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Analysis Of Convective Plane Stagnation Point Chemically Reactive Mhd Flow Past A Vertical Porous Plate With A Convective Boundary Condition In The Presence Of A Uniform Magnetic Field.

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Abstract: - The numerical investigation of a stagnation point boundary layer flow , mass and heat transfer of a steady two dimensional , incompressible , viscous electrically conducting, chemically reacting laminar fluid over a vertical convectively heated , electrically neutral flat plate exposed to a transverse uniform magnetic field has been carried out to examine the influence of the simultaneous presence of the effects of a convective boundary condition, chemical reaction, heat transfer and suction or injection. The governing coupled partial differential equations have been reduced to a set of coupled nonlinear ordinary differential equations by means of the similarity transformation , which has been solved using the classical fourth order Runge-Kutta method alongside with a shooting technique. The computational results have been presented by means of the table and discussed graphically. It has been observed that pertinent and invaluable engineering parameters such as the Skin-friction coefficient, the plate temperature distribution, the velocity , temperature and concentration profiles are appreciably influenced by flow parameters such as the magnetic field, convective heat parameters , Prandtl number, Sherwood number, Nusselt number, etc .

Keywords: - *convective boundary condition, work due to pressure, hydromagnetics, chemical reaction, porous plate, suction/ injection*

I. INTRODUCTION

Study of flow, heat and mass transfer of a boundary layer flow in the presence of oblique or transverse magnetic field has aroused the interest of many a researcher over the years and more recently consequent upon its immediate applications in extrusion of plastic and paper sheets, polymer spinning of fibres, cooling of nuclear reactors, cooling of moderately large sheets in electrically conducting liquids and molten metals, forest fire alarms, geophysical and petrochemical industries, vapour deposition on surfaces. The quality control in industrial and engineering processes is very essential for standard final products wherein the chemical deposition on surfaces is significant.

Crane [1] appeared to be the first among his pioneering contemporaries whose research incorporated the study of steady two-dimensional flow of a Newtonian fluid driven by a linearly stretching elastic flat plate. Ishak et al [2] studied the stretching sheet due to the presence of magnetic field with emphasis on buoyancy and power law effects. Nield and Bejan [3], Ingham and Pop [4] and a few others have also researched into the subject matter. Recently, a significant number of electrically conduction fluids such as liquid metals, water mixed with a little acid and others in the presence of a magnetic field on the flow and heat transfer of a viscous and incompressible fluid past a moving surface or a starching sheet in a quiescent fluid show considerable response to the influence of heat transfer rate, work done by pressure at the surface and the magnetic field. In order to accurately predict the flow, mass and heat transfer rate it is necessary to take into account not only the effect of viscosity and magnetic field but also that due to the work due to pressure at bthe surface. Chamkha [5] and Abo-Eldahab [6] considered the problem related to hydromagnetic three-dimensional flow on a stretching surface while Ishak et al [7] studied the effects of a uniform transverse magnetic field on the stagnation point flow towards a stretching sheet. Very recently, Anjali Devi and Thiyagarajan [8] investigated the effects of a transverse magnetic field on the flow and heat transfer characteristics over a stretching surface by assuming that the magnetic strength is non-linear, and he obtained similarity solution. Ibrahim and Makinde [9] examined chemically reacting MHD boundary layer flow of heat and mass transfer past a low-heat-resistant sheet moving vertically downwards in a viscous electrically conducting fluid permeated by a uniform transverse magnetic field. Adeniyan and Adigun [10] examined the effects of a chemical reaction on stagnation point MHD flow over a vertical plate with convective boundary conditions but neglected the energy due to pressure force which provides the motivation for this work. Problems of this type have invaluable applications in geothermal energy extraction and underground storage system. The effects of various thermo-magneto physical parameters on the velocity, temperature and concentration profiles are considered and discussed through tables and graphs.

II. PROBLEM FORMULATION

We consider a vertical porous plate wholly immersed in a two-dimensional stagnation point flow of a chemically reacting and electrically conducting incompressible viscous laminar MHD stream. The magnetic field of magnitude B_0 is strategically positioned in a direction which is transverse to the porous plate. We took an assumption that the magnetic Reynolds number is very small so that the Hall current effects and the induced magnetic field are negligible. T_w *and* C_w represent the wall temperature and species concentration respectively while T_∞ *and* C_∞ stand for the ambient temperature and species concentration. The flow,. Heat and mass transfer equations relevant for the model are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \gamma\frac{\partial^2 v}{\partial y^2} - \frac{\sigma}{\rho}\beta_0^2 u \qquad (2)$$

$$\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{1}{Cp}u_e\frac{du_e}{dx}(u_e - u) \qquad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} - kr(C - C_\infty) \qquad (4)$$

Outside the boundary layer, use is made of the inviscid flow assumptions to eliminate the first term on the right of eqn. (2) to obtain a new form posited as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \gamma\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}\beta_0{}^2(u - u_e) \qquad (5)$$

Boundary Conditions

$$u(x,0) = ax, v(x,0) = \pm V_0, u(x,\infty) = u_\infty = ax \qquad (6a)$$

$$\left.\begin{array}{ll} -k\dfrac{\partial T}{\partial y}(x,0) = h_f(T_f - T(x,0)), & T(x,\infty) = 0 \\ C(x,0) = C_w, & C(x,\infty) = 0 \end{array}\right\} \qquad (6b)$$

Where $T_W = T(x,0)$.

