Robot Selection Using Analytic Hierarchy Process and System Of Equations of Matrices

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Abstract: - The problem of robot selection plays an important key role concern to various fields of different applications since three decades. This problem has become more difficult in recent years due to increasing complexity of applications of the environment, features and specifications, and facilities offered by various manufactures. The primary objective of this paper is to select the suitable type of robot based on various factors such as type of application, payload, working environment, accuracy, lifespan, weight, purchasing cost etc. The selection procedure is developed for selection of particular type of robot based evaluating the various alternative selection factors using AHP technique and systems of equations of the matrices i.e. Eigen values and Eigen vectors. The ranking evolution will provide a good guidance for the robot selection to the end user. The concept of this work is an attempt has been made to create exhaustive database for identifying maximum possible number of attributes.

Keywords: - AHP, Robotics, Payload, Cost, System of Equations of matrices

I. INTRODUCTION

A Robot selection is one of the critical issues, while designing of work cells in the fields of manufacturing environment related to various types of products. Robot selection for a particular type of application is generally described based on experience, manufacturing institution and kinematic considerations like workspace, manipulability, etc. Therefore selection problem has become more difficult in recent years due to increasing complexity, available features, and facilities offered by different robotic manufactures. Systematic procedures were developed for selection of robot manipulator based on their different attributes.

II. METHODOLOGY

The objectives of this work is to develop AHP method for robot selection. The methodology of this work has been adopted from Yahya and Kingsman (1999), Tam and Tummala (2001) and Yu and Jing (2004). In order to comply with collecting quantitative and qualitative data for AHP robot selection model that could be applied by a six steps approach was performed to insure successful implementation

2.1 Robot Selection Criteria

Robots are being used increasingly in industrial workstations to enhance firm’s performance. Robots are employed to perform repetitive production jobs, hazardous jobs, multi-shift operations etc., so that it helps to reduce the delivery time, improve the work environment, lower the production cost and even increase the product range to suit market demand from time to time. When a choice must be made among several robots for a given application, it is necessary to compare their performance characteristics in a proper fashion. Some of the main performance criteria of an industrial robot are drive systems, geometrical dexterity, path measuring systems, mate-rial of robot, load-carrying capacity, velocity, weight of the robot, programming flexibility, size of the robot and accuracy of the robot. The importance of these criteria is commonly known and thus not elaborated.

2.2 AHP Method

Analytical Hierarchy Process (AHP) is one of Multi Criteria decision making method that was originally developed by Prof. Thomas L. Saaty. In short, it is a method to derive ratio scales from paired comparisons. The input can be obtained from actual measurement such as Weight, Payload, Precision, Cost, Life of robot, Process
etc., or from subjective opinion such as satisfaction feelings and preference. AHP allow some small inconsistency in judgement because human is not always consistent. The ratio scales are derived from the principal Eigen vectors and the consistency index is derived from the principal Eigen value.

2.3 System of Equation of Matrices

The eigenvalue problem is a problem of considerable theoretical interest and wide-ranging application. For example, this problem is crucial in solving systems of differential equations, analyzing population growth models, and calculating powers of matrices (in order to define the exponential matrix). Other areas such as physics, sociology, biology, economics and statistics have focused considerable attention on "eigenvalues" and "eigenvectors"-their applications and their computations. Before we give the formal definition, let us introduce these concepts

III. FIGURES AND TABLES

Fig 1: selection criteria

![Diagram with selection criteria]

Table 2: pair wise comparison in AHP preference [8]

<table>
<thead>
<tr>
<th>Verbal judgment preference</th>
<th>Numerical rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely preferred</td>
<td>9</td>
</tr>
<tr>
<td>Very strongly preferred</td>
<td>7</td>
</tr>
<tr>
<td>Strongly preferred</td>
<td>5</td>
</tr>
<tr>
<td>Moderately preferred</td>
<td>3</td>
</tr>
<tr>
<td>Equally preferred</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Pair-wise comparison matrix

<table>
<thead>
<tr>
<th>Factors</th>
<th>Spherical</th>
<th>Cylindrical</th>
<th>Scara</th>
<th>Cartesian coordinate</th>
<th>Joint arm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>1/7</td>
<td>1</td>
<td>0.14</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Scara</td>
<td>1/3</td>
<td>1/0.14</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cartesian coordinate</td>
<td>1</td>
<td>1/0.2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Joint arm</td>
<td>1</td>
<td>1/0.2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: Criterion weights obtained in AHP

<table>
<thead>
<tr>
<th>Factors</th>
<th>Spherical</th>
<th>Cylindrical</th>
<th>Scara</th>
<th>Cartesian coordinate</th>
<th>Joint arm</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>0.29</td>
<td>0.28</td>
<td>0.48</td>
<td>0.23</td>
<td>0.23</td>
<td>0.31</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>0.041</td>
<td>0.04</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Scara</td>
<td>0.09</td>
<td>0.1</td>
<td>0.16</td>
<td>0.23</td>
<td>0.23</td>
<td>0.20</td>
</tr>
<tr>
<td>Cartesian coordinate</td>
<td>0.29</td>
<td>0.2</td>
<td>0.16</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Table 5: random inconsistency indices (RI) for N=10

<table>
<thead>
<tr>
<th>Sample Size(N)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Index(RI)</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.9</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
</tr>
</tbody>
</table>

3.2. Equations

**Step 1:** An overall summation of the product of sum of each vector column for both the decision matrix and pair wise comparison matrices with the PV values of each row is carried out to obtain the principal Eigen value \( \lambda_{max} \) i.e.,

\[
\lambda_{max} = \sum_{i,j=1}^{K} C_j PV_i
\]

Where \( C_j \) is the sum of each column vector.

