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ABSTRACT: This paper presents the development of energy equations for computation of the efficiency of Parabolic-Trough Collector (PTC) using solar coordinates. The energy equations included the universal time (UT), day (n), month (M), year (Y), delta T (ΔT), longitude (L) and latitude (φ) in radian. The heliocentric longitude (H), geocentric global coordinates and local topocentric sun coordinates were considered in the modeling equations. The thermal efficiency of the PTC considered both the direct (E_{dd}) and reflected (E_{rr}) solar energy incident on the glass-cover as well as the thermal properties of the collector and the total energy losses (Q_{loss}) in the system. The developed energy equations can be used to predict the performance (efficiency) of any PTC using the meteorological and radiative data of any particular location.

Key words: Parabolic-trough Collector, Energy, solar coordinates, thermal efficiency, meteorological and radiative.

I. INTRODUCTION

1.1 Background of the study

In the design of solar energy systems, it is most important to predict the angular relationship between the sun’s rays (solar coordinates) and a vector normal to the aperture or surface of the collector at a particular location on the earth (Topocentric Location). “Topocentric” means that the sun position is calculated with respect to the observer local position at the earth surface. When the sun is observed from a position on the earth, the point of interest is to know the position of the sun relative to a coordinate system that is located at the point of observation not at the center of the earth. The position of the sun relative to these coordinates can be fixed by two angles; solar altitude angle (α) and the solar azimuth angle (A). These are as shown in figure 1[1].

Williams and Raymond, [1] stated that solar altitude angle (α) at a point on the earth is the angle between the line passing through the point and the sun and the line passing through the point tangent to the earth and passing below the sun. The solar azimuth angle (A) is the angle between the line under the sun and the local meridian pointing to the equator, or due south in the northern hemisphere. It is positive when measured to the east and negative when measured to the west. The solar zenith angle (θ_z) is the angle between a solar ray and local vertical direction and it is the complement of (α).
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The time of the year is specified by the topocentric solar declination (\( \delta_t \)). The time of the day is specified by the hour angle (\( h_z \)). The hour angle is defined as zero at local solar noon and increases by 15° for each hour. Solar time is location dependent and is generally different from local time, which is defined by time zones and other approximations. Some situations such as performance correlations, determination of true south, and tracking algorithms require an accurate knowledge of the difference between solar time and the local time [2].

The definition of a parabola provides that all paraxial rays striking the parabolic reflector are reflected toward the line-focus as shown in figure 2. Parabolic-trough Collector (PTC) consists of a reflecting surface made by bending highly polished stainless steel reflector and fixed on a parabolic contour. The reflecting surface may also be made of polished mirror. The parabolic contour is supported by steel framework and mounted on a reflector support structure. A hand-wheel operated tilting mechanism permits adjustment of declination, in order to track the sun, thus, maintains the focusing of the solar radiation on the receiver. The receiver assembly comprises of the absorber-tube covered with a glass-cover tube to reduce heat losses, is placed along the focal line of the receiver as shown in figure 3.

Concentration is achieved by using the reflector to channel natural concentration of energy on the reflector’s aperture area into a significantly smaller area, the receiver assembly that is mounted on the focal line of the parabola. Typically, concentrator systems will not work efficiently without tracking, so at least single-axis tracking is required. This is as a result of the continuous changing sun’s position in the sky with respect to time of the day as the sun moves across the sky.
The accuracy of the solar tracking depends to a great extent on the solar coordinates at a particular location and at a given instant. This would greatly influence the solar radiation collection and thus the efficiency of the system. This necessitates the need to develop energy equations for computation of PTC efficiency using solar coordinates.

Other critical factors to be considered in the analysis of the solar concentrating collector are its material properties. When radiation strikes a body, part of it is reflected, a part is absorbed and if the material is transparent, a part is transmitted. The fraction of the incident radiation reflected is the reflectance (ρ), the fraction of the incident radiation transmitted is the transmittance (τ) and the fraction absorbed is the absorptance (α) as shown schematically in figure 4.