In eqns. $(1) - (6b)$ above, the velocity field **q**= ($u(x,y), v(x,y)$) where u *and* v are the components of the velocity along and normal to the plate respectively. $T(x,y)$ is the temperature field, ɣ is the kinematic viscosity,

$\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity, p is the fluid pressure, ρ is the fluid density, k is the thermal conductivity, σ is the electrical conductivity of the fluid , Kr is the reaction rate constant of the first order homogeneous and irreversible reaction, $\boldsymbol{B} = (0, B_0)$, is the imposed magnetic field, D_m is the mass diffusivity, C_P is the specific heat at constant pressure and the left hand side of the vertical plate surface say, is heated by convection from hot fluid at temperature $T_f > T_w > T_\infty$, which provides a heat transfer coefficient h_f while the right hand of the plate is filled with electrically conducting fluid for y ≥ 0. In terms of the steam function $\psi(x, y)$, the velocity components

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

automatically satisfy eqn. (1) upon the use of the set of transformation and dimensionless quantities given by eqn. (8) below.

$$\left.\begin{aligned} &\eta = y\sqrt{\frac{a}{\gamma}}, \quad \psi(x, y) = x\sqrt{a\gamma}\, f(\eta), \\ &Pr = \frac{\gamma}{\alpha}, \qquad Bi = \frac{h_f}{k}\sqrt{\frac{\gamma}{a}} \\ &\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \qquad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \\ &M = \frac{\sigma B_0^2}{\rho a}, \; K = \frac{K_r}{a}, \quad Sc = \frac{\gamma}{D_m}, \; Ec = \frac{x^2 a^2}{C_p (T_w - T_\infty)} \end{aligned}\right\} \tag{8}$$

By means of eqn. (8), Using the similarity transformation in (8), equations (2), (3) and (4) are reduced to:

$$f''' + ff'' - (f')^2 + 1 - M(f' - 1) = 0 \tag{9}$$

$$\theta'' + PrEc(1 - f') + Prf\theta' = 0 \tag{10}$$

$$\phi'(\eta) + Scf(\eta)\phi'(\eta) - ScK\phi(\eta) = 0 \tag{11}$$

subject to the transformed boundary conditions:

$$f'(0) = 0\,, f(0) = F_w\,, \quad -\theta'(0) = B_i\big(1 - \theta(0)\big),\ \phi(0) = 1 \tag{12a}$$

$$f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0. \tag{12b}$$

Where the prime denotes differentiation with respect to η, Pr is the Prandtl number, Sc is the Schmidt number, M is the magnetic parameter , Ec is the Eckert number and the dimensionless reaction rate constant $K = \frac{K_r}{a}$.

The dimensionless suction parameter $F_w = -V_o\sqrt{a\gamma}$ and V_o is the suction velocity .

The physical quantities of relevance are Skin-friction C_f, the Nusselt number Nu and the Sherwood number Sh, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$.

The system of equations (9) – (12b) is a coupled system of non-linear boundary value ordinary differential equations and it is difficult to solve by known available analytical methods. Nevertheless the equations are solved using a reliable classical fourth order Runge-Kutta integration method with a shooting technique implemented on a computer written in Maple(15), Heck[11].

RESULTS AND DISCUSSION

Our results illustrate the influence of Prandtl number (Pr), Schmidt number (Sc), Biot number(Bi), Magnetic parameter(M), Reaction rate parameter(K), Eckert number(Ec) and Suction (Fw) which are in agreement with earlier communications [7, 9, 10] on the Skin friction coefficient as well as the heat and mass transfers at the plate surface.

