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Construction of Canonical Polynomial Basis Functions for Solving Special Nth-Order Linear Integro-Differential Equations

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Abstract: The problem of solving special nth-order linear integro-differential equations has special importance in engineering and sciences that constitutes a good model for many systems in various fields. In this paper, we construct canonical polynomial from the differential parts of special nth-order integro-differential equations and use it as our basis function for the numerical solutions of special nth-order integro-differential equations. The results obtained by this method are compared with those obtained by Adomian Decomposition method. It is also observed that the new method is an effective method with high accuracy. Some examples are given to illustrate the method.

Keyword: *Integro - differential equation, canonical polynomial, differential, collocation method*

I. INTRODUCTION

Integro - differential equation is an equation which involving both differential and integral equation. This type of problem arise in Science and Engineering because of complexity of this problem, we discover that in order to get an exact or analytical solution of the problems, numerical analyst are now to developed interest in this area and this motivated the researcher to study this class of problem. The Canonical Polynomial established by Liao [11-15] is thoroughly used by many researchers to handle a wide variety of scientific and engineering applications: linear and nonlinear, and homogeneous and inhomogeneous as well. It was shown by many authors [1, 5, 7, 8, 9, 21, 22] that this method provides improvements over existing numerical techniques. The method gives rapidly convergent series solution approximation of the exact solution if such a solution exists. Taiwo [23] motivated this researcher work due to some properties of Canonical polynomials reported in the work that:

(i) Canonical Polynomial can be generated over any given interval of consideration;
(ii) It can be easily programmed; and
(iii) It can be generated recursively.

Without loss of generality, the researcher considers special nth-order linear integro-differential equation (IDE) of the form:

$$y^{n}(x)+\sum_{j=0}^{n-1} P_j(x)y^{(j)}(x)= f(x)+\lambda_i\int_a^b K(x,t)y^{(p)}(t)dt \tag{1}$$

$$y(a)=a_0,\qquad y'(a)=a_1,.\qquad y^{(n-1)}(a)=a_{n-1} \tag{2}$$

where a_i, are real constants, n, p are positive integers, $f(x), P_j(x)$ and $K(x,t)$ are given smooth functions, while $y(x)$ is to be determined.

Eq. (1)- (2) occur in various areas of engineering, mechanics, physics, chemistry, astronomy, economics, potential theory, electrostatics, etc. Many methods are usually used to handle the high-order IDE (1)-(2) such as the successive approximations, Adomian decomposition, Homotopy perturbation method, Taylor collocation, Haar Wavelet, Tau and Walsh series methods, Monte Carlo Method, Direct method based on Fourier and block-pulse functions, etc. [2-4, 6, 10, 16-17, 21-25], but due to the problems encountered by some of these authors in integrating complex functions like e^{-x^2}, $e^{\cos\theta}$ etc, then the method serves as an advantage.

Significant of the study

The construction of a new basis called Canonical polynomials applied to the linear and non-linear problems

That a good choice of basis plays an important role in both the accuracy and efficiency of a collocation method is well-known in the literature. The extension of Canonical polynomials as a new basis for collocation method is examined and the following observation were obtained. Canonical polynomials provide some computational advantage, among which are the following;

They are generated by a single recursive formula.

They are independent of the interval of consideration.

They are independent of the associated conditions.

They ensure highly stable method (A-stability) and optional order accuracy.

One major advantage of the approach is that it is easily friendly to error estimation.

We obtained that non linear problems are solved using the collocation method in terms of canonical polynomials for the sequence of linearized approximate problems. The Newton's linearization process is used which guarantees a quadratic convergence rate of the iteration.

We also obtained that the method provides the solution in a rapidly convergent series with components that are elegantly computed.

With all these observation, the researcher conclude that canonical polynomial plays important role in term of accuracy and efficient.