Absorption of solar radiation is what solar heat collector is all about; therefore, a blackbody surface is ideal surface for making any solar collector’s absorbing surface. For this reason, the latter is usually made as black as possible so that it will have a high absorption capacity.

II. DEVELOPMENT OF SOLAR PARABOLIC-TRough ENERGY EQUATIONS USING SOLAR COORDINATES

In developing the model equations for computing the efficiency of PTC using solar coordinates, the input parameters are determined by the time scales because of the importance of using the correct time in the calculation of solar position for solar radiation application.

2.1 Determination of Delta T (ΔT)

Delta T (ΔT) is a measure of the difference between a time scale based on the rotation of the Earth (Universal time UT) and an idealized uniform time scale at the surface of the Earth (Terrestrial time TT). The values of ΔT can be obtained from the (TT – UT) smoothed data provided in Bulletin B of the International Earth Rotation and Reference Systems, (IERS) [3]. The value of ΔT was obtained for the year 2014 to be approximately 67.7 seconds.

2.2 Determination of Universal Time (UT)

The Universal Time (UT), or Greenwich civil time, is used to calculate the solar position based on the earth’s rotation and is measured in hours from 0-hour at midnight. The minutes and seconds must be converted into fraction of an hour.
2.3 Determination of Time Scale (\( t_G \) and \( t \))

The time scale, \( t_G \) and \( t \), are the Julian day and the Ephemeris Julian day respectively. These are used to overcome the drawbacks of irregularly fluctuating mean solar time due to elliptical shape of the earth’s orbit i.e. the interval between two successive passages of the sun through the meridian, is not constant. They are defined as follows [4]:

\[
t_G = \text{INT}(365.25(Y - 2000)) + \text{INT}(30.6001(M + 1)) + n + \frac{UT}{24} - 1158.5
\]

\( \text{Eqn. 1} \)

\[
t = t_G + \frac{\Delta t}{86400}
\]

\( \text{Eqn. 2} \)

Where \( \text{INT} \) is the integer of the calculated terms, \( Y \) is the year, \( M \) is the month of the year, \( n \) is the day of the month with decimal time and UT is the Universal Time.

2.4 Longitude \( \chi \) and Latitude \( \phi \)

Sun position is also very location-dependent, so it is critical that the longitude and latitude of the site are known before calculations are carried out.

2.5 Determination of Heliocentric Longitude of the Earth

According to Reda and Andreas, [4] “Heliocentric” means that the earth position is calculated with respect to the center of the sun. The heliocentric longitude (H) is the sum of four terms defined as follows:

\[
H = L_x + L_m + L_h + L_p
\]

\( \text{Eqn. 3} \)

Where:

\[
L_x = 1.74094 + 1.7202766863e^{-2t} + 3.34118e^{-2} \sin \sigma + 3.488e^{-4} \times \sin 2A_o
\]

\( \text{Eqn. 4} \)

\[
A_o = 1.72019e^{-2t} - 0.0563
\]

\( \text{Eqn. 5} \)

\[
L_m = 3.13e^{-5} \times \sin (0.2127730t - 0.585)
\]

\( \text{Eqn. 6} \)

\[
L_h = 1.26e^{-5} \times \sin(4.243e^{-1t} + 1.46) + 2.35e^{-5} \times \sin(1.0727e^{-1t} + 0.72) + 2.76e^{-5} \times \sin(1.5799e^{-2t} + 2.35) + 2.75e^{-5} \times \sin(2.1551e^{-2t} - 1.98) + 1.26e^{-4} \times \sin(3.1490e^{-2t} - 0.80)
\]

\( \text{Eqn. 7} \)

\[
L_p = (-(2.30796e^{-7}t_2 + 3.7976e^{-4}t_2 - 2.0458e^{-5}t_2 + 3.976e^{-5})t_2^2
\]

\( \text{Eqn. 8} \)

With \( t_2 = 0.001t \)

Where \( L_x, L_m, L_h, \) and \( L_p \) are linear increase with annual oscillation, moon perturbation, harmonic correction and polynomial correction respectively. The time scale \( t_2 \) is introduced in order to have more homogenous quantities in the products within the polynomial, avoiding too rough rounding approximation [5].