Table 1 shows the numerical results for various parameter variations

Pr	Sc	Bi	M	K	Ec	F_w	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.72	0.24	0.1	0.1	0.2	0.5	0.1	1.330132	0.058220	0.399302
2.71	0.24	0.1	0.1	0.2	0.5	0.1	1.330132	0.039792	0.399302
5.00	0.24	0.1	0.1	0.2	0.5	0.1	1.330132	0.026291	0.399302
7.10	0.24	0.1	0.1	0.2	0.5	0.1	1.330132	0.017352	0.399302
0.72	0.62	0.1	0. 1	0.2	0.5	0.1	1.330132	0.058220	0.610665
0.72	0.64	0.1	0.1	0.2	0.5	0.1	1.330132	0.058220	0.619710
0.72	0.78	0.1	0.1	0.2	0.5	0.1	1.330132	0.058220	0.679378
0.72	0.24	0.5	0.1	0.2	0.5	0.1	1.330132	0.180330	0.399302
0.72	0.24	1.0	0. 1	0.2	0.5	0.1	1.330132	0.244408	0.399302
0.72	0.24	10.0	0.1	0.2	0.5	0.1	1.330132	0.359320	0.399302
0.72	0.24	0.1	0.5	0.2	0.5	0.1	1.476360	0.060357	0.402593
0.72	0.24	0.1	1.0	0.2	0.5	0.1	1.640784	0.062441	0.405929
0.72	0.24	0.1	2.0	0.2	0.5	0.1	1.927601	0.065442	0.410972
0.72	0.24	0.1	0.1	0.4	0.5	0.1	1.330132	0.058220	0.448650
0.72	0.24	0.1	0.1	0.5	0.5	0.1	1.330132	0.058220	0.471947
0.72	0.24	0.1	0.1	1.0	0.5	0.1	1.330132	0.058220	0.577494
0.72	0.24	0.1	0.1	0.2	1.0	0.1	1.330132	0.031796	0.399302
0.72	0.24	0.1	0.1	0.2	5.0	0.1	1.330132	0.179595	0.399302
0.72	0.24	0.1	0.1	0.2	10.0	0.1	1.330132	0.443834	0.399302
0.72	0.24	0.1	0.1	0.2	0.5	0.5	1.578868	0.051241	0.463431
0.72	0.24	0.1	0.1	0.2	0.5	1.0	1.923517	0.078772	0.549008
0.72	0.24	0.1	0.1	0.2	0.5	2.0	2.698494	0.087938	0.734407
0.72	0.24	0.1	0.1	0.2	0.5	-0.5	1.010300	0.027260	0.311802
0.72	0.24	0.1	0.1	0.2	0.5	-1.0	0.797568	0.019326	0.248298
0.72	0.24	0.1	0.1	0.2	0.5	-2.0	0.511852	0.135360	0.151000

We observed that the magnitude of Nusselt number which represents the heat transfer is decreased with an increase in the Prandtl number. An increase in the Sherwood number which stands for the mass transfer rate is noticed when the Schmidt number is increased. Furthermore, an increased variation in the Biot number produces a corresponding increase in the Nusselt number.

It is also of note that the Skin friction coefficient, which shows the level of viscous drag in the boundary layer, the Nusselt and Sherwood numbers are all increased with an increase in the Magnetic parameter. An increase in the reaction rate parameter produces a corresponding increase in the Sherwood number.

When the Eckert number is increased, a increase in the Nusselt number is observed. The table also shows that the skin friction together with the heat and mass transfer rate at the porous plate surface increase with increasing magnitude of fluid Suction(F_w) at the plate surface.

Effects of parameter variation on velocity profiles

The effects of various parameters on velocity profiles in the boundary layer region are depicted in Figures 1 to 2. It is observed that the velocity starts minimum value of zero at the plate surface and increases exponentially to a free stream value of unity located far from the plate surface, satisfying the far field boundary conditions for all parameter values. In **Figure 1** , the magnetic parameter(M) enhanced the fluid velocity thereby increasing the velocity boundary layer thickness.

Moreover, it is interesting to note that in **Figure 2.** , the effect of increasing the magnitude of fluid suction (F_w) at the plate surface increases the fluid velocity and also further thickens the velocity boundary layer thickness. Reverse is the case for the response due to increase in injection or fluid blowing (-F_w) as seen in **table 1**.

Effects of parameter variation on temperature profile

It is observed in **Figure 3**, that the thermal boundary layer thickness decreases with an increase in the Prandtl number. The opposite is observed in the fluid temperature as it is seen that the temperature increases with increasing Prandtl number. Nevertheless **Figures 4** and **5** revealed that there is an increase in the thermal boundary layer and rise in fluid temperature as the values of Biot and Eckert number are increased respectively. Lastly, in **Figure 6**, we obtained a slightly similar result to that of **Figure 3**, in terms of decreased thermal boundary layer but there is a decrease in the fluid temperature as the magnitude of the suction is increased.

Effects of parameter variation on concentration profile

Generally, the species concentration in the fluid has a maximum value at the plate surface and decreases exponentially to the free stream zero value far from the plate. In the case of concentration boundary layer, we have observe a decrease with an increase in Schmidt number, rate constant and increasing magnitude of suction (see **figures 7** to **9**).

