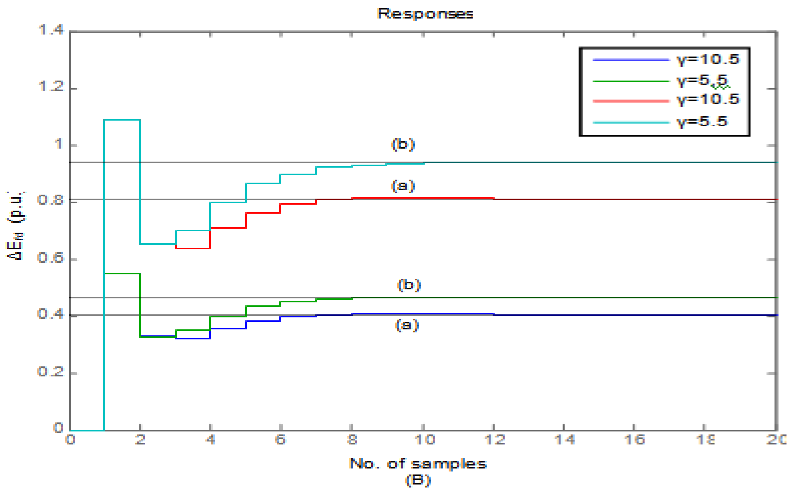


Fig. 3. Responses of Δδ, Δω, Δv_t, ΔP_t, Δi_f, ΔE_{fd}, of:

(A): discrete open-loop system model subject to step input changes $\Delta V_{ref.}$=0.05 p.u.
(B): designed discrete closed-loop model:
 (a): with γ=10.5 and to step input changes, $\Delta V_{ref.}$=10.5 p.u. & $\Delta V_{ref.}$=0.10 p.u. respectively.
 (b): with γ=5.5 and to step input changes, $\Delta V_{ref.}$=0.05 p.u. & $\Delta V_{ref.}$=0.10 p.u. respectively.

From Figs. 3 it is clear that the dynamic stability characteristics of the designed discrete closed-loop system-models are far more superior than the corresponding ones of the original open-loop model, which attests in favour of the proposed H^{∞}-control technique.
It is to be noted that the solution results of the discrete system models , i.e. eigenvalues, eigenvectors, responses of system variables etc., for zero initial conditions were obtained using a special software program, which is based on the theory of & II and runs on MATLAB program environment.

In Fig. 4 , the maximum singular value of $T_{qy_c}(z)$ is depicted, as a function of the frequency ω.

Clearly, the design requirement $\left\| T_{qy_c}(z)_{\infty} \right\| \geq 10.5$, is satisfied. Moreover, as it can be easily checked the poles of the closed loop system, (see, Table 2), lie inside the unit circle. Therefore, the requirement for the stability of the closed-loop system is also satisfied.
Not that, the H^{∞}-norm of the open-loop system transfer function between disturbances and controlled outputs has the value $\left\| \mathbf{C}(j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \right\|_{\infty} = 479$.

