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Bernoulli-Euler Beam Response to Constant Bi-parametric Elastic Foundation Carrying Moving Distributed Loads

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ABSTRACT: *A study of the dynamic response of Bernoulli-Euler beam to constant bi-parametric elastic foundation to moving distributed forces is presented. The transformed equation governing the system is obtained by means of the Galerkin's technique. The cases of the dynamic response of the beam to distributed loads of equal magnitude are studied. Numerical examples are given in order to determine the effects of various parameters on the response of the simply-supported Bernoulli-Euler beam.*

Keywords- Deflection, foundation modulus, Resonance, Shear force, Simply supported beam.

I. INTRODUCTION

Beams are of great importance in construction engineering. This may be due to their light weight and this has contributed to their widely usage.

The dynamic response of Bernoulli-Euler beam have been extensively studied especially for simply supported beams [1], [2], [3], [4], [5]. In most of the previous works, the problem of assessing the dynamic response of Bernoulli-Euler beam carry moving loads, has been restricted to the case when the loads are simplified as moving concentrated forces [6], [7], [8] and mostly placed on Winkler elastic foundation [9], [10].

The classical Winkler model has various applications in construction engineering and this model suffers several criticisms as it has some shortcomings due to the discontinuity of the adjacent displacement [11]. To tackle this deficiency, a better model that introduced shear interaction between adjacent Winkler spring elements was introduced [12].

Several authors in the area of structural dynamics have thoroughly investigated the dynamics and stability of the Winkler-type foundation model by both approximate methods [13] and exact approaches [14]. In 1991, [15] presented some finite element models for the static analysis of Euler-Bernoulli beam resting on a Winkler-type foundation. In 2010, Omolofe [16] investigated the dynamic response to moving load of an elastically supported non-prismatic Bernoulli-Euler beam on variable elastic foundation and obtained analytical solutions for which the numerical solutions are displayed in plotted curves.

In this paper, the problem of dynamical analysis of Bernoulli-Euler when it is simply supported and resting on constants bi-parametric elastic foundation under moving distributed forces are presented. All the components of inertia terms are considered in the analysis. While section II describes the theory and brief description of the problem under investigation, section III focuses on the technique involved in the transformation of the fourth order partial differential equation of the dynamical system. Some remarks on the analytical solution so obtained are reported in section IV and finally, numerical results are displayed in plotted curves in section V.

II. THEORY AND FORMULATION

The problem of the displacement response of simply-supported Bernoulli-Euler beam resting on constant bi-parametric elastic foundation carry moving distributed loads is governed by the fourth order partial differential equation

$$EI\frac{\partial^4}{\partial x^4}Y(x,t)+\mu\frac{\partial^2}{\partial t^2}Y(x,t)-N\frac{\partial^2 Y(x,t)}{\partial x^2}+Q_m(x)Y(x,t)+MH(x-ct)P_fY(x,t)=P(x,t)$$
(2.1)

where E is the young modulus, $Y(x,t)$ is the transverse displacement, $Q_m(x)$ is the bi-parametric elastic foundation, $P(x,t)$ is the moving load. Also, $H(x,ct)$ is the Heaviside function defined as

$$H(x,ct)=\begin{cases}0, & \text{for } x<0\\ 1, & \text{for } x>0\end{cases} \tag{2.2}$$

In this paper, when the effect of the mass of the moving load on the beam is considered, $P(x,t)$ takes the form

$$P_f(x,t)=MH(x,ct)\left[g-\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x^2}\right)Y(x,t)\right] \tag{2.3}$$

The boundary conditions for our dynamical system are arbitrary and the initial conditions without any loss of generality are taken to be

$$Y(x,0)=0=\frac{\partial}{\partial t}Y(x,0) \tag{2.4}$$

The relationship that exists between the foundation reaction and the lateral deflection $Y(x,t)$ is given by

$$Q_m(x)=GmY(x,t)-\frac{\partial}{\partial x}\left(Km\frac{\partial}{\partial x}Y(x,t)\right) \tag{2.5}$$

where Gm and Km are two constant parameter of elastic foundation. Thus, Gm is the constant foundation stiffness and Km is the variable shear modulus. To this end, equation (2.5) can be written as

$$Q_m(x,t)=GmY(x,t)-K'm\frac{\partial Y(x,t)}{\partial x}-Km\frac{\partial^2 Y(x,t)}{\partial x^2} \tag{2.6}$$

Substituting equations (2.2) to (2.6) into equation (2.1), one obtains

$$EI\frac{\partial^4}{\partial x^4}Y(x,t)+\mu\frac{\partial^2}{\partial t^2}Y(x,t)-N\frac{\partial^2}{\partial x^2}Y(x,t)+GmY(x,t)-\frac{d}{dx}\left[Km\frac{d}{dx}Y(x,t)\right]MH(x-ct)$$
$$\left[\frac{\partial^2}{\partial t^2}Y(x,t)+2C\frac{\partial^2}{\partial x\partial t}Y(x,t)+C^2\frac{\partial}{\partial x^2}Y(x,t)\right]=MgH(x-ct) \tag{2.7}$$