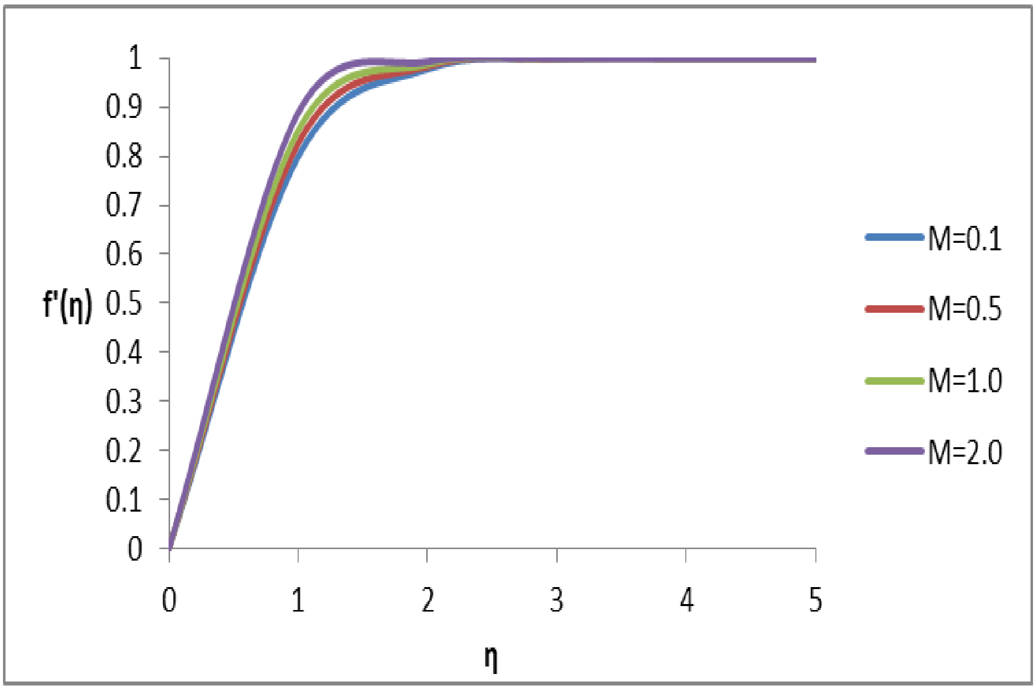

Figure 1: Effect of Magnetic parameter on the velocity profile for Pr=0.72, Sc = 0.24, Bi=0.1, K = 0.2, Ec=0.5 and $F_w = 0.1$.

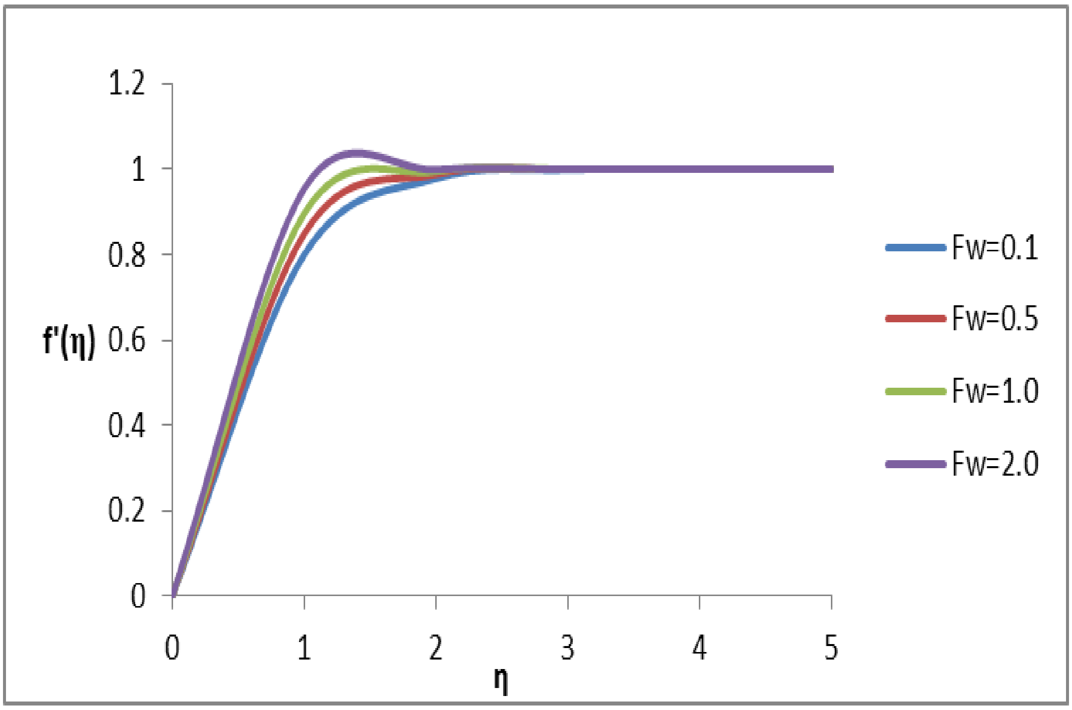

Figure 2: Effect of Suction on the velocity profile for Pr=0.72, Sc = 0.24, Bi=0.1, M = 0.1, K = 0.2 and Ec=0.5.