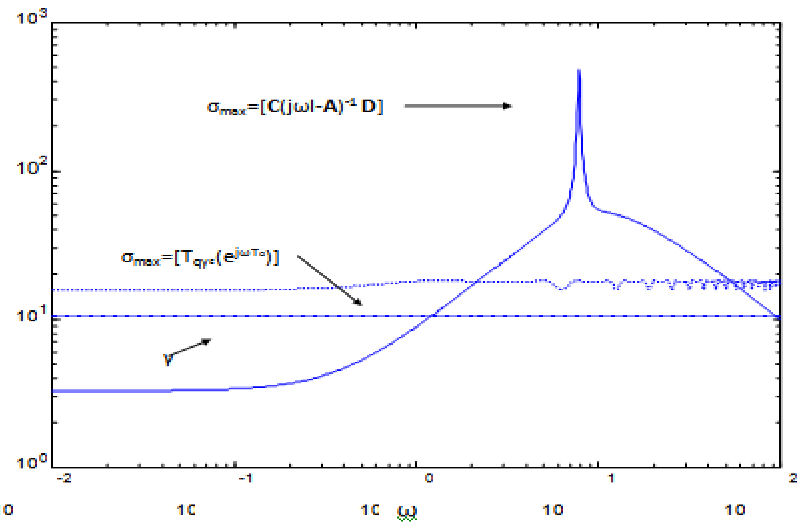


Fig. 4. The maximum singular value of $T_{qy_c}(z)$ over ω, for the unsaturated machine and for γ=10.5

In Fig. 5 , the maximum singular value of $T_{qy_c}(z)$ is depicted, as a function of the frequency ω.

Clearly, the design requirement $\left\| T_{qy_c}(z)_\infty \right\| \geq 5.5$, is satisfied. Moreover, as it can be easily checked the poles of the closed loop system, (see, Table 2), lie inside the unit circle. Therefore, the requirement for the stability of the closed-loop system is also satisfied.

Not that, the H$^\infty$-norm of the open-loop system transfer function between disturbances and controlled outputs has the value $\left\| \mathbf{C}(j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} \right\|_\infty = 479$.

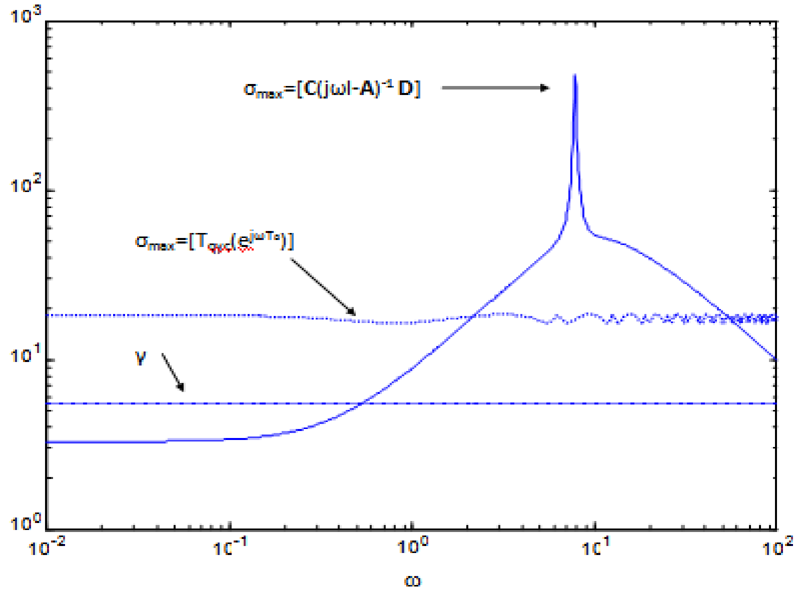


Fig. 5. The maximum singular value of $T_{qy_c}(z)$ over ω, for the unsaturated machine for γ=5.5

V. CONCLUSIONS

The method, H$^\infty$ -control was applied successfully to a discrete open-loop power system model, which was computed from an original continuous linearized open-loop one, resulting in the design of an associated discrete closed-loop power system model. The results of the simulations performed on the discrete open- and closed-loop power system models demonstrated clearly the significant enhancement of the dynamic stability characteristics achieved by the designed closed-loop model. Thus this H$^\infty$ -control technique was proved to be a reliable tool for the design of implementable MOCs. Moreover, it has been shown that the control effort in attenuating disturbances is decreased if the sampling period related to the multirate mechanism is increased and vice versa.

REFERENCES

[1] I. Yaesh and U. Shaked, *Minimum H$^\infty$-norm regulation of linear discrete-time systems and its relation to linear quadratic discrete games*, IEEE Trans. Autom. Control, AC-35, 1990, 1061-1064.