III. MATHERIALS AND METHOD

A close form solution to equation (2.7) does not exist, so the elegant Galerkin's method described in [10] is employed to tackle the fourth order partial differential equation. The method is presented in the form

$$Y_j(x,t)=\sum_{i=1}^{j}W_i(t)U_i(x) \tag{3.1}$$

where $U_i(x)$ is chosen such that the desired boundary conditions are satisfied. And since we are considering simply supported boundary condition, $U_i(x)$ is defined thus,

$$U_i(x)=\frac{\sin i\pi x}{L} \tag{3.2}$$

Substituting equation (3.1) into equation (2.7), taking note of equation (3.2) one obtains

$$\sum_{i=1}^{j}\Big[EIU_i^{IV}(x)W_i(t)+\mu U_i(x)\ddot{W}_i(t)+GmU_i(x)W_i(t)-KmU_i'(x)W_i(t)-KmU_i''(x)W_i(t)$$

$$+M\left(\frac{x}{L}+\frac{2}{i\pi L}\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L}\cos\frac{i\pi t}{L}+C_0\right)\Big[\ddot{W}_i(t)U_i(x)2C\dot{W}_i(t)U_i'(x)+C^2W_i(t)U_i''(x)\Big]\Big]$$

$$=\frac{MgL}{\pi i}\left[\cos\frac{\pi ict}{L}-\cos\pi i\right] \tag{3.3}$$

where $H(x-ct)$ have been defined as

$$H(x-ct)=\frac{x}{L}+\frac{2}{i\pi L}\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L}\cos\frac{i\pi t}{L}+C_0 \tag{3.4}$$

imposing orthogonaliy condition, equation (3.3) after simplification becomes

$$\int_0^L\Bigg\{\sum_{i=1}^{j}\Big[EIW_j(t)U_i^{iv}(x)+\mu\ddot{W}_j(t)U_i^{iv}(x)+GmW_i(t)U_i(x)-KmW_i(t)U_i'(x)-KmW_i(t)U_i''(x)$$

$$+\frac{M}{\mu}\left(\frac{x}{L}+\frac{2}{i\pi L}\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L}\cos\frac{i\pi ct}{L}+C_0\right)\Big[\ddot{W}_i(t)U_i(x)+2C\dot{W}_i(t)U_i'(x)+C^2W_i(t)U_i''(x)\Big]\Big]\Bigg\}$$

$$=U_k(x)\frac{MgL}{\pi i}\left[\cos\frac{\pi ict}{L}-\cos\pi i\right] \tag{3.5}$$

which can be re-written as

$$\sum_{i=1}^{j}\Bigg[\ddot{W}_i(t)H_{i1}(i,k)+H_{i2}\left(\frac{EI}{\mu}H_{i2}(i,k)+\frac{S(x)}{\mu}H_{i3}(i,k)-\frac{K(x)}{\mu}H_{i4}(i,k)-\frac{K(x)}{\mu}H_{i5}(i,k)\right)W_i(t)$$

$$+\frac{M}{\mu L}H_{i6}(i,k)\ddot{W}_i(t)+\frac{2M}{\mu i\pi L}\sum_{i=1}^{\infty}\cos\frac{i\pi ct}{L}H_{i7}(i,k)\ddot{W}_i(t)+\frac{MC_0}{\mu}\ddot{W}_i(t)H_{i8}(i,k)$$

$$+\frac{2cM}{\mu}\dot{W}_i(t)H_{i9}(i,k)+\frac{2M}{\mu i\pi L}\sum_{i=1}^{\infty}\cos\frac{i\pi ct}{L}H_{i10}(i,k)\ddot{W}_i(t)+\frac{C_0 2c}{\mu}\dot{W}_i(t)H_{i11}(i,k)+\frac{Mc^2}{\mu}\dot{W}_i(t)H_{i12}(i,k)$$

$$\frac{2Mc^2}{\mu i\pi L}\sum_{i=1}^{\infty}\cos\frac{i\pi ct}{L}H_{i13}(i,k)+\frac{C_0c^2}{\mu}H_{i14}(i,k)=\frac{MgL}{\pi i}\left[\cos\frac{\pi ict}{L}-\cos\pi i\right] \tag{3.6}$$

where

$$H_{i1}(i,k)=\int_0^L U_k(x)U_i(x)dx \quad ; \quad H_{i2}(i,k)=\int_0^L U_k(x)U_i^{iv}(x)dx$$