2.4 Correction to Geocentric Longitude Due to Nutation (\( \Delta \chi \))

“Geocentric” means that the sun position is calculated with respect to the earth center [4]. One problem is that the sun’s apparent diurnal and annual motions are not completely regular, due to the ellipticity of the Earth’s orbit and its continuous disturbance by the moon and planets. Consequently, the complex interactions between the bodies produce a wobbly motion of the earth rather than completely smooth motion, thus the need to compute the correction to geocentric longitude defined by the following relation [5]:

\[
\Delta \chi = 8.33e^{-5} \times \sin (9.252e^{-4t} - 1.173)
\]

\( \text{Eqn. 10} \)

2.5 Determination of the Earth Axis Inclination (\( \epsilon \))

This is the inclination of the center of the earth axis to the sun and is correlated as follows [5]:

\[
\epsilon = -6.21e^{-8} + 0.409086 + 4.46e^{-5} \times \sin(9.262e^{-4t} + 0.397)
\]

\( \text{Eqn. 11} \)

2.6 Determination of Geocentric Global Solar Coordinates

This is the coordinates of the sun with respect to the earth’s center.

2.6.1 Computation of geocentric solar longitude \( (\chi_g) \)

The geocentric solar longitude may be estimated using the following relation [5]:

\[
\chi_g = H + \pi + \Delta \chi - 9.932e^{-5}
\]

\( \text{Eqn. 12} \)

2.6.2 Computation of geocentric right ascension \( (\alpha_a) \)

The ascension angle \( (\alpha_a) \) is defined by [5]:

\[
\alpha_a = \arctan (\frac{\sin y_g \cos \epsilon}{\cos y_g})
\]

\( \text{Eqn. 13} \)
2.6.3 Computation of geocentric declination ($\delta$)

The geocentric solar declination is the declination of the sun with respect to the earth’s center. The angle between the earth’s equatorial plane and the earth-sun line varies between $\pm 23.45^\circ$ throughout the year. This angle is called the declination ($\delta$). At the time of the year when the northern part of the earth’s rotational axis is inclined toward the sun; the earth’s equatorial plane is inclined $23.45^\circ$ to the earth-sun line. At this time, the solar noon is at its highest point in the sky and the declination angle ($\delta$) is $\pm 23.45^\circ$ as illustrated in figure 5. This condition is called the summer solstice [6]. Thus, the declination can be computed at any given instant by equation 14.

$$\delta = \arcsin (\sin \phi \sin \gamma_g)$$

Eqn. 14

Fig. 5: Solar Declination Angle

2.7 Computation of Local Hour Angle of the Sun ($h$)

The hour angle is the angle through which the earth must turn to bring the meridian of a point directly in line with the sun’s rays. It is measured positively westward from the observer and it may be expressed in hours, minutes and seconds or degrees, minutes radians. One hour is equivalent to $\frac{2\pi}{24} = 0.262\text{radians}$, $\frac{360^\circ}{24} = 15^\circ$ [4];

$$h = 6.3003880990t_\alpha + 4.8824623 + 0.9174\Delta \gamma + \gamma - \alpha_0$$

Eqn. 15

2.8 Computation of Parallax Correction to Right Ascension ($\Delta \alpha$)

The parallax correction may be computed using the following correlation [5]:

$$\Delta \alpha = -4.26e^{-5} \times \cos \phi \sin h$$

Eqn. 16

2.9 Computation of Topocentric Sun Coordinates

According to Reda and Andreas, [5] topocentric sun coordinates means that the sun position is calculated with respect to the observer local position at the earth surface. Grena [5] mentioned that topocentric sun coordinates affects the sun positions by many seconds of arc.