[2] J. C, Doyle, K.Glover, P. P. Khargonekar and B.A.Francis, *State space solutions to standard H^2 and H$^\infty$-control problems,* IEEE Trans. Autom. Control, AC-34, 1989, 831-847.

[3] P. Iglesias and K. Glover, *State-space approach to discrete-time H$^\infty$-control*, Int. J. Control, 54, 1991, 1031-1074.

[4] A. A .Stoorvogel, *The H$^\infty$-Control Problem.(* Englewood Cliffs, NJ: Prentice-Hall, 1992).

[5] A. A. Stoorvogel, *The discrete time H$^\infty$-control problem with measurement feedback*, SIAM J. Control Optim., 30, 1992, 180-202.

[6] T. Chen and L. Qiu, *H$^\infty$ design of general multirate sampled-data control systems,* Automatica, 30(7), 1994, 1139-1152.

[7] T. Chen and B. A. Francis, H$^\infty$-optimal sample-data control: Cmputation and design, Automatica, 32(2), 1996, 223-228.

[8] Mats F. Sagfors, H. T. Toivonem and B. Lennartsons, *H$^\infty$ control of multirate sampled-data systems: A state-space approach,* Automatica, 34, 1998, 415-428.

[9] H. Katayama and A. Ichikawa, H^{∞}-control with output feedback for time-varying discrete systems, International Journal of Control, 63(6), 1996, 1167-1178.

[10] K. G. Arvanitis and P. N. Paraskevopoulos, *Sampled-data minimum H^{∞}-norm regulation of linear continuous-time systems using multirate-output controllers*, J. Optim.Theory Appls., 87, 1995, 235-267.

[11] A.M. Azad and T. Hesketh, H^{∞} -optimal control of multirate sampled-data systems,Proc. Of the American control conference, 1, 2002, 459-464.

[12] M.Cimino and P. R. Pagilla, Design of linear time-invariant controllers for multirate systems, Automatica, 46, Issue 8, 2010, 1315-1319.

[13] E. J. Davison and N. S. Rau, *The optimal output feedback control of a synchronous machine*, IEEE Trans. Power App. Syst., PAS-90, 1971, 2123-2134. .

[14] A. K. Boglou and D.P. Papadopoulos, Dynamic Performance Improvement of Hydrogenerator With Modern Pole-Assignment Control Methods, Journal Electrical Engineering, 46(3), 1995, 81-89.

Appendix A

Numerical values of the system parameters and the operating point (p.u. values on generator ratings).

Hydrogenerator: 117MVA, kV=15.75, RPM=125, H=3.0, x_d=0.935 p.u., x_q=0.574 p.u., x_D=0.992 p.u., x_Q=0.551p.u., x_f=0.221 p.u., i_q=0.665 p.u., i_d=0.746 p.u., v_q=0.924 p.u., v_d=0.381 p.u.
External system: R_e=0.015 p.u., X_e=0.40 p.u. (on a 117MVA base).
Operating point: v_{to}=1.0 p.u., P_{t0}=1.1 p.u., Q_{t0}=0.5 p.u., δ_{nom} =0.9604 rad., ω_{nom}=100π rad./sec, $i_{fnom.}$=1.9634 p.u., $E_{fdnom.}$=1.7720 p.u.

Appendix B

Numerical values of matrices **A**, **B**, **C** and **D** of the original continuous 6th-order system

$$\mathbf{A}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -150.5484 & 0 & -196.0696 & 38.49705 & 0 & 0 \\ -2.5353 & -0.1258 & -7.9768 & -0.0193 & 2.1392 & 0.0401 \\ 11.4595 & 1.3822 & 0.2009 & -7.9565 & 4.6095 & 0.0865 \\ 26.2471 & 0.2898 & 55.2988 & -5.9203 & -12.1345 & 0.6411 \\ 0 & 0 & -1000 & 0 & 0 & -20 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix}^T$$

$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{D}=\mathbf{B}$$