II. CONSTRUCTION OF CANONICAL POLYNOMIAL

From the general equations, stated in (1)-(2), we define D as follows;

$$L \equiv \sum_{i=0}^{n} P_i \frac{d^i}{dx^i}$$

implies

$$L \equiv P_n \frac{d^n}{dx^n} + \cdots + P_3 \frac{d^3}{dx^3} + P_2 \frac{d^2}{dx^2} + P_1 \frac{d}{dx} + P_0$$

and let

$$L\Phi_i(x) = x^i$$

$$Lx^i = P_n i(i-1)(i-2)\cdots(i-n)x^{i-n} + \cdots + P_3 i(i-1)(i-2)x^{i-3} + P_2 i(i-1)x^{i-2} + P_1 i x^{i-1} + P_0 x^i$$

$$L[L\Phi_i(x)] = P_n i(i-1)(i-2)\cdots(i-n)L\Phi_{i-n}(x) + \cdots + P_3 i(i-1)(i-2)L\Phi_{i-3}(x) + P_2 i(i-1)L\Phi_{i-2}(x) + P_1 i L\Phi_{i-1}(x) + P_0 L\Phi_i(x)$$

$$x^i = P_n i(i-1)(i-2)\cdots(i-n)\Phi_{i-n}(x) + \cdots + P_3 i(i-1)(i-2)\Phi_{i-3}(x) + P_2 i(i-1)\Phi_{i-2}(x) + P_1 i\Phi_{i-1}(x) + P_o\Phi_i(x)$$

$$\Phi_i(x) = \frac{1}{P_0}\left[x^i - P_1 i\Phi_{i-1}(x) - P_2 i(i-1)\Phi_{i-2}(x) - P_3 i(i-1)(i-2)\Phi_{i-3}(x) - \cdots - P_n i(i-1)(i-2)\cdots(i-n)\Phi_{i-n}(x)\right]$$

$$i \geq 0; P_0 \neq 0 \qquad (3)$$

For the case **n = 3**, we define our operator as:

$$L \equiv P_3 \frac{d^3}{dx^3} + P_2 \frac{d^2}{dx^2} + P_1 \frac{d}{dx} + P_0$$

$$L\Phi_i(x) = x^i$$

$$Lx^i = P_3 i(i-1)(i-2)x^{i-3} + P_2 i(i-1)x^{i-2} + P_1 i x^{i-1} + P_0 x^i$$

$$L[L\Phi_i(x)] = P_3 i(i-1)(i-2)L\Phi_{i-3}(x) + P_2 i(i-1)L\Phi_{i-2}(x) + P_1 i L\Phi_{i-1}(x) + P_0 L\Phi_i(x)$$

$$x^i = P_3 i(i-1)(i-2)\Phi_{i-3}(x) + P_2 i(i-1)\Phi_{i-2}(x) + P_1 i\Phi_{i-1}(x) + P_o\Phi_i(x)$$

$$\Phi_i(x)=\frac{1}{P_0}\left[x^i-P_1 i\Phi_{i-1}(x)-P_2 i(i-1)\Phi_{i-2}(x)-P_3 i(i-1)(i-2)\Phi_{i-3}(x)\right], i\geq 0; P_0\neq 0 \qquad (4)$$

For i = 0: $$\Phi_0(x)=\frac{1}{P_0}$$

For i =1: $$\Phi_1(x)=\frac{1}{P_0}(x-P_1\Phi_0(x))=\frac{x}{P_0}-\frac{P_1}{P_0^2}$$

For i = 2: $$\Phi_2(x)=\frac{1}{P_0}\left[x^2-2P_1\Phi_1(x)-2P_2\Phi_0(x)\right]=\frac{x^2}{P_0}-2x\frac{P_1}{P_0^2}+2\frac{P_1^2}{P_0^3}-2\frac{P_2}{P_0^2}$$

For i = 3:

$$\Phi_3(x)=\frac{1}{P_0}\left[x^3-3P_1\Phi_2(x)-6P_2\Phi_1(x)-6P_3\Phi_0(x)\right]$$

$$\Phi_3(x)=\frac{x^3}{P_0}-\frac{3x^2P_1}{P_0^2}+\frac{6xP_1^2}{P_0^3}-\frac{6P_1^3}{P_0^4}-\frac{6xP_2}{P_0^2}-\frac{6P_3}{P_0^2}$$