2.9.1 Topocentric right ascension ($\alpha_t$)

The topocentric ascension angle is correlated as [5]:

$$\alpha_t = \alpha_0 + \Delta \alpha$$

Eqn. 17

2.9.2 Topocentric solar declination ($\delta_t$)

This is the declination as expressed in section 2.6.3 with respect to the local observer position. This angle is given as [5]:

$$\delta_t = \delta - 4.26e^{-5} \times (\sin \phi - \delta \cos \phi)$$

Eqn. 18

Where $\delta$-geocentric declination and $\phi$-observer local latitude.

2.9.3 Topocentric hour angle

This is as explained in section 2.7 and is determined with respect to the local observer position [5]:

$$h_t = h - \Delta \alpha$$

Eqn. 19

$$c h_t = \cosh + \Delta \sinh$$ (Approximate cosine of $h_t$)  

Eqn. 20

$$s h_t = \sinh + \Delta \cosh$$ (Approximate sine of $h_t$)  

Eqn. 21

2.10 Determination of Solar Elevation Angle, Without Refraction Correction ($e_0$)

Atmospheric refraction is the deviation of light or other electromagnetic wave from a straight line as it passes through the atmosphere due to the variation in air density as a function of altitude and is given by [7]:

$$e_0 = \arcsin (\sin \phi \sin \delta_t + \cos \phi \cos \delta_t \cosh)$$

Eqn. 22
2.11 Atmospheric Refraction Correction to the Solar Elevation (Δe)
This correction factor is correlated as [8]:

\[ Δe = \frac{0.0842}{(273 + T) \tan \left( \frac{90 - e_0}{2} \right)} \]

Eqn. 23

Where T is Temperature at a particular time of the day and P is the pressure at that same time.

2.12 Local topocentric sun coordinates

2.12.1 Topocentric Zenith angle (θz)
The solar zenith angle θz is the angle between a solar ray and local vertical direction and it is the complement of the altitude as illustrated in figure 1.

\[ θz = \frac{π}{2} - e_0 - Δe \]

Eqn. 24

2.12.2 Topocentric Azimuth angle (A)
The solar azimuth is the angle between the line under the sun and the local meridian pointing to the equator, or due south in the northern hemisphere. It is positive when measured to the east and negative when measured to the west as illustrated in figure 1.

\[ A = \arctan \left( \frac{\sin δ \sin \phi - \tan δ \cos \phi}{\cos δ} \right) \]

Eqn. 25

2.12.3 Topocentric Altitude angle (α)
The solar altitude at a point on the earth is the angle between the line passing through the point and the sun and the line passing through the point tangent to the earth and passing below the sun as shown in figure 1.

\[ α = \sin δ \sin \phi + \cos δ \cos \phi \cos h \]

Where δ is the local latitude.

2.12.4 Solar incidence angle
In the design of solar energy systems, it is most important to be able to predict the angle between the sun’s rays and a vector normal to the aperture or surface of the collector. This angle is called the angle of incidence θi; the maximum amount of solar radiation reaching the surface (or geometric aperture) is reduced by cosine of this angle. The other angle of importance is the tracking angle ρt.

Fig. 6: Solar Parabolic-trough Tracking Aperture where tracking rotation is about the axis.

Most type of medium and high-temperature collectors require a tracking drive system to align at least one and often both axes of the collector aperture perpendicular to the sun’s central ray. This is illustrated in figure 6.