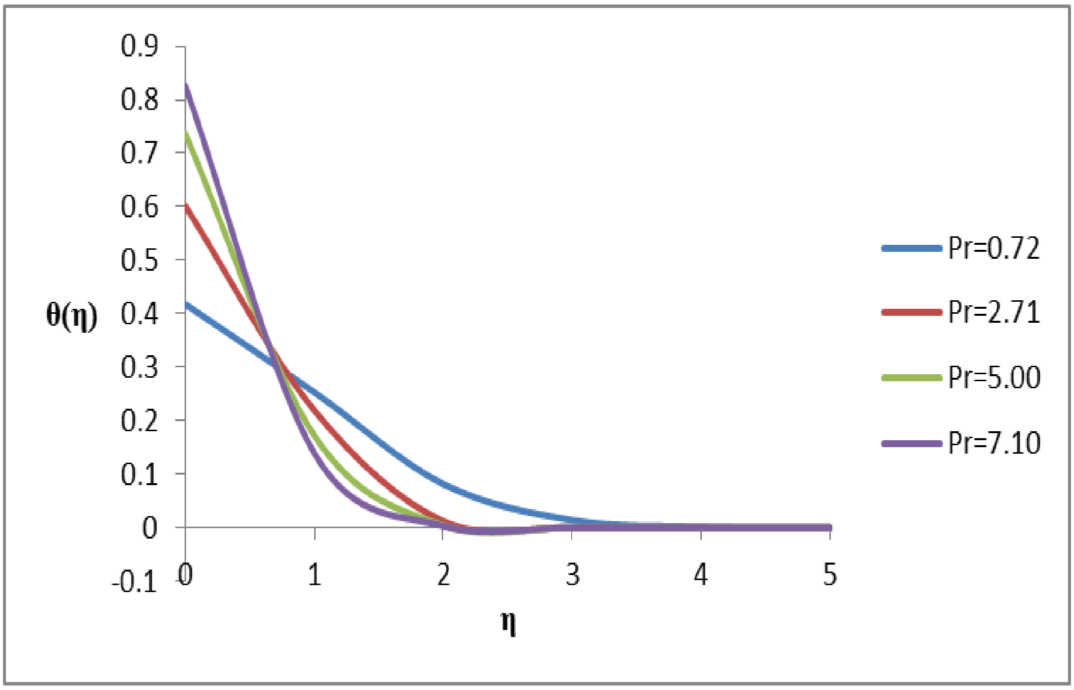

Figure 3: Effect of Prandtl number on the temperature profile for Sc = 0.24, Bi=0.1, M = 0.1, K = 0.2, Ec=0.5 and $F_w = 0.1$.