For i = 4:

$$\Phi_4(x)=\frac{1}{P_0}\left[x^4-4P_1\Phi_3(x)-12P_2\Phi_2(x)-24P_3\Phi_1(x)\right]$$

$$=\left[\frac{x^4}{P_0}-\frac{4x^3P_1}{P_0^2}+\frac{12x^2P_1^2}{P_0^3}-\frac{24xP_1^3}{P_0^4}+\frac{24P_1^4}{P_0^5}-\frac{72P_1^2P_2}{P_0^4}+\frac{48P_1P_3}{P_0^3}-\frac{12x^2P_2}{P_0^2}-\frac{24xP_3}{P_0^2}+\frac{24P_2^2}{P_0^3}\right]$$

Thus, from equation (4), we obtain the following

$\Phi_0(x)=1,$

$\Phi_1(x)=x-1,$

$\Phi_2(x)=x^2-2x,$

$\Phi_3(x)=x^3-3x^2,$

$\Phi_4(x)=x^4-4x^3+24,$

.

.

.

etc.

For the case **n = 4**, we define our operator as:

$$L\equiv P_4\frac{d^4}{dx^4}+P_3\frac{d^3}{dx^3}+P_2\frac{d^2}{dx^2}+P_1\frac{d}{dx}+P_0$$

$$L\Phi_i(x)=x^i$$

$$Lx^i=P_4 i(i-1)(i-2)(i-3)x^{i-4}+P_3 i(i-1)(i-2)x^{i-3}+P_2 i(i-1)x^{i-2}+P_1 ix^{i-1}+P_0 x^i$$

$$L[L\Phi_i(x)]=P_4 i(i-1)(i-2)(i-3)L\Phi_{i-4}(x)+P_3 i(i-1)(i-2)L\Phi_{i-3}(x)+P_2 i(i-1)L\Phi_{i-2}(x)$$
$$+P_1 iL\Phi_{i-1}(x)+P_0 L\Phi_i(x)$$

$$x^i=P_4 i(i-1)(i-2)(i-3)\Phi_{i-4}(x)+P_3 i(i-1)(i-2)\Phi_{i-3}(x)+P_2 i(i-1)\Phi_{i-2}(x)+P_1 i\Phi_{i-1}(x)+P_o\Phi_i(x)$$

$$\Phi_i(x)=\frac{1}{P_0}\left[x^i-P_1 i\Phi_{i-1}(x)-P_2 i(i-1)\Phi_{i-2}(x)-P_3 i(i-1)(i-2)\Phi_{i-3}(x)+P_4 i(i-1)(i-2)(i-3)\Phi_{i-4}(x)\right],$$

$$i\geq 0; P_0\neq 0 \qquad (5)$$

For i = 0: $$\Phi_0(x)=\frac{1}{P_0}$$

For i =1: $$\Phi_1(x)=\frac{1}{P_0}(x-P_1\Phi_0(x))=\frac{x}{P_0}-\frac{P_1}{P_0^2}$$

For i = 2: $$\Phi_2(x)=\frac{1}{P_0}\left[x^2-2P_1\Phi_1(x)-2P_2\Phi_0(x)\right]=\frac{x^2}{P_0}-2x\frac{P_1}{P_0^2}+2\frac{P_1^2}{P_0^3}-2\frac{P_2}{P_0^2}$$

$$\Phi_3(x)=\frac{1}{P_0}\left[x^3-3P_1\Phi_2(x)-6P_2\Phi_1(x)-6P_3\Phi_0(x)\right]$$

For i = 3:

$$\Phi_3(x)=\frac{x^3}{P_0}-\frac{3x^2P_1}{P_0^2}+\frac{6xP_1^2}{P_0^3}-\frac{6P_1^3}{P_0^4}-\frac{6xP_2}{P_0^2}+\frac{6P_1P_2}{P_0^3}-\frac{6P_3}{P_0^2}$$