$$H_{i3}(i,k)=\int_0^L U_k(x)U_i(x)dx \quad ; \quad H_{i4}(i,k)=\int_0^L U_k(x)U_i'(x)dx$$

$$H_{i5}(i,k)=\int_0^L U_k(x)U_i''(x)dx \quad ; \quad H_{i6}(i,k)=\int_0^L U_k(x)U_i(x)dx$$

$$H_{i7}(i,k)=\int_0^L U_k(x)U_i(x)\sin\frac{i\pi x}{L}dx \quad ; \qquad H_{i8}(i,k)=\int_0^L U_k(x)U_i(x)dx$$

$$H_{i10}(i,k)=\int_0^L U_k(x)U_i'(x)\sin\frac{i\pi x}{L}dx \quad ; \qquad H_{i11}(i,k)=\int_0^L U_k(x)U_i'(x)dx=H_{i4}(i,k)$$

$$H_{i12}(i,k)=\int_0^L U_k(x)U_i''(x)dx \quad ; \qquad H_{i13}(i,k)=\int_0^L U_k(x)U_i''(x)\sin\frac{i\pi x}{L}dx$$

$$H_{i14}(i,k)=\int_0^L U_k(x)U_i''(x)dx \tag{3.7}$$

equation (3.5) when re-arranged gives

$$\ddot{W}_i(t)+\varepsilon_0^2W_i(t)+\Gamma_0\Bigg[\left(H_{i6}(i,k)+\frac{2}{i\pi}\sum_{i=1}^{\infty}\cos\frac{i\pi ct}{L}H_{i7}(i,k)+LC_0H_{i8}(i,k)\right)\ddot{W}_i(t)$$

$$+\left(2CLH_{i9}(i,k)+\frac{2}{i\pi}\sum_{i=1}^{\infty}\cos\frac{i\pi ct}{L}H_{i10}(i,k)+2C_0LH_{i11}(i,k)\right)\dot{W}_i(t)$$

$$+\left(C^2LH_{i12}(i,k)+\frac{2C^2}{i\pi}\sum_{i=1}^{\infty}\cos\frac{i\pi ct}{L}H_{i13}(i,k)+C_0C^2LH_{i14}(i,k)\right)\Bigg]$$

$$=\frac{MgL}{\mu\pi i}\left[\cos\frac{\pi ict}{L}-\cos\pi i\right] \tag{3.8}$$

so that

$$\ddot{W}_i(t)+\varepsilon_0^2W_i(t)+\Gamma_0\left[\frac{L^2}{\pi^2}\left(\frac{1}{(i+k)^2}-\frac{1}{\pi^2(i-k)^2}\right)+\left(\frac{8ikL}{\pi^2\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}\right)\right.$$

$$\left.+\frac{L^2c_o}{2}\right]\ddot{W}_i(t)+\left[2cL+\frac{4i^2\pi(n^2+k^2-i^2)}{\pi^2\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}-\frac{4n^2c_oL}{k^2-n^2}\right]\dot{W}_i(t)$$

$$+\left[c^2n^2L\left(\frac{1}{(n+k)^2}-\frac{1}{(n+k)^2}\right)+\left(\frac{8ikL}{\pi^2\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}\right)+\left(\frac{c^2\pi^2n^2c_o}{2}\right)\right]W_i(t)$$

$$=\frac{MgL}{\mu\pi i}\left[\cos\frac{\pi ict}{L}-\cos\pi i\right] \tag{3.9}$$

Further arrangements of equation (3.9) give

$$\ddot{W}_i(t)+\varepsilon_0^2W_i(t)=\frac{MgL}{H_{i1}(i,k)\mu i\pi}\left[1-(-1)^i+\cos\frac{i\pi ct}{L}\right] \tag{3.10}$$

where

$$\varepsilon_0^2 = \frac{1}{H_{i1}(i,k)}\left[\frac{EI}{\mu}H_{i2}(i,k) + \frac{Gm}{\mu}H_{i3}(i,k) - \frac{Km}{\mu}H_{i4}(i,k) - \frac{Km}{\mu}H_{i5}(i,k)\right] \quad (3.11)$$

and by Laplace methods, one obtains

$$W_i(t)S^2 + \varepsilon_0^2 W_i(t) = \frac{MgL}{H_{i1}(i,k)\mu i\pi}\left[\frac{S}{S^2+\Omega_i^2} - \frac{(-1)^i}{S}\right] \quad (3.12)$$

where

$$\Omega_i = \frac{ic\pi}{L} \quad (3.13)$$

then from equation (3.12)

$$W_i(t)\left[S^2 + \varepsilon_0^2\right] = \frac{MgL}{H_{i1}(i,k)\mu i\pi}\left[T_a - T_b\right] \quad (3.14)$$

