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An inverse steady state thermal stresses in a thin clamped circular plate with internal heat generation

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Abstract: - This paper deals with the determination of temperature, displacement and thermal stresses in a thin clamped circular plate with internal heat generation. A clamped circular plate is subjected to arbitrary known interior temperature. Under steady state, the fixed circular edge and lower surface of circular plate are thermally insulated. Here we modify Kulkarni (2008) and designed most general solution of displacement potential, radial stresses and angular stresses. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.

Keywords: - *Inverse problem, thin clamped circular plate and internal heat generation.*

I. INTRODUCTION

During the second half of the twentieth century, nonisothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure.

The inverse thermoelastic problem consists of determination of the temperature of the heating medium, the heat flux on the boundary surfaces of the thin clamped circular plate when the conditions of the displacement and stresses are known at the some points of the thin clamped circular plate under consideration. Noda *et al.* (1989) discussed an analytical method for an inverse problem of three dimensional transient thermoelasticity in a transversely isotropic solid by integral transform technique with newly designed potential function and illustrated practical applicability of the method in engineering problem. Sabherwal K. C. (1965) studied an inverse problem of heat conduction. Greysa *et al*. (1989) investigated an inverse temperature field problem of theory of thermal stresses. Deshmukh and Wankhede (1998) studied an inverse transient problem of quasi static thermal deflection of a thin clamped circular plate. Ashida *et al.* (2002) studied the inverse transient thermoelasticity problem for a composite circular disc constructed of transversely isotropic layer. Most recently Bhongade and Durge (2013) considered thick circular plate and discuss, effect of Michell function on steady state behavior of thick circular plate, now here we consider a thin clamped circular plate with internal heat generation subjected to arbitrary known interior temperature. Under steady state, the fixed circular edge and lower surface of circular plate are thermally insulated. Here we modify Kulkarni (2008) and designed most general solution of displacement potential, radial stresses and angular stresses. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. A mathematical model has been constructed for thin clamped circular plate with the help of numerical illustration by considering aluminum (pure) circular plate. No one previously studied such type of problem. This is new contribution to the field.

The inverse problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill.

II. FORMULATION OF THE PROBLEM

Consider a thin clamped circular plate circular plate of thickness $2h$ defined by $0 \leq r \leq a, -h \leq z \leq h$. Let the plate be subjected to arbitrary known interior temperature $f(r)$ within region$-h < z < h$. With lower surface $z = -h$ and circular surface $r = a$ are thermally insulated. Under these more realistic prescribed conditions, the unknown temperature $g(r)$, which is at upper surface of the plate. Temperature, displacement and stresses in a thin clamped circular plate with internal heat generation are required to be determined.

Following Roy Choudhuri (1972), assume that for small thickness h plate is in a plane state of stress. In fact the smaller the thickness of the plate compared to its diameter, the nearer to a plane state of stress is the actual state. Then the displacement equations of thermoelasticity have the form

$$U_{i,kk} + \left(\frac{1+v}{1-v}\right) e_{,i} = 2\left(\frac{1+v}{1-v}\right) a_t T_{,i} \tag{1}$$

$$e = U_{k,k}\ ; k, i = 1,2. \tag{2}$$

where

U_i - displacement component

e - dilatation

T – temperature

and v and a_t are respectively, the Poisson's ratio and the linear coefficient of thermal expansion of the plate material.

Introducing

$$U_i = U_{,i} \qquad i = 1, 2.$$

we have

$$\nabla_1{}^2 U = (1+v) a_t\, T \tag{3}$$

$$\nabla_1{}^2 = \frac{\partial^2}{\partial k_1{}^2} + \frac{\partial^2}{\partial k_2{}^2}$$

$$\sigma_{ij} = 2\mu\left(U_{,ij} - \delta_{ij}\, U_{,kk}\right), \qquad i,j,k = 1, 2. \tag{4}$$

where μ is the Lames constant and δ_{ij} is the Kronecker symbol.