For i = 4:

$$\Phi_4(x)=\frac{1}{P_0}\left[x^4-4P_1\Phi_3(x)-12P_2\Phi_2(x)-24P_3\Phi_1(x)\right]$$

$$=[\frac{x^4}{P_0}-\frac{4x^3P_1}{P_0^2}+\frac{12x^2P_1^2}{P_0^3}-\frac{24xP_1^3}{P_0^4}+\frac{24P_1^3}{P_0^5}-\frac{48P_1P_2}{P_0^4}+\frac{24xP_2}{P_0^3}+\frac{24P_3}{P_0^3}-\frac{12x^2P_2}{P_0^2}+\frac{24xP_1}{P_0^3}-\frac{24P_1^2P_2}{P_0^4}$$

$$+\frac{24P_2^2}{P_0^3}-\frac{24xP_3}{P_0^2}+\frac{24P_1P_3}{P_0^3}-\frac{24P_4}{P_0^2}]$$

Thus, from equation (5), we obtain the following

$\Phi_n(x)=1,$

$\Phi_1(x)=x-1,$

$\Phi_2(x)=x^2-2x,$

$\Phi_3(x)=x^3-3x^2,$

$\Phi_4(x)=x^4-4x^3,$

.

$.\,etc$

Let $y(x)$ be the exact solution of the integro-differential equation,

$$Dy(x)-\lambda\int_a^b m(x,t)y(t)dt=f(x),\qquad x\in[a,b] \tag{6}$$

with

$$\sum_{m=1}^{v}\left[c_{jm}^{(1)}y^{(m-1)}(a)+c_{jm}^{(2)}y^{(m-1)}(b)\right]=d_j,\quad j=1,\ldots,v, \tag{7}$$

where $f(x)$ and $m(x,t)$ are given continuous functions $\lambda,a,b,c_{jm}^1,c_{jm}^2$ and d_j some given constants.

III. MATRIX REPRESENTATION FOR THE DIFFERENT PARTS

Let $\underline{V}:=\{v_0(x),v_1(x),\ldots\}$ be a polynomial basis by $\underline{V}:=V\underline{X}$, where V is a non-singular lower triangular matrix and degree $(v_i(x))\le i$, for $i=0,1,2,\ldots.$. Also for any matrix P, $P_v=VPV^{-1}$.

Now we convert the Eq. (6) and (7) to the corresponding linear algebraic equations in three parts; (a), (b) and (c).

(a). Matrix representation for $Dy(x)$:

Ortiz and Samara proposed in [18] an alternative for the Tau technique which they called the operational approach as it reduces differential problems to linear algebraic problems. The effect of differentiation, shifting and integration on the coefficients vector

$\underline{\underline{\tilde{a}}}_n := (\tilde{a}_0, \tilde{a}_1, \ldots, \tilde{a}_n, 0, 0, \ldots)$

Of a polynomial $u_n(x) = \underline{\underline{\tilde{a}_n}}\underline{X}$ is the same as that of post-multiplication of $\underline{\underline{\tilde{a}}}_n$ by the matrices η, μ and i respectively,

$$\frac{du_n(x)}{dx} = \underline{\underline{\tilde{a}_n}}\eta\underline{X}, \qquad u_n(x) = \underline{\underline{\tilde{a}_n}}\mu\underline{X} \qquad \text{and } \int_0^x u_n(t)dt = \underline{\underline{\tilde{a}_n}}i\underline{X}$$

where

$$\eta = \begin{bmatrix} 0 & 0 & \cdots & \cdots \\ 1 & 0 & 0 & \cdots \\ 0 & 2 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \mu = \begin{bmatrix} 0 & 1 & \cdots\cdots \\ 0 & 0 & 1\cdots \\ \cdots & \cdots & \cdots\cdots \end{bmatrix}, i = \begin{bmatrix} 0 & 1 & \cdots\cdots \\ 0 & 0 & \tfrac{1}{2}\cdots \\ \cdots & \cdots & \cdots\cdots \end{bmatrix}.$$

We recall now the following theorem given by Ortiz and Samara [18].

