

PAPR Reduction in OFDM Systems Using PTS: With New Phase Sequences

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Abstract: - Along with the advantages of OFDM signals like high spectral efficiency and robustness against ISI, main drawback of OFDM signals is high PAPR. Many techniques are proposed to reduce PAPR including amplitude clipping, clipping and filtering, coding, tone reservation (TR), tone injection (TI), active constellation extension (ACE), partial transmit sequence (PTS), selected mapping (SLM) and interleaving. One of the effective methods, Partial Transmit Sequence is described in this article. In the conventional PTS (C-PTS) several inverse fast Fourier transform (IFFT) operations and complicated calculations to obtain optimum phase sequence, increase computational complexity of C-PTS. But in new proposed phase sequence the number of IFFT operations is reduced to half at the expense of a slight PAPR degradation. In this paper, a technique is proposed to reduce the number of IFFT operations to half at the expense of a slight PAPR degradation. Simulations are performed in Matlab with QPSK modulation with OFDM signal.

Keywords: - Bit error rate, computational complexity, Multi-carrier signal, PAPR Reduction, PTS

I. INTRODUCTION

Multicarrier transmission, also known as orthogonal frequency-division multiplexing (OFDM) or discrete multi tone (DMT), is a technique with a long history that has recently seen rising popularity in wireless and wireline applications. The recent interest in this technique is mainly due to the recent advances in digital signal processing technology. International standards making use of OFDM for high-speed wireless communications are already established or being established by IEEE 802.11, IEEE 802.16, IEEE 802.20, and European Telecommunications Standards Institute (ETSI) Broadcast Radio Access Network (BRAN) committees.

For wireless applications, an OFDM-based system can be of interest because it provides greater immunity to multipath fading and impulse noise, and eliminates the need for equalizers, while efficient hardware implementation can be realized using fast Fourier transform (FFT) techniques. One of the major drawbacks of multicarrier transmission is the peak-to-average power ratio (PAPR) of transmit signal. If the peak transmit power is limited by either regulatory or application constraints, the effect is to reduce average power allowed under multicarrier transmission relative to that under constant power modulation techniques. This in turn reduces range of multicarrier transmission. Moreover, to prevent spectral growth of the multicarrier signal in the form of intermodulation among subcarriers and out-of-band radiation, the transmit power amplifier must be operated in its linear region (i.e. with a large input backoff), where the power conversion is inefficient. This may have deleterious effect on battery lifetime in mobile applications. In many low-cost applications, drawback of high PAPR may outweigh all potential benefits of multicarrier transmission systems. A number of approaches have been proposed to deal with PAPR problem. These techniques include amplitude clipping [1], clipping and filtering [2, 3], coding [4], tone reservation (TR) [5], tone injection (TI) [5], active constellation extension (ACE) [6], and multiple signal representation techniques such as partial transmit sequence (PTS) [8-11], selected mapping (SLM) [12-13] and interleaving [15]. These techniques achieve PAPR reduction at the expense of transmit signal power increase, bit error rate (BER) increase, data rate loss, computational complexity increase, and so on. In this article some important PAPR reduction techniques for multicarrier transmission are described.

II. PAPR OF A MULTICARRIER SIGNAL

A multicarrier signal is the sum of many independent signals modulated onto subchannels of equal bandwidth. Let us denote collection of all data symbols X_n , $n = 0, 1, \dots, N-1$, as a vector $X = [X_0, X_1, \dots, X_{N-1}]^T$ that will be termed a data block. The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n \Delta f t}, \quad 0 \leq t \leq T \quad (1)$$

where $j = \sqrt{-1}$, Δf is subcarrier spacing and NT denotes useful data block period. In OFDM subcarriers are chosen to be orthogonal (i.e., $\Delta f = 1/NT$). PAPR of transmit signal is:

$$PAPR = \frac{\max_{0 \leq t < NT} |x(t)|^2}{\frac{1}{NT} \int_0^{NT} |x(t)|^2 dt} \quad (2)$$

In remaining part of this article, approximation will be made in that only NL equidistant samples of $x(t)$ will be considered where L is an integer that is larger than or equal to 1. These " L -times oversampled" time-domain signal samples are represented as vector $x = [x_0, x_1, \dots, x_{NL-1}]^T$ and obtained as:

$$x_k = x(k.T/L) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n k T/L} \quad (3)$$