where

$$T_a = \frac{S}{S^2+\Omega_i^2}\times\frac{1}{S^2+\varepsilon_0^2} \quad ; \quad T_b = \frac{R_0}{S}\times\frac{1}{S^2+\varepsilon_0^2} \text{ and } R_i = \frac{(-1)i}{S} \quad (3.15)$$

when solutions of T_a and T_b are substituted into equation (3.14), one obtains

$$W_i(t) = \frac{MgL}{H_{i1}(i,k)\mu i\pi}\left[\frac{\cos\Omega_i t - \cos\varepsilon_0 t}{\varepsilon_0^2 - \Omega_i^2} + \frac{1-\cos\varepsilon_0 t}{\varepsilon_0}\right] \quad (3.16)$$

which when inverted gives,

$$Y(x,t) = \frac{2}{L}\sum_{i=1}^{j}\frac{MgL}{H_{i1}(i,k)\mu i\pi}\left[\frac{\cos\Omega_i t - \cos\varepsilon_0 t}{\varepsilon_0^2 - \Omega_i^2} + \frac{1-\cos\varepsilon_0 t}{\varepsilon_0}\right]\times\frac{\sin i\pi x}{L} \quad (3.17)$$

which represents the displacement response to moving distributed force of simply supported Bernoulli-Euler beam resting on constant bi-parametric elastic foundation.

Next, it is pertinent to seek solution to the moving distributed mass of the problem. If Γ_0 is not equal to zero in equation (3.9), it means that the inertia term is retained and it is evident that an exact solution to the equation (3.9) is not possible. A modification of Strubles technique extensively discussed in [10] is used to obtain the modified frequency. To this end, equation (3.9) is written in the form

$$\left[1 + \Gamma_0\left(\frac{L^2}{\pi^2(i+k)^2} - \frac{L^2}{\pi^2(i-k)^2}\right) + \left(\frac{8ikL}{\pi^2\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}\right) + \left(\frac{L^2 c_o}{2}\right)\right]\ddot{W}_i(t)$$

$$+\Gamma_0\left[2cL + \frac{4i^2\pi(n^2+k^2-i^2)}{\pi^2\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]} - \frac{4n^2 c_o L}{k^2-n^2}\right]\dot{W}_i(t) + \left(\varepsilon_0^2 + \Gamma_0\left[c^2 n^2 L\left(\frac{1}{(n+k)^2} - \frac{1}{(n+k)^2}\right)\right.\right.$$

$$\left.\left.+\frac{8ikL}{\pi^2\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]} + \frac{c^2\pi^2 n^2 c_o}{2}\right]\right)W_i(t) == \frac{MgL}{\mu\pi i}\left[\cos\frac{\pi ict}{L} - \cos\pi i\right] \quad (3.18)$$

Further re-arrangements give

$$\ddot{W}_i(t)+\Gamma_1\left(\left[2cL+\frac{4i^2\pi(n^2+k^2-i^2)}{\pi^2\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}-\frac{4n^2c_oL}{k^2-n^2}\right]\right)\dot{W}_i(t)$$

$$\left[\varepsilon_0^2-\varepsilon_0^2\Gamma_1\left(\frac{L^2}{\pi^2(i+k)^2}-\frac{L^2}{\pi^2(i-k)^2}+\frac{8ikL}{\pi^2\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}\right)\right]$$

$$+\Gamma_1\left(c^2n^2L\left(\frac{1}{(n+k)^2}-\frac{1}{(n+k)^2}\right)+\left(\frac{8ikL}{\pi^2\left[(n-k)^2-m^2\right]\left[(n+k)^2-m^2\right]}\right)+\left(\frac{c^2\pi^2n^2c_o}{2}\right)\right)\Bigg]W_i(t)$$

$$=\frac{MgL}{\mu\pi i}\left[\cos\frac{\pi ict}{L}-\cos\pi i\right] \qquad (3.19)$$

when Γ_1 ,the homogenous part of equation (3.19) can be written as

$$W_i(t)=\gamma_i(t)\cos\left[\varepsilon_0 t-\phi_i\right]+\Gamma_1 W_{1i}(t)+0\left(\Gamma_1^2\right) \qquad (3.20)$$

where $\gamma_i(t)$ and ϕ_i are slowly varying functions or equivalently.