(b). Matrix representation for the integral term:
Let us assume that

$$m(x,t) = \sum_{i=0}^{n}\sum_{j=0}^{n} m_{ij}v_i(x)v_j(t), \quad \text{and } y(x) = \sum_{i=0}^{\infty} a_i v_i(x) = \underline{\underline{a}}\underline{V}. \tag{8}$$

Then, we can write

$$\int_a^b m(x,t)y(t)dt = \sum_{1=0}^{\infty}\sum_{i=0}^{n}\sum_{j=0}^{n} m_{ij}a_i v_i(x)\int_a^b v_j(t)v_i(t)dt = \underline{\underline{aM}}\underline{V}, \tag{9}$$

where,

$$\underline{\underline{M}} = \begin{bmatrix} \sum_{j=0}^{n} m_{0j}\alpha_{jo} & \cdots & \sum_{j=0}^{n} m_{nj}\alpha_{jo} & 0 & 0 & \\ . & & . & . & . & \cdots \\ \sum_{j=0}^{n} m_{0j}\alpha_{jn} & \cdots & \sum_{j=0}^{n} m_{nj}\alpha_{jn} & 0 & 0 & \cdots \\ . & & . & . & . & \cdots \end{bmatrix}$$

With,

$$\alpha_{ji} = \int_a^b v_j(t)v_1(t), \qquad \text{for} \qquad \text{j}, \ l = 0, \ldots, n.$$

(c). Matrix representation for the supplementary conditions:

Replacing $y(x) = \sum_{i=0}^{\infty} a_i v_i(x)$ in the left hand side of (7), it can be written as

$$\sum_{m=1}^{v}\left[c_{jm}^{(1)}y^{(m-1)}(a) + c_{jm}^{(2)}y^{(m-1)}(b)\right] = \sum_{i=0}^{\infty}\sum_{m=1}^{v}\left[c_{jm}^{(1)}v_i^{m-1}(a) + c_{jm}^{(2)}v_i^{(m-1)}(b)\right] = \underline{\underline{a}}B_j, \tag{10}$$

where for j = 1, …,v,

$$B_j = \begin{bmatrix} c_{jm}^{(1)}v_0(a) + c_{jm}^{(2)}v_0(b) \\ \sum_{m=1}^{2}\left[c_{jm}^{(1)}v_1^{m-1}(a) + c_{jm}^{(2)}v_1^{(m-1)}(b)\right] \\ \vdots \\ \sum_{m=1}^{v}\left[c_{jm}^{(1)}v_{v-1}^{(m-1)}(a) + c_{jm}^{(2)}v_{v-1}^{(m-1)}(b)\right] \\ \vdots \end{bmatrix} \tag{11}$$

We refer to B as the matrix representation of the supplementary conditions and B_j as its jth column. The following relations for computing the elements of the matrix B can be deduced from (10):

$$b_{ij} = \sum_{m=1}^{v}\left[c_{jm}^{(1)} v_{v-1}^{(m-1)}(a) + c_{jm}^{(2)} v_{v-1}^{(m-1)}(b)\right], \qquad \text{for } i, j = 1,2,\ldots,v, \tag{12}$$

and,

$$b_{ij} = \sum_{m=1}^{v}\left[c_{jkm}^{(1)} v_{v-1}^{(m-1)}(a) + c_{jm}^{(2)} v_{v-1}^{(m-1)}(b)\right] \qquad \text{for } i = v+1, v+2,\ldots, j = 1,2,\ldots,v. \tag{13}$$

We introduce $\underline{\underline{d}} = (d_1, d_2, \ldots, d_v)$, the vector that contains right hand sides of conditions. Then the supplementary conditions take the form

$$\underline{\underline{a}}B = \underline{\underline{d}}. \tag{14}$$

It follows from (8) and (9) that

$$Dy(x) - \lambda\int_a^b m(x,t)y(t)d = \underline{\underline{a}}(\prod{}_v - \lambda M)\underline{V}. \tag{15}$$