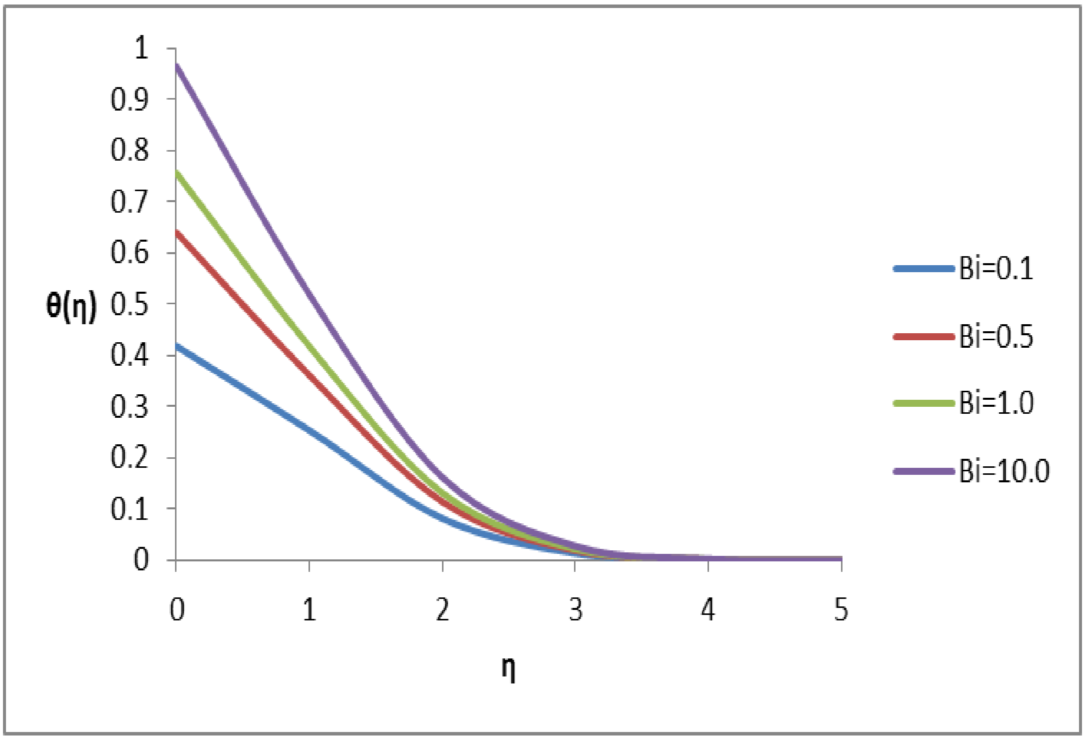

Figure 4: Effect of Biot number on the temperature profile for Pr=0.72, Sc=0.24, M = 0.1, K = 0.2, Ec=0.5 and $F_w = 0.1$.

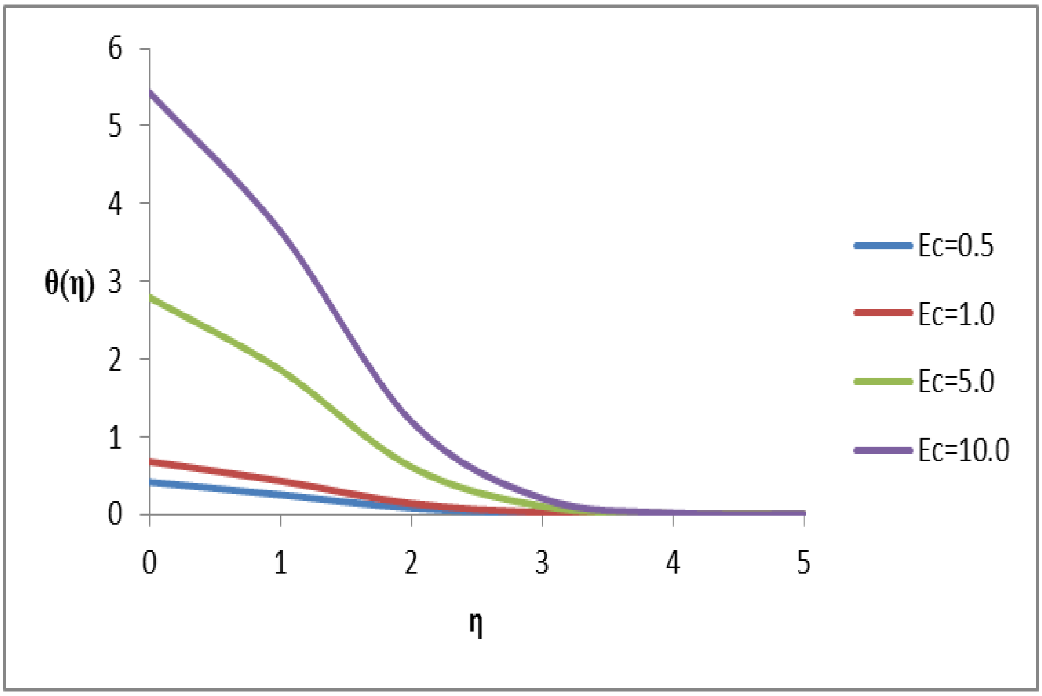

Figure 5: Effect of Eckert on the temperature profile for Pr=0.72, Sc = 0.24, Bi=0.1, M = 0.1, K = 0.2, Ec=0.5 and $F_w = 0.1$.