**Step 2:** Comparison of Eigen values and Eigen vectors

Eigen Value

\[
A - \lambda_i = 0
\]

Eigen Vector

\[
(A - \lambda_i)X = 0
\]

**Step 3:** The level of inconsistency in both decision and pair wise compression matrix is checked using the following equation.

\[
I.I = \frac{\lambda - N}{N - 1}
\]

Where I.I is the inconsistency index, N is the number of element of each of matrix.

**Step 4:** Prof. saaty proved that for consistent reciprocal matrix, the largest Eigen value is equal to the number of comparisons, or \( \lambda_{max}=n \). then he gave a measure of consistency, called consistency index as deviation or degree of consistency using the following formula

\[
C.I = \frac{\lambda_{max} - n}{n - 1}
\]

**Step 5:** Random inconsistency indices (RI) are then determined for each of the square matrices equation

\[
R.I = \frac{1.98(N - 2)}{N}
\]

**Step 6:** consistency ratio (C.I) which is a comparison between consistency index and random consistency index, or in formula

\[
C.R = \frac{C.I}{R.I}
\]

**IV. RESULTS**

AHP Technique

\[
\lambda_{max} = (3.48*0.31) + (25*0.04) + (6.14*0.20) + (4.20*0.23) + (4.20*0.23)
\]

\[
\lambda_{max}=5.23
\]

Having a comparison matrix, now compute priority vector, which is the normalized Eigen vector of the matrix. To know what are Eigen vector, Eigen values and how to compute manually. The following method will give a detailed explanation of getting an approximation of Eigen vector (and Eigen value) of reciprocal matrix. This approximation is actually worked well for small matrix and there is no guarantee that the rank will not reverse because of the approximation error. Nevertheless it is easy to compute because all we need to do is just to normalize each column of matrix.
System of Equation of Matrices

Eigen value 

\[ \lambda = [5.17, 0.93, 0.93, 0.07, 0] \]

Eigen Vector

<table>
<thead>
<tr>
<th></th>
<th>0.6433</th>
<th>0.8065</th>
<th>0.8065</th>
<th>0.5556</th>
<th>0.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0793</td>
<td>0.0487</td>
<td>0.0487</td>
<td>0.1608</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.4057</td>
<td>0.4417</td>
<td>0.4417</td>
<td>0.2010</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.4557</td>
<td>0.2758</td>
<td>0.2758</td>
<td>0.5590</td>
<td>0.7071</td>
</tr>
<tr>
<td></td>
<td>0.4557</td>
<td>0.2758</td>
<td>0.2758</td>
<td>0.5590</td>
<td>0.7071</td>
</tr>
</tbody>
</table>

The largest Eigen value is called the principal Eigen value, that is \( \lambda_{\text{max}} = 5.17 \)

Which is very close to our approximation \( \lambda_{\text{max}} = 5.23 \). The principal Eigen vector is the Eigen vector that corresponds to the highest Eigen value.

Thus in the previous example, we have \( \lambda_{\text{max}} = 5.23 \) for five comparisons, or \( n=5 \), thus the consistency index is

\[ C.I = \frac{5.23 - 5}{5 - 1} \]

\[ C.I = 0.05 \]

Knowing the consistency index, prof. saaty, T. (1980) proposed that consistency index by comparing it with the appropriate one. The appropriate consistency index is called Random Consistency Index (R.I)

Then, proposed what is called consistency ratio, which is a comparison between consistency index and random consistency index, or in formula

\[ C.R = \frac{0.057}{1.12} \]

\[ C.R = 0.5 \]

If the value of consistency ratio is smaller or equal to 10%, the inconsistency is acceptable. If the consistency ratio is greater than 10%, we need to revise the subjective judgment.

1.1 Graphs

![Graph showing criterion weights obtained via AHP.](image)
V. CONCLUSION

The priorities obtained from the group decision makers’ judgments are depicted. It shows that reliability of robot, the best robot selection criterion, followed by quality of product, life time of the robot, process, work environment, accuracy, life span, weight and cost of product. Thus, suggesting that the decision makers in the case of manufacturing firms should integrate the preceding criteria into robot selection decision. The inconsistency referred as Consistency Ratio is $0.5 < 10$ reported by the Mat lab Software. This implies that the group decision maker’s evaluation is consistent.

VI. REFERENCES