Let $M_v := \prod{}_v - \lambda \underline{\underline{M}}$ and M_{vi} stands for its ith column and let $f(x) = \sum_{i=0}^{n} f_i v_i(x) = f\underline{V}$ with $f = (f_0, \ldots, f_n, 0, 0, \ldots)$. . Then the coefficient of exact solution $y = \underline{\underline{a}}\underline{V}$ of problem (6) and (7) satisfies the following infinite algebraic system:

$$\begin{cases} \underline{\underline{a}}M_{vi} = f_i; & i = 0,\ldots, n, \\ \underline{\underline{a}}M_{vi} = 0; & i \geq n+1, \\ \underline{\underline{a}}B_j = d_i; & j = 1,2,\ldots, v. \end{cases} \tag{16}$$

setting,

$$G = (B_1, \ldots, B_v, M_{v0}, M_{v1}, \ldots),$$

and,

$$g = (d_1, \ldots, d_v, f_0, f_1, \ldots),$$

We can write instead of (16)

$$\underline{\underline{a}}G = g. \tag{17}$$

Remark:

For $v = 0$ and $G_0(x) = 1$, Eq. (6) is transformed into a Fredholm integral equation of second kind and for $\lambda = 0$, it is transformed into a differential equation.

IV. DESCRIPTION OF THE METHOD

For the purpose of our discussion, we assume an approximate solution of the form

$$y_N(x) = \sum_{i=0}^{N} a_i \Phi_i(x) \tag{18}$$

Where a_i are constants to be determined and Φ_i are the canonical polynomials constructed above

We write equation (1) in the form:

$$y^n(x) + D(x) = f(x) + \lambda_i V(x) \qquad \text{or } D(x) = I(x); i = 1,2,\ldots \tag{19}$$

So that

$$D(x) = \sum_{j=0}^{n} P_j(x) y^{(j)}(x), \tag{20}$$

$$V(x) = \int_a^b K(x,t)y(t)dt, \tag{21}$$

and

$$I(x) = f(x) + \lambda_i V(x) \qquad (22)$$

Then, putting equation (18) into equation (1), we obtain

$$\sum_{i=0}^{N} a_i Q_i^{n}(x) + \sum_{j=0}^{n-1} \sum_{i=0}^{N} a_j(x) a_i Q_i^{n}(x) = f(x) + \lambda \int_a^b k(x,t) \sum_{i=0}^{N} a_i Q_i(t) dt \qquad (23)$$

$a_j(x)$ are known functions to be supplied,

a_i are unknown constants to be determined;

$\Phi_i(x)$ are canonical polynomial generated in section 2,

together with the following conditions:

$$\sum_{i=0}^{N} a_i \Phi_i(a) = a_0$$

$$\sum_{i=0}^{N} a_i \Phi_i^{'}(a) = a_1$$

$$\sum_{i=0}^{N} a_i \Phi_i^{(n-1)}(a) = a_{n-1}$$

In equation (23) the integral part has to be evaluated after which the left over are then collocated at point $x = x_k$, to obtain

$$\sum_{i=0}^{N} a_i \Phi_i^{n}(x_k) + \sum_{j=0}^{n-1} \sum_{i=0}^{N} a_j(x_k) a_i \Phi_i^{n}(x_k) = f(x_k) + \lambda \int_a^b k(x_k,t) \sum_{i=0}^{N} a_i \Phi_i(t) dt \qquad (24)$$

where,

$$x_k = a + \frac{(b-a)k}{N-2}; \qquad k = 1, \ldots, N-3$$

Thus, equation (24) give rise to (N-3) algebraic linear system of equations in (N+1) unknown constants. The remaining equations are obtained using the boundary conditions stated in equation (2).

These equations are then solved to obtain the unknown constants $a_i (i \geq 0)$ which are then substituted into equation (18) to obtain our approximate solution.

Remark: all these procedure discussed above have been translated and the entire process is automated by the use of symbolic algebraic program MATLAB 7.9 and no manual computation is required.