It can be seen that the sequence $\{x_k\}$ can be interpreted as the inverse discrete Fourier transform (IDFT) of data block X with $(L-1)N$ zero padding. It is well known that the PAPR of the continuous-time signal cannot be obtained precisely by the use of Nyquist rate sampling, which corresponds to the case of $L = 1$. It is shown in [15] that $L = 4$ can provide sufficiently accurate PAPR results. The PAPR computed from the L times oversampled time domain signal samples is given by:

$$PAPR = \frac{\max_{0 \leq k < NL-1} |x_k|^2}{E[|x_k|^2]} \quad (4)$$

where $E[\cdot]$ denotes expectation.

III. PAPR REDUCTION METHODOLOGIES

As discussed in introduction there are a number of techniques developed for PAPR reduction. The conventional PTS (C-PTS) is a promising technique that can reduce the PAPR, while its complexity increases rapidly as the number of subblocks increases.

3.1 Partial Transmit Sequence (PTS)

3.1.1 Conventional PTS (C-PTS)

Let X denote random input signal in frequency domain with length N . X is partitioned into V disjoint sub blocks $X_v = [X_{v,0}, X_{v,1}, \dots, X_{v,N-1}]^T$, $v=1, 2, \dots, V$ such that $\sum_{v=1}^V X_v = X$ and then these sub blocks are combined to minimize the

PAPR in time domain [8]. The sub block partition is based on interleaving in which the computational complexity is less compared to adjacent and Pseudo-random, however it has the worst PAPR performance among them.

By applying the phase rotation factor $b_v = e^{j\phi_v}$, $v = 1, 2, \dots, V$ to the IFFT of the v th subblock X_v , the time domain signal after combining is given by:

$$x'(b) = \sum_{v=1}^V b_v x_v \quad (7)$$

Where $x'(b) = [x'_0(b), x'_1(b), \dots, x'_{NL-1}(b)]^T$ and L is the oversampling factor. The objective is to find the optimum signal $x'(b)$ with lowest PAPR.

Both b and x can be shown in matrix form as follows:

$$b = \begin{bmatrix} b_1 & b_1 & \dots & b_1 \\ \vdots & \vdots & & \vdots \\ b_v & b_v & \dots & b_v \end{bmatrix}_{V \times N} \quad (8)$$

$$x = \begin{bmatrix} x_{1,0}, & x_{1,1}, & \dots & x_{1,NL-1} \\ \vdots & \vdots & & \vdots \\ x_{v,0}, & x_{v,1}, & \dots & x_{v,NL-1} \end{bmatrix}_{V \times NL} \tag{9}$$

It should be noted that all elements of each row of matrix b are of same values and this is in accordance with C-PTS method. It should be noted that in order to have exact PAPR calculation, at least 4 times oversampling is necessary. As oversampling of x , add zeros to vector, hence number of phase sequence to multiply to matrix x will remain same. Now, the process is performed by choosing optimization parameter \tilde{b} with following condition:

$$\tilde{b} = \arg \min \left(\max_{0 \leq k \leq NL-1} \left| \sum_{v=1}^V b_v x_v \right| \right) \tag{10}$$

After finding the optimum \tilde{b} then the optimum signal is transmitted to the next block. For finding optimum \tilde{b} , we should perform exhaustive search for $(V-1)$ phase factors since one phase factor can remain fixed, $b_1=1$. Hence to find optimum phase factor, W^{V-1} iteration should be performed, where W is the number of allowed phase factors.