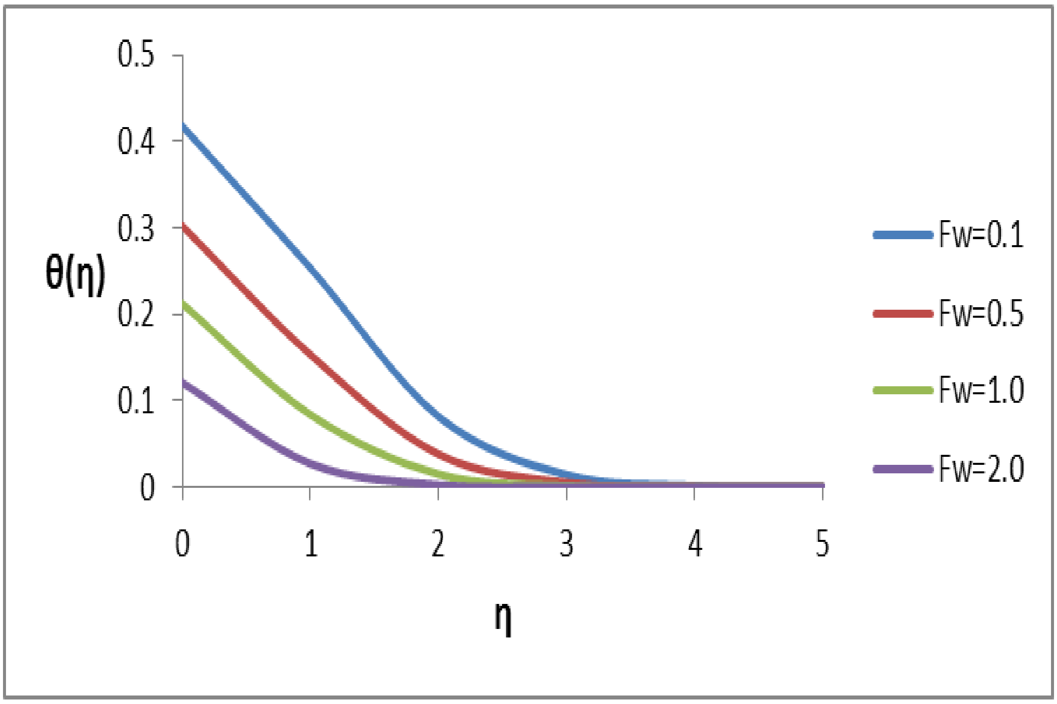

Figure 6: **Effect of Injection on the Temperature profile for Pr=0.72, Sc = 0.24, Bi=0.1, M = 0.1, K = 0.2, and Ec=0.5.**

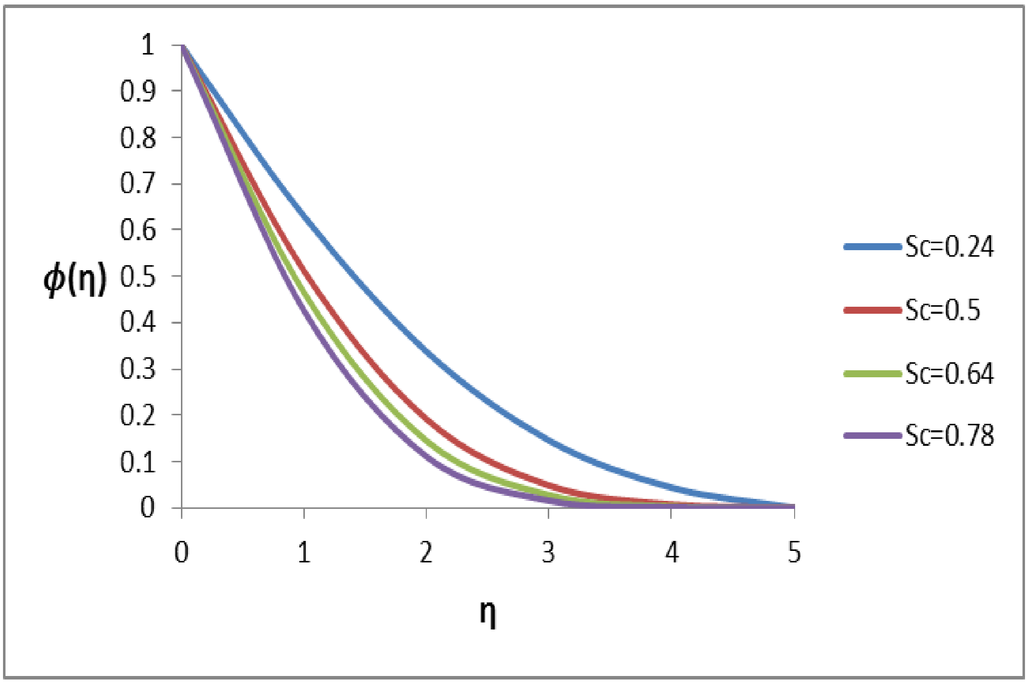

Figure 7: **Effect of Schmidt number on the concentration profile for Pr=0.72, Bi=0.1, M = 0.1, K = 0.2, Ec=0.5 and $F_w = 0.1$**

