# Numerical study of the three-dimensional mixed convection around an inclined cone of revolution 

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#### Abstract

A numerical study of mixed convection transfers is presented. We consider the predominance of forced axial convection on the one hand and that of natural convection on the other hand, around a cone of revolution and inclined relative to the vertical. The vertical flow of the fluid supposed Newtonian develops at a flow of boundary layer type around the cone. This vertical flow is assumed to occur in the presence of ascending natural convection. The conservation equations are solved by an implicit finite difference method. The influence of the angle of inclination of the cone on the transfers is analyzed. The results are presented by dimensionless velocity and temperature profiles as well as local Nusselt number and friction coefficients.


Keywords: three-dimensional mixed convection, boundary layer, inclined cone of revolution, heat transfer, numerical study.

## NOMENCLATURE

## Roman letter symbols

$\mathrm{a}: \quad$ thermal diffusivity of the fluid, $\left(\mathrm{m}^{2} . \mathrm{s}^{-1}\right)$
Cfu: meridian friction coefficient
Cfw: azimuthal friction coefficient
Cp : specifique heat capacity at constant pressure of the fluid, $\left(\mathrm{J}^{\left(\mathrm{Kg}^{-1} \mathrm{~K}^{-1}\right)}\right.$
$\mathrm{E}_{\mathrm{k}}$ : Eker number
$\mathrm{g}: \quad$ acceleration due to gravity, $\left(\mathrm{m} . \mathrm{s}^{-2}\right)$
Gr: Grashoft number
L: length generative, (m)
Nu: local Nusselt number
Pr: Prandlt number
R: normal distance from the projected M of a
point $P$ of the fluid to the axis of revolution of the cone, (m)
Re, $\operatorname{Re}_{\infty}$ : Reynold number
$S_{\mathrm{X}}, \mathrm{S}_{\varphi}:$ factors of geometric configuration
$\mathrm{T}_{\mathrm{P}:} \quad$ temperature of the wall, $(\mathrm{K})$

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T
    wall, (K)
V
    directions, (m.s.-1)
x,y : meridian and normal coordinates,(m)
Greek letter symbols
\alpha: angle of inclination, (')
\beta: volumetric coefficient and thermal
    expansion, (K-1)
    \varphi: azimuthal coordinate, (}\mp@subsup{}{}{\circ}
\lambda: thermal conductivity, (W.m.'. . K-1)
\mu: dynamic viscosity, (Kg.m. - . .s )
v: kinematic viscosity, (m}\mp@subsup{\textrm{m}}{}{2}.\mp@subsup{\textrm{s}}{}{-1}
\Theta0: demi-angle of opening of cone ( }\mp@subsup{}{}{\circ
\rho: density of the fluid, (Kg.m3)
\Omega: Richardison number
Indices/Exponents
+: dimensionless variables
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## I. INTRODUCTION

Numerous theoretical and experimental studies have been carried out on convective transfers in the vicinity of a cone of revolution. For example, the study of the influence of the angle of inclination of the cone on the thermal transfers between the wall and fluid, by forced convection which was initiated by F.A. Rakotomanga et E. Alidina [1], they showed that the increase in the angle of inclination attenuates the heat exchange between the wall and the fluid on the one hand and induces a slight increase in the thickness of the boundary layer. Similarly, for U. Canissius and E. Alidina [2], they confirmed the results shown in [1] that the effect of the taper of the cone is relatively very weak on the heat exchange and on the thickness of the boundary layer. In addition, they also highlighted the existence of a privileged point. In this work, we consider a mixed laminar flow of a Newtonian fluid around a smooth-walled cone of revolution in the presence of natural convection.

The purpose of this study is to analyze the influence of the inclination angle of the cone on the transfers, take place in the boundary layer through forced convection. The conservation equations are discretized using an implicit finite difference scheme, velocity fields and temperature is determined from Thomas algorithm.

## II. MATHEMATICAL FORMULATION OF THE PROBLEM

A cone of revolution of generative length L , inclined at an angle $\alpha$ with respect to the vertical and plunged into a forced flow of a Newtonian fluid of ascending vertical direction, is considered. The temperature of the surface of the wall Tp of the cone is assumed constant and different from the temperature $\mathrm{T} \propto$, also constant, of the fluid away from the wall.


Figure 1: physical model

## 2-1. Simplifying assumptions

As part of this work, in addition to the classical assumptions of the boundary layer, we admit the following conditions:

- the cone is fixed and does not undergo any rotation,
- the flow is laminar and permanent,
-the physical properties of the fluid, supposed to be air, are constant, except for its density in the equation of motion, the variations of which are at the origin of a natural convection,
- the radiative transfers and the dissipation of viscous energy are negligible.