In the axially-symmetric case

$$U = U(r,z),\ T = T(r,z)$$

and the differential equation governing the displacement potential function $U(r,z)$ is given as

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+v) a_t\, T \tag{5}$$

$$U = \frac{\partial U}{\partial r} = 0, \quad r = a \text{ for all time } t. \tag{6}$$

The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = \frac{-2\mu}{r}\frac{\partial U}{\partial r} \tag{7}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{8}$$

In the plane state of stress within the plate

$$\sigma_{rz} = \sigma_{zz} = \sigma_{\theta z} = 0 \tag{9}$$

Temperature $T(r,z)$ of the circular plate satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = 0 \tag{10}$$

with the conditions

$$\frac{\partial T}{\partial r} = 0 \ \ at \ \ r = a,\ -h \le z \le h \tag{11}$$

$$\frac{\partial T}{\partial z} = 0 \ \ at\, z = -h,\ 0 \le r \le a \tag{12}$$

$$T = f(r)\ (known)\ at \ \ z = \xi\ ,\ -h \le \xi \le h,\ 0 \le r \le a \tag{13}$$

and

$$T = g(r)\ (unknown)\ at \ \ z = h\ ,\ 0 \le r \le a \tag{14}$$

where k is the thermal conductivity of the material of the plate, q is the internal heat generation.
Equations (1) to (14) constitute the mathematical formulation of the problem.

III. SOLUTION

To obtain the expression for temperature T (r, z), we introduce the finite Hankel transform over the variable r and its inverse transform defined as in Ozisik (1968)

$$\bar{T}(\beta_m, z) = \int_{r'=0}^{a} r' K_0(\beta_m, r)\ T(r,z)\, dr' \tag{15}$$

$$T(r,z) = \sum_{m=1}^{\infty} K_0(\beta_m, r)\ \bar{T}(\beta_m, z) \tag{16}$$

where $K_0(\beta_m, r) = \frac{\sqrt{2}}{a} \frac{J_0(\beta_m r)}{J_0(\beta_m a)}$
and $\beta_1, \beta_2 \ldots\ldots$ are roots of the transcendental equation

$$J_1(\beta_m a) = 0 \tag{17}$$

where $J_n(x)$ is Bessel function of the first kind of order n.
On applying the finite Hankel transform defined in the Eq. (15) and its inverse transform defined in Eq. (16) to the Eq. (10), one obtains the expression for temperature as

$$T(r,z) = \sum_{m=1}^{\infty} \frac{\sqrt{2}}{a} \frac{J_0(\beta_m r)}{J_0(\beta_m a)} \left\{ \begin{array}{c} -[A(\beta_m, \xi) - F(\beta_m)] \frac{\cosh[\beta_m (z+h)]}{\cosh[\beta_m (h+\xi)]} \\ -\frac{1}{\beta_m} \frac{dA(\beta_m, -h)}{dz} \frac{\sinh[\beta_m (z-\xi)]}{\cosh[\beta_m (h+\xi)]} + A(\beta_m, z) \end{array} \right\} \tag{18}$$

where $A(\beta_m, z)$ is particular integral of differential Eq. (10) and $F(\beta_m)$ is Hankel transform of $f(r)$.
The unknown temperature $g(r)$ can be obtained by substituting $z = h$ in Eq. (18) as

$$g(r) = \sum_{m=1}^{\infty} \frac{\sqrt{2}}{a} \frac{J_0(\beta_m r)}{J_0(\beta_m a)} \left\{ \begin{array}{c} -[A(\beta_m, \xi) - F(\beta_m)] \frac{\cos h[2\beta_m h]}{\cos h[\beta_m (h+\xi)]} \\ -\frac{1}{\beta_m} \frac{dA(\beta_m, -h)}{dz} \frac{\sin h[\beta_m (h-\xi)]}{\cos h[\beta_m (h+\xi)]} + A(\beta_m, h) \end{array} \right\} \tag{19}$$

To obtain displacement potential $U(r,z)$ using Eq. (18) and Eq. (5), one obtain,

$$U(r,z) = \frac{\sqrt{2}}{a}(1+v) a_t \sum_{m=1}^{\infty} \frac{[J_0(\beta_m r) - J_0(\beta_m a)]}{\beta_m{}^2 J_0(\beta_m a)} \times \left\{ \begin{array}{c} [A(\beta_m, \xi) - F(\beta_m)] \frac{\cosh[\beta_m (z+h)]}{\cosh[\beta_m (h+\xi)]} \\ -\frac{1}{\beta_m} \frac{dA(\beta_m, -h)}{dz} \frac{\sinh[\beta_m (z-\xi)]}{\cosh[\beta_m (h+\xi)]} + A(\beta_m, z) \end{array} \right\} \tag{20}$$