For practical application when the tracking axis is oriented in the north-south direction the incidence angle θi, and the tracking angle ρt, and are given by equation 27 and 28 respectively [9]:

\[ \cos θ_i = \frac{1 - \cos^2 h}{\tan θ_i} \]

Eqn. 27

\[ \tan ρ_t = \frac{\tan θ_i}{\tan θ_i} \]

Eqn. 28

2.13 Heat Gain, Heat Transfer and Heat Absorbed by the PTC
The solar radiation incident on the concentrator is reflected on to the receiver (absorber tube) located at the focal line through which working fluid flows and which is covered by concentric-transparent glass cover. As the temperature of the receiver increases, heat transfer processes take place. Energy in transition under the motive force of a temperature difference between components of the collector forms the basis for the determination of heat gained and heat losses to and from one component to another.

The basic components of the PTC are as follows:
The concentrator (reflecting surface)

A receiver assembly comprising of a circular absorber tube with suitable coating, enclosed inside a concentric-transparent glass-cover; and

The working fluid.

These are illustrated in figure 7 below:

Energy balance equations for a PTC by Egbo et al.[10] considered the heat-energy-gain, the heat-energy-loss and the heat-energy-transfer between the components, i.e. the reflecting surface, the glass-cover and the absorber-tube, the thermal properties of the materials of the components and geometric dimensions of the SPTC. The equation for the enveloping glass-cover temperature developed by Egbo et al. [10] is given as follows:

### 2.13.1 Energy Equations for the Enveloping Glass-Cover

The energy balance equation for the enveloping glass-cover can be written as follows:

$$2\alpha_g R L [U_{beam} + I_{diff}] + \rho_g \alpha_g \left[ \frac{(W - D)L}{\pi} \right] (U_{beam} + R_b) + \frac{A_t \sigma (T_t^4 - T_g^4)}{\gamma_t + \frac{\Delta T}{\gamma_g} \left[ \frac{1}{\gamma_g} - 1 \right]} =$$

$$\sigma \varepsilon_g A_g (T_g^4 - T_{sky}) - A_g h_c (T_g - T_{surf}) = m_g c_p g A g \frac{dT_g}{dt}$$

Eqn. 29

Where: absorptance of the glass-cover material ($\alpha_g$), outer radius of the enveloping glass-cover ($R_g$), outer diameter of the enveloping glass-cover ($D_g$), length of the concentrator ($L$), tilt factor ($R_b$), beam radiation on a horizontal surface ($I_{beam}$), diffuse radiation ($I_{diff}$), reflectance of the concentrator ($\rho_c$), aperture width of the concentrator ($W$), surface area of the enveloping glass-cover ($A_g$), mass of the enveloping-glass-cover material ($m_g$), specific heat capacity of the enveloping-glass-cover material ($C_p_g$), surface area of the absorber-tube ($A_t$), Stefan-Boltzmann’s constant of radiation ($\sigma$), temperature of the absorber-tube ($T_t$), temperature of the enveloping glass-cover ($T_g$), emittance of the enveloping glass-cover material ($\varepsilon_g$), emittance of the coating material on the absorber-tube ($\varepsilon_c$), sky temperature ($T_{sky}$), ambient temperature ($T_{surf}$) and convective heat transfer coefficient ($h$), temperature gradient of the enveloping-glass-cover ($\frac{dT_g}{dt}$) as shown in figure 9.

![Image of the enveloping glass-cover showing heat gain and heat loss](image-url)
For purpose of solar concentrator design, it is often necessary to convert the beam radiation data on a horizontal surface to radiation on a tilt surface by using a conversion ratio \( R_b \) [10]. The value of \( R_b \) depends solely on the solar topocentric angles. Consequently, the beam radiation incident on the aperture of the concentrating collector becomes \( I_{beam} \ast R_b \).

\[
R_b = \frac{\cos \epsilon \cos \theta \cos \delta - \sin \epsilon \sin \theta \sin \delta}{\cos \epsilon \cos \theta \sin \delta - \sin \epsilon \sin \theta \sin \delta}
\]

Eqn. 30

### 2.13.2 Heat Gained and Heat Lost by the Absorber-Tube

The energy balance for the absorber-tube is as shown in Figure 10 and can be written as in equation 31 follows;