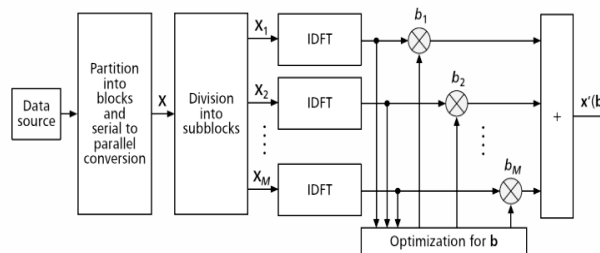


Fig. 1: Block diagram of PTS technique

3.1.2 Proposed Phase Sequence

A new phase sequence is proposed in [9], in order to decrease computational complexity of the C-PTS technique. As mentioned in [12], total complexity of C-PTS is more than SLM because of high number of iterations for finding optimum phase sequence. With this new phase sequence, we can have less complexity compared to C-PTS while PAPR reduction is almost the same. The new phase sequence is based on the generation of N random values from the possible phase factors $\{1 -1\}$, if we consider the number of allowed phase factor $W=2$. Therefore the new phase sequence can be constructed as follows:

$$\hat{b} = \begin{bmatrix} b_{1,1}, & b_{1,2}, & \dots & b_{1,N} \\ \vdots & \vdots & & \vdots \\ b_{v,1}, & b_{v,2}, & \dots & b_{v,N} \end{bmatrix}_{V \times N} \tag{11}$$

Where N is number of subcarriers and V is number of sub blocks partitioning.

As mentioned in (9), matrix x has $V \times NL$ elements and each row consists of NL elements, after oversampling by L . As oversampling will add zeros to the vector, then after multiplying phase sequence b with x , the only section that counts in the multiplying will be N elements, hence the new phase sequence matrix in (11) still has N rows and the oversampling factor does not have any effect on that.

In (11), N random phase sequence is generated periodically V times. As compared with (8), new phase sequence matrix has N different random values while each row of phase sequence in (9) is with same value. These values are randomly selected from the allowed phase factors. E.g. in case of number of allowed phase factors $W=2$, then phase sequence can be chosen from $\{1, -1\}$ and when $W=4$ then $\{1, j, -1, -j\}$. The time domain signal from the new phase sequence that is based on (7) can be obtained from phase sequence matrix in (11) multiplies point-wise with matrix in (9). In C-PTS, it was mentioned that for searching the optimum phase factor, W^{V-1} iteration is required. It means that, if for example $W=2$ and $V=4$, then we need 8 iterations to find the optimum phase sequence. For the new phase sequence format, the way to find the optimum phase factor will be different. In this case, first N different random phase sequence is generated and this is continued V times according to (11), hence the optimum phase factor is each row of this matrix. But for finding the optimum phase factor, matrix in (11) should be randomly generated several times. We constrain the number of times that the matrix would be generated to be the same as in C-PTS for fair comparison. Hence for the case of $W=2$ and $V=4$, C-PTS has 8 iterations and therefore (11) should be generated 8 times. In this case we have 8 possibilities, because the first bit is fixed, $\{1,1,1\}, \{1,1,-1\}, \{1,-1,1\}, \{1,-1,-1\}, \{-1,1,1\}, \{-1,-1,1\}, \{-1,1,-1\}, \{-1,-1,-1\}$. Optimum phase factor will be chosen from these 8 phase sequences. In our proposed method, because there are

N different random phase factors, to search for the optimum phase sequence it requires N8 iterations which is not practical. But here, we only apply the same iteration as was applied in C-PTS and later it will be shown through simulations, that good PAPR performance is achieved, and it is also possible to have less iteration while keeping the PAPR performance same as C-PTS but with reduced complexity.

Hence the matrix in (11) can be extended as follows:

$$\hat{b} = \begin{bmatrix} b_{1,1} & \dots & b_{1,N} \\ \vdots & \vdots & \vdots \\ b_{v,1} & \dots & b_{v,N} \\ b_{v+1,1} & \dots & b_{v+1,N} \\ \vdots & \vdots & \vdots \\ b_{p-1,1} & \dots & b_{p-1,N} \end{bmatrix}_{[P \times N]} \tag{12}$$

where P is the number of iterations that should be set in accordance with the number of iterations of the C-PTS. The value of P can be calculated as follows:

$$P = DW^{V-1}, D = 1, 2, \dots, D_N \tag{13}$$

where D is coefficient that can be specified based on the PAPR reduction and complexity and D_N is amount that is specified by user. Value of P explicitly depends on the number of subblocks V if assuming the number of allowed phase factor is constant.