By Struble's method, one obtains

$$2\varepsilon_0\dot{Y}_i(t)+\Gamma_1 2c\varepsilon_0 LY_i(t)-\Gamma_1\frac{4i^2\pi^2+k^2-i^2}{n\left[(n+k)^2-m^2\right]\left[(n-k)^2-m^2\right]}Y_i(t)+\frac{\Gamma_1 4c_o\varepsilon_0 n^2L}{k^2-n^2}Y_i(t)=0 \qquad (3.21)$$

and

$$Y_i(t)\varepsilon_0\dot{\phi}_i-\Gamma_1\frac{L^2\varepsilon_0^2c_o}{2}Y_i(t)=0 \qquad (3.22)$$

which when solved gives

$$\gamma_i(t)=C_{mm}\ell\frac{\Gamma_1 B_{mm}}{2\varepsilon_0} \qquad (3.23)$$

and

$$\phi_i(t)=\left(\Gamma_1\varepsilon_0\frac{L^2c_o}{2}\right)t+\phi_{mm} \qquad (3.24)$$

where ϕ_{mm} is a constant.

Substituting (3.23) and (3.24) into equation (3.20), one obtains

$$W_i(t)=C_{mm}\frac{\Gamma_1 B_{mm}L}{2\varepsilon_0}\cos\left[\varepsilon_m t-\phi_{mm}\right] \qquad (3.25)$$

Therefore when the mass effect of the particle is considered, the first approximation to the homogenous system is given as

$$W_i(t)=\gamma_i(t)\cos\left[\varepsilon_m t-\phi_{mm}\right] \qquad (3.26)$$

where

$$\varepsilon_m = \varepsilon_0\left[1-\left(\Gamma_1\varepsilon_0\frac{L^2c_o}{2}\right)\right] \tag{3.27}$$

is the modified frequency corresponding the frequency of the free system due to the presence of the moving distributed mass. Thus,

$$\ddot{W}_i(t)+\varepsilon_m^2 W_i(t) = \frac{MgL}{\mu\pi_i}\left[\cos\frac{i\pi ct}{L}-\cos i\pi\right] \tag{3.28}$$

Equation (3.28) is a prototype of equation (3.10) and following similar arguments, one obtains

$$\dot{W}_i(t) = \frac{MgL}{\mu\pi_i}\left[\frac{\cos\Omega_i t-\cos\varepsilon_m t}{\varepsilon_m^2-\Omega_i^2}+1-\frac{\cos\varepsilon_m t}{\varepsilon_m}\right] \tag{3.29}$$

which when inverted gives,

$$Y_i(x,t) = \frac{2}{L}\sum_{i=1}^{j}\frac{MgL}{H_{i1}(i,k)\mu i\pi}\left[\frac{\cos\Omega_i t-\cos\varepsilon_0 t}{\varepsilon_0^2-\Omega_i^2}+\frac{1-\cos\varepsilon_0 t}{\varepsilon_0}\right]\times\sin\frac{i\pi x}{L} \tag{3.30}$$

which represents the displacement response to moving distributed mass of simply supported Bernoulli-Euler beam resting on constant elastic foundation.

IV. REMARKS ON ANALYTICAL SOLUTION

4.1 Effect of Resonance

When an undamped system such as this is considered, it is pertinent to examine the resonance condition of the structure. Following [10], and from equation (3.17), the Bernoulli- Euler beam traversed by a moving distributed force will be in a state resonance when

$$\varepsilon_0 = \frac{i\pi c}{L} \tag{4.1}$$

while equation (3.30) shows that the same Bernoulli- Euler beam reaches resonance effect at

$$\varepsilon_m = \frac{i\pi c}{L} \tag{4.2}$$

where

$$\varepsilon_m = \varepsilon_0-\left(\Gamma_1\varepsilon_0^2L^2C^0\right) \tag{4.3}$$

Thus,

$$\varepsilon_m = 2\varepsilon_0-\Gamma_1\varepsilon_0^2L^2C^0\big/2 = \frac{i\pi c}{L} \tag{4.4}$$

Clearly, $1-\frac{1}{2}\left[\Gamma_1\varepsilon_0^2L^2C^0\right]<1\forall_i$

V. NUMERICAL RESULTS

From the analytical solutions, calculations of practical interests in structural engineering and physics are presented in this section. The simply supported Bernoulli-Euler beam of length L 12.123m, velocity c=8.12m/s, flexural rigidity $EJ = 6.068\times10^{18}\,N/m^2$, and for the shear modulus K_m, the values are varied between $0\,N/m^3$ and $5\times10^5\,N/m^2$, for foundation stiffness G_m varies between 0N/ m^3 and $3\times10^6\,N/m^5$, axial force Nm values varied between $0\,N/m^3$ and $\times10^5\,N/m^2$ and the mass per unit length of the structure is 4501.537g/m. The results are shown on the various curves below for the simply supported Bernoulli-Euler beam on constant bi-parametric elastic foundation.

Figure 1and 2 depict the flexural deflections of the beam resting on constant bi-parametric elastic foundation at constant velocity for both moving distributed force and moving distributed mass. It is clearly seen that for fixed values of the shear modulus Km and foundation stiffness Gm, the displacement response of the beam decreases as the values of the prestress function increases.