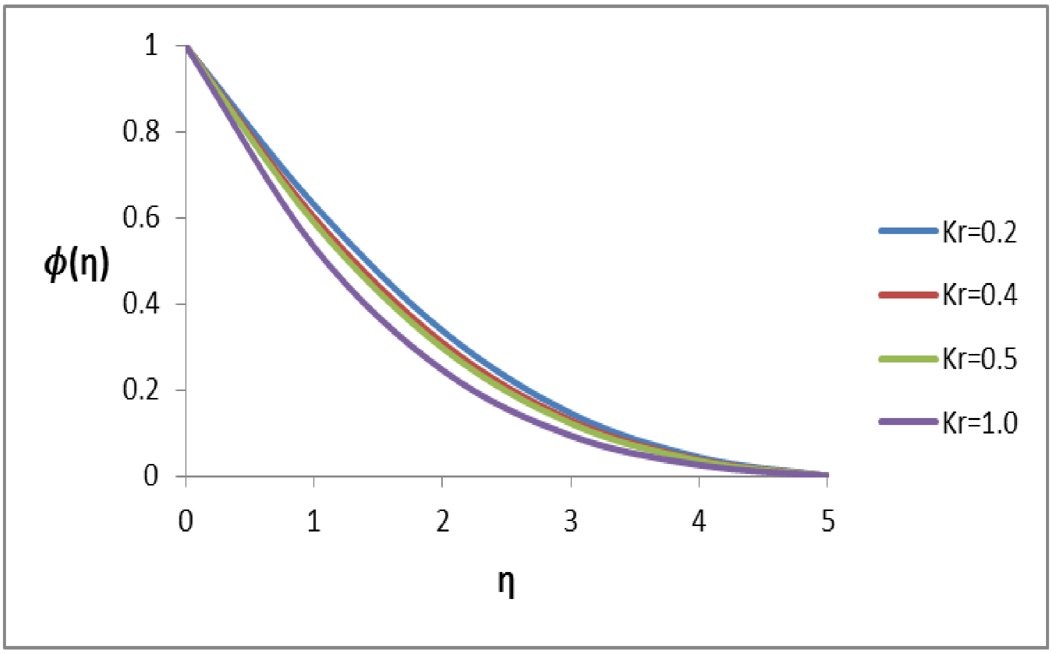

Figure 8: Effect of Reaction rate on the concentration profile for Pr=0.72, Sc = 0.24, Bi=0.1, M = 0.1, Ec=0.5 and $F_w = 0.1$.

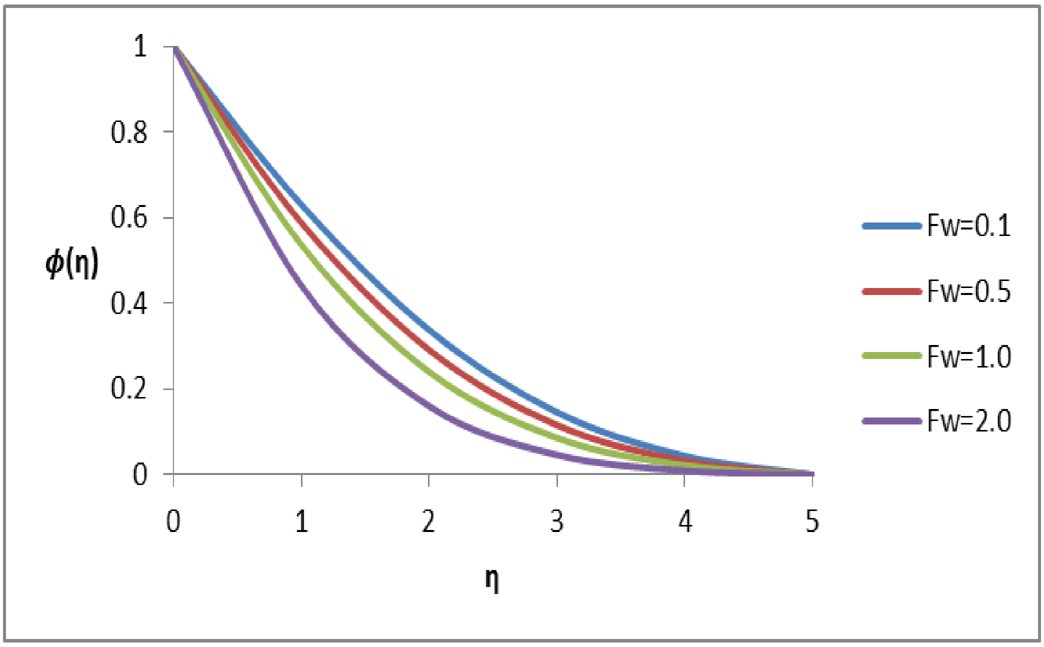

Figure 9: Effect of Injection on the concentration profile for Pr=0.72, Sc = 0.24, Bi=0.1, M = 0.1, K = 0.2, and Ec=0.5.

CONCLUSION

This paper studies the effects of chemical reaction and magnetic field on the convective stagnation point MHD flow over a plate which is vertically placed and with convective boundary conditions. The non-linear and coupled governing differential equations have been solved numerically using the fourth order Runge-Kutta shooting technique. Numerical results have been presented whilst the velocity, temperature and concentration profiles illustrated graphically and interpreted for the physical meanings. Our results show that the convective boundary conditions, the stagnation point, chemical reaction and the magnetic field have significant effects on the heat and mass transfer rate, velocity boundary layer thickness and the thermal boundary layer thickness respectively.

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