There is a tradeoff for choosing the value of D, the higher D leads to higher PAPR reduction but at the expense of higher complexity; while lower D gives smaller PAPR reduction but with less complexity. If W=2 and V=4, then in C-PTS there are 8 iterations and hence P=8D. If D=2 then P=16 and both methods have the same number of iterations. But when D=1 then number of iterations to find the optimum phase factor will be reduced to 4 and this will result in complexity reduction. The main advantage of this method over C-PTS is the reduction of complexity while at the same time maintaining the same PAPR performance.

With the proposed method, this scenario is possible and reason is use of NxP random phase sequence which has more capability to reduce PAPR further, whereas in C-PTS this value is only W^{V-1}.

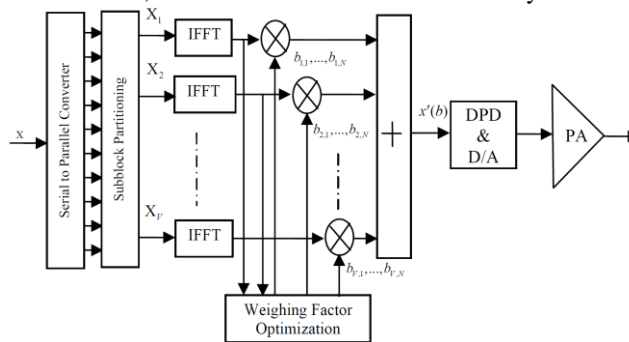


Fig. 2 Block diagram of the proposed PTS technique

The other formats that matrix in (12) can be expressed are the interleaved and adjacent phase sequences matrix respectively. The optimum phase sequence of (12) can be derived by successive search of phase sequence based on the value derived from (13) and then multiplied with input signal according to (9). In this paper random phase sequence matrix as (12) is used because it has the highest PAPR performance among them by considering W=2.

3.1.3 Computational Complexity

The total complexity of the C-PTS when oversampling factor L=1, is given by [14]:

$$T_{C-PTS} = 3VN/2 \log N + 2VW^{V-1}N \tag{14}$$

Whereas for the proposed method this value is as follows:

$$T_{Proposed\ PTS} = 3/4VN \log N + PVN \tag{15}$$

where P is the number of iterations and V is the number of subblocks. It can be observed that (14) and (15) consist of two parts; the first part is actually the complexity of the IFFT itself and the second part is the complexity of the searching algorithm. Most of the papers did not consider the second part which causes miscalculation of the complexity. It should be noted that the number of IFFT in (15) is halved which basically is concluded from the simulation results. From the simulation results in the following section the PAPR performance of the proposed method when number of IFFT is half of the C-PTS, is almost same. This is shown for different number of subblocks which proved that in the proposed method the number of IFFT is halved compared to the C-PTS because it leads to the same PAPR performance.

The computational complexity reduction ratio (CCRR) of the proposed technique over the C-PTS is defined as [5]:

$$CCRR = (1 - \frac{\text{Complexity of the Proposed PTS}}{\text{Complexity of the C-PTS}}) \times 100\% \tag{16}$$

Table 1 below presents the computational complexity of C-PTS and proposed method, for N=512 and W=2.

Table 1 Computational Complexity of DSI-PTS Technique and Conventional PTS When N=512 And W=2

	No. of sub blocks	C-PTS	Proposed PTS		CCRR	
			D=1	D=2	D=1	D=2
Total complexity	V=4	60416	30208	46592	50%	23%
	V=8	1103872	551936	1076224	50%	3%

It is clear that the CCRR is improved for both V=4 and V=8. It should be noted that when D increases, the complexity reduction becomes less while PAPR performance improves, as shown in the simulations.

3.1.4 Side Information

The other important factor in studying the PAPR method is the side information which has to be transmitted to the receiver to extract the original signal. One method is the side information can be transmitted with a separate channel at the expense of spectrum efficiency. The number of required side information bits in C-PTS is $\lceil \log_2 W^{y-1} \rceil$ where W is the number of allowed phase factors and the sign [y] denotes the smallest integer less than y. In the proposed method the required side information will not change, however, the only drawback of this method is that, because of the increase in the phase sequence matrix, higher memory space is required.

IV. RESULTS AND DISCUSSION

In order to evaluate and compare the performance of the proposed method with C-PTS, simulations have been performed as per parameters shown in Table 2.