Now using Eqs. (18) and (20) in Eq. (7) and (8), one obtains the expressions for stresses respectively as

$$\sigma_{rr} = \frac{2\sqrt{2}\mu}{a\, r}(1+v) a_t \sum_{m=1}^{\infty} \frac{J_0{}'(\beta_m r)}{\beta_m J_0(\beta_m a)} \times \left\{ \begin{array}{c} [A(\beta_m, \xi) - F(\beta_m)] \frac{\cosh[\beta_m (z+h)]}{\cosh[\beta_m (h+\xi)]} \\ -\frac{1}{\beta_m} \frac{dA(\beta_m, -h)}{dz} \frac{\sinh[\beta_m (z-\xi)]}{\cosh[\beta_m (h+\xi)]} + A(\beta_m, z) \end{array} \right\} \tag{21}$$

$$\sigma_{\theta\theta} = -\frac{2\sqrt{2}\mu}{a}(1+v) a_t \sum_{m=1}^{\infty} \frac{J_1{}'(\beta_m r)}{J_0(\beta_m a)} \times \left\{ \begin{array}{c} [A(\beta_m, \xi) - F(\beta_m)] \frac{\cosh[\beta_m (z+h)]}{\cosh[\beta_m (h+\xi)]} \\ -\frac{1}{\beta_m} \frac{dA(\beta_m, -h)}{dz} \frac{\sinh[\beta_m (z-\xi)]}{\cosh[\beta_m (h+\xi)]} + A(\beta_m, z) \end{array} \right\} \tag{22}$$

IV. SPECIAL CASE AND NUMERICAL CALCULATIONS

1. $f(r) = r^2$ (23)

applying finite Hankel transform as defined in eq.(15) to the eq.(23), one obtains

$$F(\beta_m) = \frac{\sqrt{2}\, a}{J_0(\beta_m a)} [a J_1(\beta_m a) - 2 J_2(\beta_m a)]$$

2. $q(r,z) = \delta(r - r_0)\delta(z - z_0)$

$$\bar{q}(\beta_m, z) = \frac{\sqrt{2}}{a} r_0 \delta(z - z_0) \frac{J_0(\beta_m r_0)}{J_0(\beta_m a)}$$

where $\delta(r)$ is well known dirac delta function of argument r.
$a = 1m, h = 0.1m, r_0 = 0.5m \; and \; z_0 = 0.05m.$

4.1 *Material Properties*

The numerical calculation has been carried out for aluminum (pure) plate with the material properties defined as
Thermal diffusivity $\alpha = 84.18 \times 10^{-6}\ \mathrm{m^2 s^{-1}}$,
Specific heat $c_\rho = 896$ J/kg,
Thermal conductivity k = 204.2 W/m K,
Shear modulus $G = 25.5\ G$ pa,
Poisson ratio $\vartheta = 0.281$,

4.2 *Roots of transcendental equation*

The $\beta_1 = 3.8317$, $\beta_2 = 7.0156$, $\beta_3 = 10.1735$, $\beta_4 = 13.3237$, $\beta_5 = 16.4704$, $\beta_6 = 19.6159$ are the roots of transcendental equation $J_1(\beta_m a) = 0$. The numerical calculation and the graph has been carried out with the help of mathematical software Mat lab.

V. DISCUSSION

In this problem, thin clamped circular plate is considered which is subjected to arbitrary known interior temperature and determined the expressions for unknown temperature, displacement and stresses. As a special case mathematical model is constructed for $f(r) = r^2$ and performed numerical calculations. The thermoelastic behavior is examined such as temperature, displacement and stresses in a thin clamped circular plate with internal heat generation.