V. ERROR

In this section, we have defined our error as

$$e_N(x) = y(x) - y_N(x),$$

where $y(x)$ is the exact solution and $y_N(x)$ is the approximate solution computed for various values of N.

VI. NUMERICAL EXAMPLES

In this section, we consider some examples of third and fourth order linear integro-differential equations.

Reason:

Because of frequent occurrence of problem in fluid dynamics and biological model in science and engineering we decided to pick some problem which are commonly used and compared the result obtained by analytic solution result available.

- Mathematical modelling of real life, physics and engineering problems usually results in these classes.
- Many mathematical formulations of physical phenomena contains integro-differential equation, these equations arise in fluid dynamics, biological models and chemical kinetics.
- Integro-differential equations are usually difficult to solve analytically so it is required to obtain an efficient approximate solution.
- Therefore the need of this study, and also to discuss the existence and uniqueness of the solutions for these classes of problems.
- The present work is motivated by the desire to obtain analytical and numerical solutions to boundary value problems for high-order integro-differential equations.

Example 1: Consider the third order linear integro differential equation

$$y'''(x) - xy''(x - \pi/2) - y'(x - \pi) = x\sin(x) + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [xy'(t) - ty(t) + ty''(t - \pi)]dt$$

with the conditions

$$y(0) = 1, y'(0) = 0, y''(0) = -1$$

and exact solution is $y(x) = \cos(x)$. We use the absolute error to measure the difference between the numerical and exact solutions. In table 1 result obtained for N=6, 7, 8 are given with the exact solution.

Table 1a: Table of result of Example 1

X	Exact solution	N=6	N=7	N=8
		New method	New method	New method
$-\pi/2$	0	0	0	0
$-2\pi/5$	0.30902	0.308796	0.3089668	0.309015916
$-3\pi/10$	0.58779	0.583085	0.5877655	0.587784962
$-\pi/5$	0.80902	0.807776	0.80901055	0.809015988
$-\pi/10$	0.95106	0.950922	0.95104963	0.951055998
0	1	1	1	1
$\pi/10$	0.95106	0.950922	0.95105059	0.951055998
$\pi/5$	0.80902	0.807776	0.80901055	0.809015988
$3\pi/10$	0.58779	0.583085	0.5877655	0.587784962
$2\pi/5$	0.30902	0.308796	0.3089668	0.309015916
$\pi/2$	0	0	0	0

Table 1b: Table of errors for example 1

X	Exact solution	N=6	N=7	N=8
		Error of New method	Error of New method	Error of New method
$-\pi/2$	0	0.0000E+00	0.0000E+00	0.0000E+00
$-2\pi/5$	0.30902	2.2400E-04	5.3200E-05	4.0840E-06
$-3\pi/10$	0.58779	4.7050E-03	2.4500E-05	5.0380E-06
$-\pi/5$	0.80902	1.2440E-03	9.4500E-06	4.0120E-06
$-\pi/10$	0.95106	1.3800E-04	1.0370E-05	4.0020E-06
0	1	0.0000E+00	0.0000E+00	0.0000E+00
$\pi/10$	0.95106	1.3800E-04	9.4100E-06	4.0020E-06
$\pi/5$	0.80902	1.2440E-03	9.4500E-06	4.0120E-06
$3\pi/10$	0.58779	4.7050E-03	2.4500E-05	5.0380E-06
$2\pi/5$	0.30902	2.2400E-04	5.3200E-05	4.0840E-06
$\pi/2$	0	0.0000E+00	0.0000E+00	0.0000E+00