Table 2 Simulation parameters

Parameter	Value
Modulation	QPSK
OFDM samples	1e5
FFT length 'N'	512
Number of subblocks 'V'	4
Oversampling factor 'L'	1,4
Allowed phase factors 'W'	4 i.e. (1,j,-1,-j)
User defined variable 'D'	1 or 2

Fig. 3 shows the CCDF of three different types of phase sequences, interleaved, adjacent and random. From this figure PAPR reduction with random phase sequence outperforms the other types and hence this type of phase sequence is applied in the following simulations.

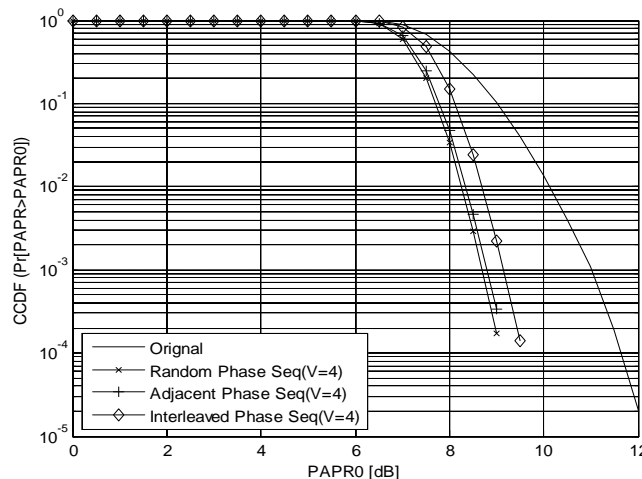


Fig. 3 CCDF of PAPR of the proposed method for different phase sequences when V=4 and D=2

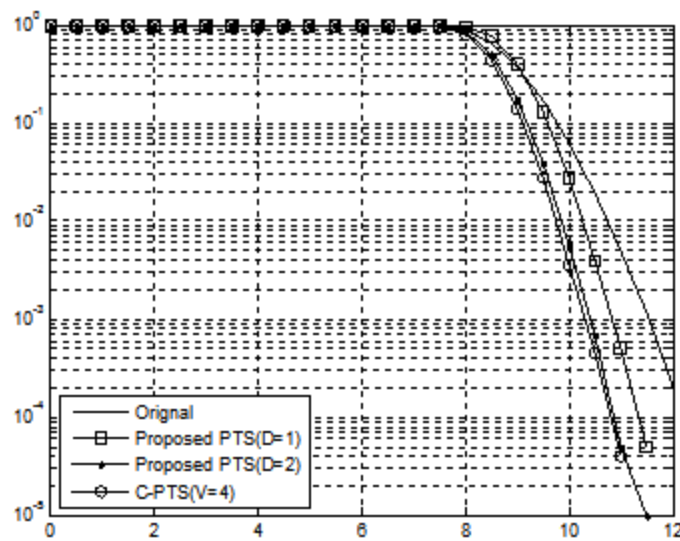


Fig.4 CCDF of PAPR of the proposed method compared to C-PTS for V=4

Fig. 4 shows a comparison of CCDF of the proposed method and C-PTS. It can be observed that the PAPR reduction for the proposed PTS scheme degrades slightly compared to C-PTS even though complexity is improved as shown in Table 1. The simulation is examined for V=4 and 8 and D=1 and 2. It is clear that when D=2 the PAPR reduction is improved and comparable to C-PTS for V=4, but this achievement is at the expense of complexity, nevertheless the CCRR is still positive which shows an improvement over that of C-PTS.

V. CONCLUSION AND FUTURE SCOPE

A new phase sequence of PTS scheme has been proposed in this paper. In this approach matrix of possible random phase factors are first generated and then multiplied point-wise with the input signal. By applying this technique the number of IFFT operation is halved which results in lower complexity compared to C-PTS at the expense of slightly PAPR degradation.

The performance of the out-of-band distortion can be also examined with the existence of nonlinear power amplifier (PA). By applying both PAPR reduction and digital predistortion (DPD) the power spectral density (PSD) of the output signal can further suppressed. This results in enhancement of power efficiency and therefore less power consumption and more battery life. The proposed method can be applied in recent wireless communications systems such as WiMAX and long term evolution (LTE).

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