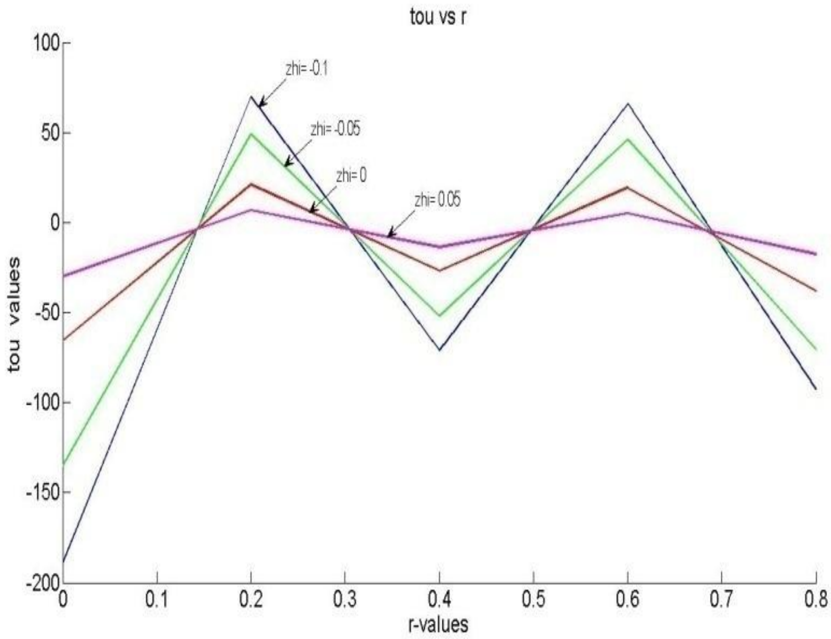

Fig. 1 Temperature $T(r, z)$ in radial direction

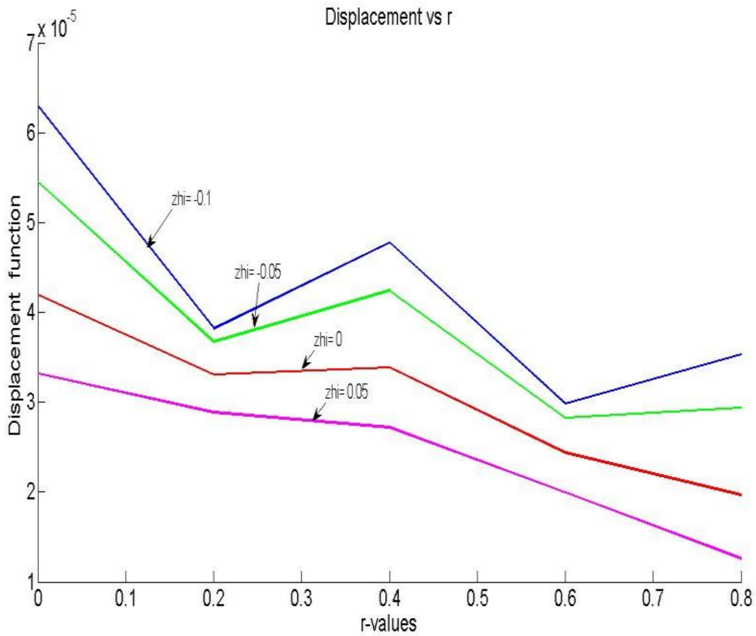

Fig. 2 Displacement $U(r, z)$ in radial direction

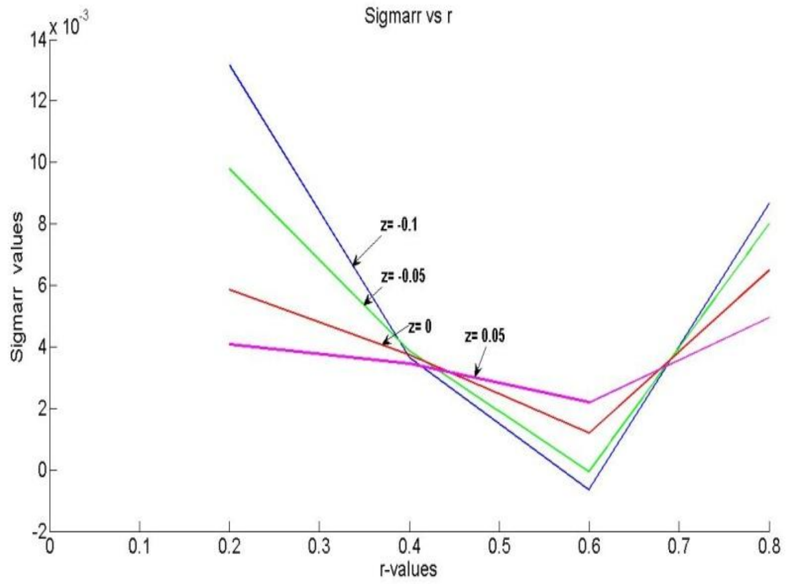

Fig. 3 Radial stress σ_{rr} in radial direction

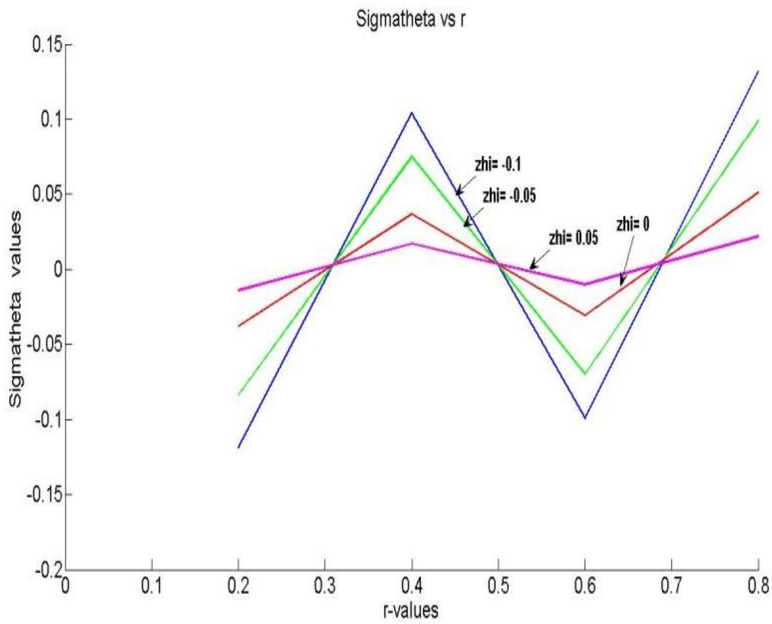

Fig. 4 Angular stress $\sigma_{\theta\theta}$ in radial direction

From Fig. 1 Due to internal heat generation temperature is increasing for $0 \leq r \leq 0.2,\ 0.4 \leq r \leq 0.6$ and decreasing for $0.2 \leq r \leq 0.4,\ 0.6 \leq r \leq 0.8$ along radial direction. The overall behavior of temperature is decreasing and it is inversely vary with arbitrary known interior temperature along radial direction.
From Fig. 2 Due to internal heat generation displacement is decreasing for $0 \leq r \leq 0.2,\ 0.4 \leq r \leq 0.6$ and increasing for $0.2 \leq r \leq 0.4,\ 0.6 \leq r \leq 0.8$ along radial direction. The overall behavior of displacement is decreasing and it is inversely vary with arbitrary known interior temperature along radial direction.
From Fig. 3 Due to internal heat generation the radial stress is decreasing for $0.2 \leq r \leq 0.6$ and increasing for $0.6 \leq r \leq 0.8$ along radial direction. The overall behavior of radial stress is compressive and it is inversely vary with arbitrary known interior temperature along radial direction.
From Fig. 4 Due to internal heat generation the angular stress is increasing for $0.2 \leq r \leq 0.4,\ 0.6 \leq r \leq 0.8$ and decreasing for $0.4 \leq r \leq 0.6$ along radial direction. The overall behavior of angular stress is tensile and it is inversely vary with arbitrary known interior temperature along radial direction.

VI. CONCLUSION

Due to internal heat generation temperature and displacement are decreasing and it is inversely vary with arbitrary known interior temperature along radial direction. Due to internal heat generation the radial stresses are compressive and the angular stresses are tensile and It is inversely vary with arbitrary known interior temperature along radial direction.

The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thin clamped circular plate, base of furnace of boiler of a thermal power plant and gas power plant.

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