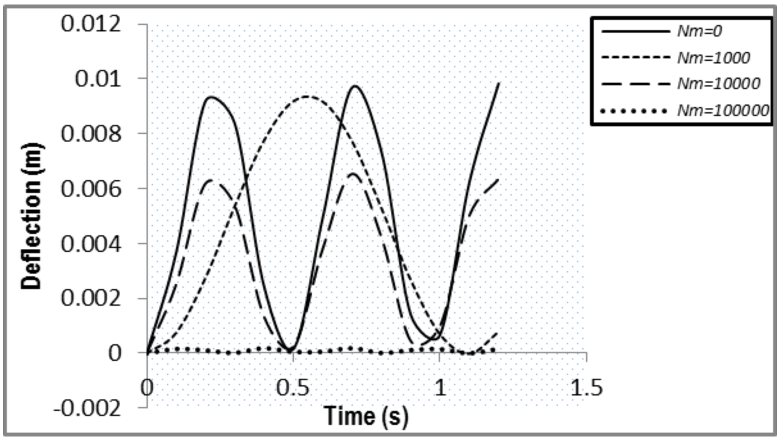

Fig.1: Displacement response of simply supported Bernoulli-Euler beam for moving distributed force for various values of axial force Nm and fixed values of shear modulus and foundation stiffness.

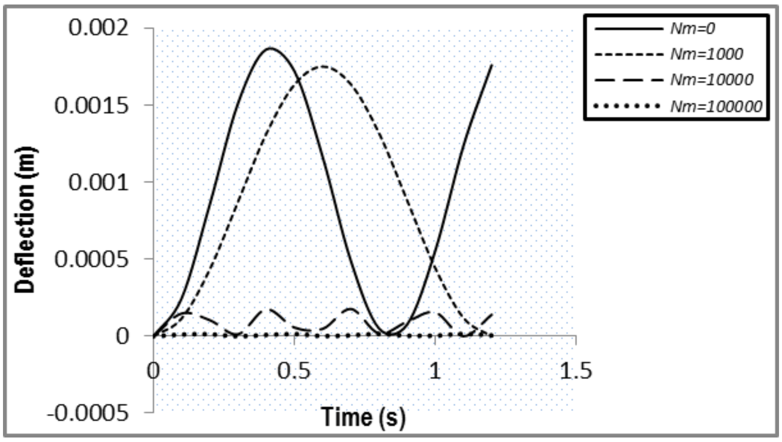

Fig.2: Displacement response of simply supported Bernoulli-Euler beam for moving distributed mass for various values of axial force Nm and fixed values of shear modulus and foundation stiffness.

Figure 3 and 4 displays the deflection profile of the simply supported Bernoulli-Euler beam resting on constant bi-parametric elastic foundation at constant velocity for both moving distributed force and moving distributed mass respectively. It is found that the dynamic deflections of the structure decreases as the values of the shear modulus Km increases for fixed values of the prestress function Nm and foundation stiffness Gm.

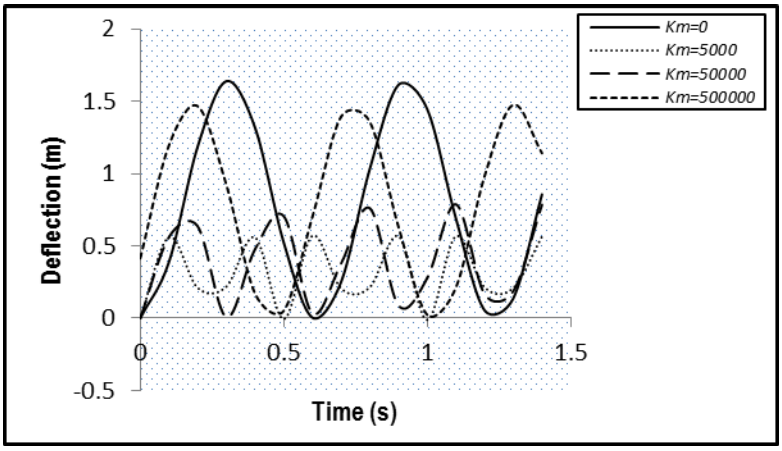

Fig.3: Deflection profile of simply supported Bernoulli-Euler beam for moving distributed force for various values of shear modulus Km and fixed values of axial force and foundation stiffness.