Example 2: Consider the third order linear integro differential equation

$$y'''(x) - xy''(x - \pi/2) - y(x - \pi/2) = 2 - x\cos(x) + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [xy'(t) - ty(t) + ty''(t - \pi/2) + xy(t - \pi/2)]dt$$

with the conditions

$$y(0) = 1, y'(0) = 0, y''(0) = 0$$

and exact solution is $y(x) = \sin(x)$

Table 2a: Table of result of Example 2 for the value of N

X	Exact solution	N=5	N=7
		New method	New method
$-\pi/2$	-1	-1.00E+00	-1.00E+ 00
$-2\pi/5$	-0.9511	-9.63E-01	-9.67E- 01
$-3\pi/10$	-0.8090	-9.16E-01	-9.04E- 01
$-\pi/5$	-0.5878	-6.78E-01	-7.85E- 01
$-\pi/10$	-0.3090	-4.26E-01	-4.58E- 01
0	0	0.00E+00	0.00E+ 00
$\pi/10$	0.3090	4.26E-01	4.58E- 01
$\pi/5$	0.5878	6.78E-01	7.85E- 01
$3\pi/10$	0.8090	9.16E-01	9.04E- 01
$2\pi/5$	0.9511	9.63E-01	9.67E- 01
$\pi/2$	1	1	1

Table 2b: Table of errors of Example 2 for the value of N

X	Exact solution	N=5	N=7
		Error of New method	Error of New method
$-\pi/2$	-1	0.0000E+00	0.0000E+00
$-2\pi/5$	-0.9511	1.1900E-02	1.5900E-02
$-3\pi/10$	-0.809	1.0700E-01	9.5000E-02
$-\pi/5$	-0.5878	9.0200E-02	1.9700E-01
$-\pi/10$	-0.309	1.1700E-01	1.4900E-01
0	0	0.0000E+00	0.0000E+00
$\pi/10$	0.309	1.1700E-01	1.4900E-01
$\pi/5$	0.5878	9.0200E-02	1.9700E-01
$3\pi/10$	0.809	1.0700E-01	9.5000E-02
$2\pi/5$	0.9511	1.1900E-02	1.5900E-02
$\pi/2$	1	0.0000E+00	0.0000E+00

Example 3: Consider the linear boundary value problem for the fourth-order integro differential equation.

$$y^{iv}(x) = x\left(1+e^{x}\right)+3e^{x}+y(x)-\int_0^1 y(t)dt \qquad a<x<b$$

with the conditions

$$y(0)=1, y'(0)=1, y(1)=1+e, y'(1)=2e$$

The exact solution of the above boundary value problem is $y(x)=1+xe^{x}$

Table 3a: Table of results of Example3 for various value of N

X	Exact solution	N=6	N=8
		New method	New method
1	1	1	1
0.1	1.111	1.10856	1.11062115
0.2	1.244	1.23912	1.24375129
0.3	1.405	1.40154	1.40295678
0.4	1.597	1.58745	1.58921410
0.5	1.824	1.81309	1.82056380
0.6	2.093	2.06289	2.09263967
0.7	2.410	2.25290	2.36349645
0.8	2.780	2.66534	2.76798420
0.9	3.214	3.20256	3.21389340
1	3.718	3.71528	3.71681830

Table 3b: Table of errors of example 3 for various value of N

X	Exact solution	N=6	N=8
		Error of new method	Error of new method
1	1	0.00000E+00	0.00000E+00
0.1	1.111	2.44000E-3	3.788500E-4
0.2	1.244	4.88000E-3	2.487100E-4
0.3	1.405	3.46000E-3	2.043220E-3
0.4	1.597	9.55000E-3	7.785900E-3
0.5	1.824	1.09100E-2	3.436200E-3
0.6	2.093	3.01100E-2	3.603300E-4
0.7	2.41	1.57100E-1	4.650355E-2
0.8	2.78	1.14660E-1	1.201580E-2
0.9	3.214	1.14400E-2	1.066000E-4
1	3.718	2.72200E-3	1.181700E-3

VII. DISCUSSION AND CONCLUSION

In this paper, Canonical polynomial has been successfully used as a basis function for the numerical solution of special n^{th}-order integro-differential equations. The solution obtained by means of the canonical polynomial is an infinite power series for appropriate conditions, which can be in turn, expressed in a closed form. The results obtained here are compared with result of Sezer and Gulsu [21] and revealed that Canonical polynomial is a powerful mathematical tool for the numerical solutions of special nth-order linear integro differential equations in terms of accuracy achieved.

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