![Fig. 10: Representation of heat gain and heat loss by absorber-tube.](image)

\[
\alpha_t \tau_g \left( \frac{2R_0}{\pi} \right) \left( I_{beam} \ast R_b \right) + I_{diff} + \alpha_t \tau_g \alpha_b \left[ \frac{W \cdot dE}{\pi^2} \right] \left( I_{beam} \ast R_b \right) - \frac{\epsilon_t \sigma (T_e^4 - T_f^4)}{\frac{1}{2} \frac{A_m (T_e^4 - T_f^4)}{\frac{1}{2} \frac{A_m (T_e^4 - T_f^4)}}} = m_f c_{pf} \frac{dT_f}{dt}
\]

Eqn. 31

Where: absorptance of the absorber-tube material (\( \alpha_t \)), transmittance of the enveloping glass-cover material (\( \tau_g \)), surface area of the tube based on internal diameter of absorber-tube (\( A_{in} \)) and temperature of the fluid (\( T_f \)), convective heat transfer coefficient of the fluid (\( h_f \)), inner radius of the absorber-tube (\( r_i \)) and outer radius of the absorber-tube (\( r_o \)), mass of the absorber-tube material (\( m_t \)), specific heat capacity of the enveloping-glass-cover material (\( C_{pg} \)) and temperature gradient of the absorber-tube (\( \frac{dT_f}{dt} \)).

### 2.13.3 Energy Equation for the Working Fluid

The equation for the fluid temperature developed by Egbo et al. [10] is given as follows;

\[
\frac{A_m (T_e^4 - T_f^4)}{\frac{1}{2} \frac{A_m (T_e^4 - T_f^4)}} = m_f c_{pf} \frac{dT_f}{dt}
\]

Eqn. 32

Where: mass of the working fluid (\( m_f \)), specific heat capacity of the working fluid (\( C_{pf} \)) and temperature gradient of the working fluid (\( \frac{dT_f}{dt} \)).

### 2.13.4 The Thermal Efficiency of the Parabolic-Trough Collector (\( \eta_{th} \))

The thermal efficiency of the Parabolic-trough Collector is given by;

\[
\eta_{th} = 1 - \frac{Q_{losses}}{Q_{input}}
\]

Eqn. 33

Where \( Q_{losses} \) is the total heat losses and \( Q_{input} \) is the total heat supplied to the receiver.

The hourly heat supplied to the receiver (\( Q_{input} \)) can be computed as follows:

\[
Q_{input} = \left( I_{beam} \ast R_b \right) + I_{diff} \left[ WL \right]
\]

Eqn. 34

The total heat losses in the system (\( Q_{losses} \)) is considered as the sum of the radiative heat-loss from the surface...
of the enveloping glass-cover to the surroundings \( q_r \), the convective heat loss from the surface of the enveloping glass-cover to the surroundings \( q_c \) and the conductive/convective heat loss from the fluid to the surroundings \( q_1 \). This can be expressed as follows;

\[
Q_{\text{losses}} = q_r + q_c + q_1
\]

\( \text{Eqn. 35} \)

III. CONCLUSION

The energy equations for computation of Solar Parabolic-trough Collector Efficiency using solar coordinates in Bauchi were developed. The input data considered for the model equations are the Universal Time \( (UT) \), Delta T \( (\Delta T) \), Date (day \( n \), months \( M \), and years \( Y \)), longitude \( \gamma \) and latitude \( \phi \) (in radians) of Bauchi. The data also include the geometric global coordinates and local topocentric sun coordinates. The thermal efficiency \( \eta_{th} \) of the Parabolic-trough collector, considered both the total energy supplied to the receiver \( Q_{\text{input}} \) and the total energy losses \( Q_{\text{losses}} \) in the system. The developed energy equations can be used to predict the efficiency of the solar PTC at any location.

REFERENCE