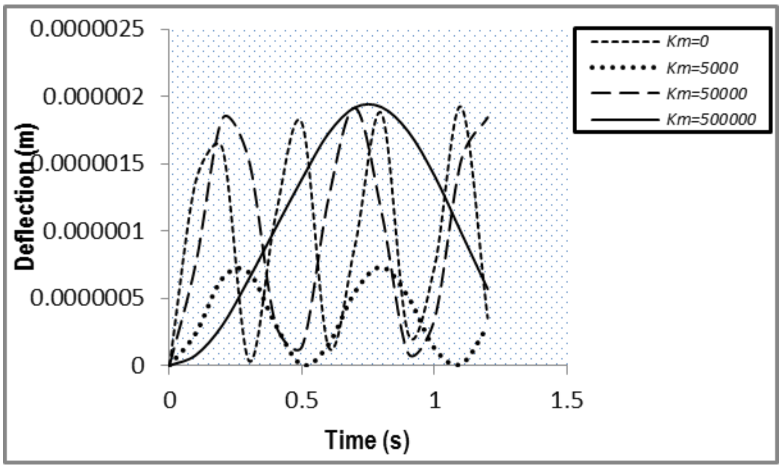

Fig.4: Deflection profile of simply supported Bernoulli-Euler beam for moving distributed mass for various values of shear modulus Km and fixed values of axial force and foundation stiffness.

Similarly, figure 3 and 4 displays the dynamic deflection of the structure resting on constant bi-parametric elastic foundation at constant velocity for both moving distributed force and moving distributed mass. For fixed values of prestress Nm, shear force Km and various values of foundation modulus Gm, it is found that the dynamic deflections of the beam decreases as the values of the foundation stiffness Gm increases.

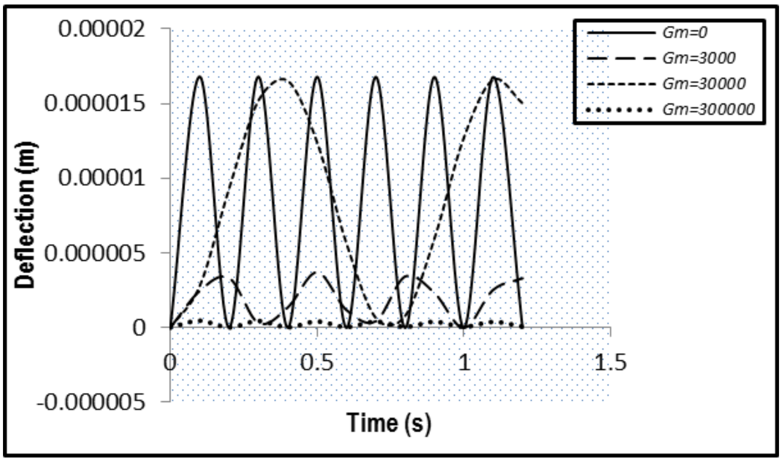

Fig.5: Deflection profile of simply supported Bernoulli-Euler beam for moving distributed force for various values of foundation stiffness and fixed values of axial force and shear modulus.

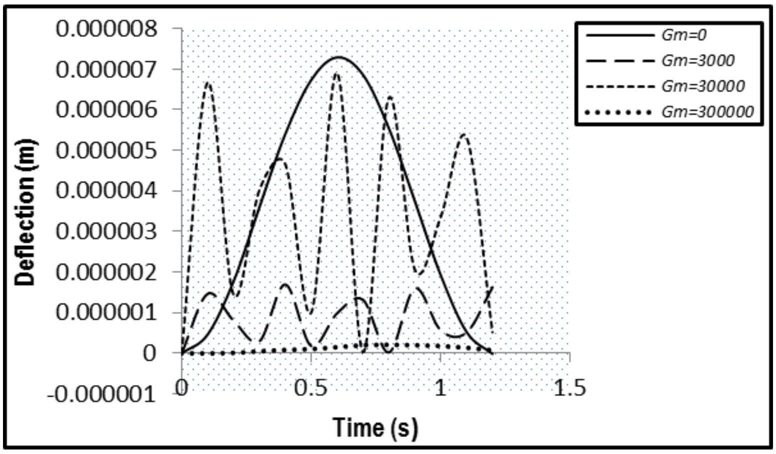

Fig.6: Deflection profile of simply supported Bernoulli-Euler beam for moving distributed mass for various values of foundation stiffness and fixed values of axial force and shear modulus.

The comparison of the displacement response of the simply supported Bernoulli-Euler beam resting on constant bi-parametric elastic foundation at constant velocity for moving distributed force and moving distributed mass. It is observed that the moving distributed force solution is not an upper bound for moving distributed mass solution as shown in the figure below

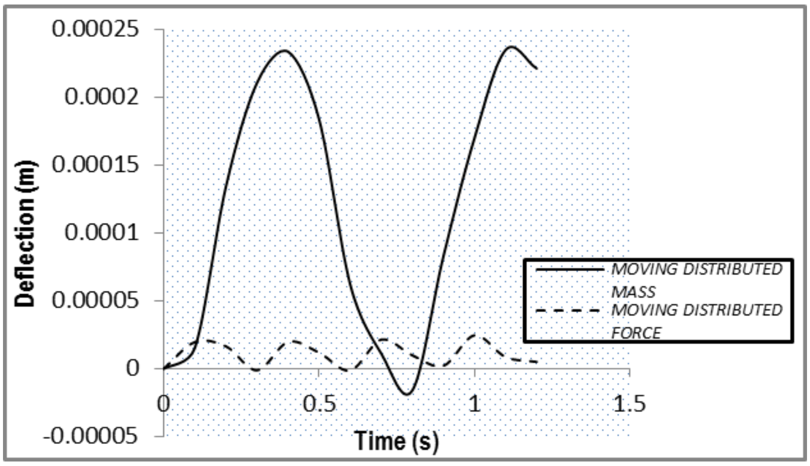

Fig.7: Comparison of the displacement response of simply supported Bernoulli-Euler beam resting on bi-parametric elastic foundation for fixed values of foundation stiffness, axial force and shear modulus.

VI. CONCLUSION

The problem of dynamic analysis under a moving distributed load of a simply supported Bernoulli-Euler beam on constant bi-parametric elastic foundation has been solved. And in view of the condition of resonance established in section VI, it is deduced that for the same natural frequency, the critical speed for the moving distributed force problem is greater than that of the moving distributed mass problem. Hence, for the same natural frequency, resonance is reached earlier in the moving distributed mass problem than in the moving distributed force.

REFERENCES

[1] L. Fryba, *Vibration of solids and structures under moving loads* (Groninggen: Noordhoof, (196), 1972).

[2] M. R. Shandnam, F. R Rofooei, M. Mofid, and B. Mehri, Periodicity in the response of non–linear moving Mass, *Thin-walled structures, 40,* 2002, 283-295.

[3] S.Timoshenko, On the correction for shear of the differential equation for transverse vibration of prismatic bars, *Phil. Mag*. S, *6(41),* l921, 744-776.

[4] G. Muscolino and A. Palmeri, Response of beams resting on viscoelastically damped foundation to moving Oscillators, *International Journal of Solids and Structures, 44(5)*, 2007, 1317-1336.

[5] K. Achawakorn and T. Jearsiripongkul, Vibration analysis of Exponential Cross-Section Beam Using Galerkin's Method, *International Journal of Applied Science and Technology, 2(6)*, 2012.

[6] S .T. Oni, Response of a non-uniform beam resting on an elastic foundation to several masses, *ABACUS, Journal of Mathematical Association of Nigeria, 24(2),*1996.

[7] B. Omolofe, S. T. Oni and J. M. Tolorunshagba, On the Transverse Motions of Non-prismatic Deep beam under the actions of variable magnitude moving loads, *Latin American Journal of Solid and Structures, (6)*, 2009, 153-167.

[8] S. Sadiku and H. H. E. Leipholz, On the Dynamics of elastic systems with moving Concentrated masses, *Ingenieur Archive, (57)*, 1981, 223-242.

[9] S.T. Oni and T.O. Awodola, Dynamic response to moving concentrated masses of uniform Rayleigh beams resting on variable Winkler elastic foundation, *Journal of the Nigerian Association of Mathematical Physics. (9)*, 2005, 151-162.

[10] S. T. Oni and S. N. Ogunyebi, Dynamical Analysis of finite prestressed Bernoulli-Euler beams with general Boundary conditions under travelling distributed loads, *Journal of the Nigerian Association of mathematical Physics,(12)*, 2008, 87-102.

[11] H. Tanahashi, characteristics of rigid foundations on Pasternak model (in case of two-dimensional plane Strain condition), *Summaries of Technical Papers of Annual Meeting, AIJ* (in Japanese), 2001, 519-520.

[12] P.L Pasternak, *On a new method of analysis of an elastic foundation by means of two foundation constants*, Gosudarstvennoe Izdatelstvo LIteraturi Po Stroitelstuve in Arkhitekture, Moscow (in Russian) 1954.

[13] M.A. De Rosa, Free Vibrations of Timoshenko beams on Two-Parameter Elastic Foundation, *Computer and Structures, 57(1)*, 1995, 151-156.

[14] S.H. Farghaly and K.M.Zeid, An exact frequency Equation For an Axially loaded Beam-mass-spring System Resting on a Winkler Elastic Foundation, *Journal Of Sound and Vibration, 185(2)*, 1995, 357-363.

[15] A.G Razaqpur, and K.R Shah, Exact analysis of Beams on Two-Parameter Elastic Foundations, *International Journal of Solids Structures,27(4)*, 1991, 435-454.

[16] B.Omolofe, Dynamic response under a moving load of an elastically supported non-prismatic Bernoulli-Euler beam on variable elastic foundation, *Latin American Journal of Solid and Structures,7*, 2010, 3-20.