## 2-2. Conservation equations in the boundary layer

## - Equation of continuity

$\frac{\partial V_{x}}{\partial x}+\frac{\partial V_{y}}{\partial y}+\frac{1}{r} \frac{\partial V_{\varphi}}{\partial \varphi}+\frac{V_{x}}{r} \frac{d r}{d x}=0$

- Momentum equation


## Component in $\mathbf{x}$ direction

$V_{x} \frac{\partial V_{x}}{\partial x}+V_{y} \frac{\partial V_{x}}{\partial y}+\frac{V_{\varphi}}{r} \frac{\partial V_{x}}{\partial \varphi}-\frac{V_{\varphi}^{2}}{r} \frac{d r}{d x}=U e \frac{\partial U e}{\partial x}+v \frac{\partial^{2} V_{x}}{\partial y^{2}}+g \beta\left(T-T_{\infty}\right) S_{x}$
Component in $\varphi$ direction
$V_{x} \frac{\partial V_{\varphi}}{\partial x}+V_{y} \frac{\partial V_{\varphi}}{\partial y}+\frac{V_{\varphi}}{r} \frac{\partial V_{\varphi}}{\partial \varphi}+\frac{V_{x} V_{\varphi}}{r} \frac{d r}{d x}=\frac{U e}{r} \frac{\partial U e}{\partial \varphi}+v \frac{\partial^{2} V_{\varphi}}{\partial y^{2}}+g \beta\left(T-T_{\infty}\right) S_{\varphi}$
$U e=\sqrt{U e_{x}^{2}+U e_{\varphi}^{2}} \quad:$ modulus of external speed [1]
the coefficients $S_{x}$ and $S_{\varphi}$ are the factors of geometric configuration defined by [2] :

$$
\begin{align*}
& S_{x}=\sin \alpha \cdot \cos \varphi \cdot \sin \theta_{0}+\cos \alpha \cdot \cos \theta_{0}  \tag{4}\\
& S_{\varphi}=-\sin \alpha \cdot \sin \varphi \tag{5}
\end{align*}
$$

- Heat equation

$$
\begin{equation*}
V_{x} \frac{\partial T}{\partial x}+V_{y} \frac{\partial T}{\partial y}+\frac{V_{\varphi}}{r} \frac{\partial T}{\partial \varphi}=a \frac{\partial^{2} T}{\partial y^{2}} \tag{6}
\end{equation*}
$$

2-2-3. theboundary conditions associatedwith these equations is:
on the wall $\mathrm{y}=0$;
$\mathrm{T}=\mathrm{T}_{\mathrm{p}}, \quad \mathrm{V}_{\mathrm{x}}=0 \quad ; \mathrm{V}_{\mathrm{y}}=0 \quad ; \quad \mathrm{V}_{\varphi}=0$
away from the wall $y \longrightarrow \infty$;
$\mathrm{T}=\mathrm{T}_{\infty} ; \mathrm{V}_{\mathrm{x}}=\mathrm{Uex} ; \mathrm{V}_{\varphi}=\mathrm{Ue} \varphi$
Uex et Ueழare the components meridian and azimuthal of external speed [1].
$U e_{x}=U_{\infty}\left(A_{\varphi} \cdot \sin \alpha \cdot \sin \varphi\right)$
$U e_{\varphi}=U_{\infty}\left(A_{x} \cdot \cos \alpha+B_{x} \cdot \sin \alpha \cdot \cos \varphi\right)$
$A_{x}(x)=0,68+3,03296 x-25,44074 x^{2}+121,069 x^{3}-318,64541 x^{4}+466,99471 x^{5}-356,01959 x^{6}+$
$+110,24752 x^{7}$
$B_{x}(x)=-0,80834+2,69424 x-21,37757 x^{2}+98,83137 x^{3}-252,98221 x^{4}+363,05621 x^{5}-272,50282 x^{6}+$
$+83,5537 x^{7}$
$A_{\varphi}=2,3181-2,29665 x+5,87104 x^{2}-10,90766 x^{3}+10,3346 x^{4}-4,06092 x^{5}$

## 2-3. Main physical quantities

- Nusselt number : $N u=-\frac{L}{\Delta T}\left(\frac{\partial T}{\partial y}\right)_{y=0} ; \operatorname{avec} \Delta T=T_{P}-T_{\infty}$
- Reynolds number: $R e_{\infty}=\frac{L U_{\infty}}{v}$
- Grashoft number : $G r=\frac{g \beta \Delta T L^{3}}{v^{2}}$
- Richardson number : $\Omega=\frac{G r}{\operatorname{Re}_{\infty}^{2}}$
- Eker number : $E_{k}=\frac{U_{\infty}^{2}}{C_{p} \Delta T}$
- Prandtl number: $\operatorname{Pr}=\frac{v}{a}$
- Frictional stresses:

$$
\begin{equation*}
\tau_{x}=\mu\left(\frac{\partial V_{x}}{\partial y}\right)_{y=0} \text { and } \tau_{\varphi}=\mu\left(\frac{\partial V_{\varphi}}{\partial y}\right)_{y=0} \tag{16}
\end{equation*}
$$

- Friction coefficients
$C f_{u}=\frac{\tau_{x}}{\frac{1}{2} \rho U_{0}^{2}}$ and $C f_{w}=\frac{\tau_{\varphi}}{\frac{1}{2} \rho U_{0}^{2}}$


## 2-4. The dimensionless equation of mixed convection

The predominance of each type of convection must be considered separately. But Ch.R.
RAMINOSOA [4] has shown, in his thesis, that it is possible to write a unique system of equations, by introducing two coefficients, $\mathrm{C}_{\text {Nat }}$ for natural convection and $\mathrm{C}_{\text {For }}$ for that of forced convection as follows:

- If one of the convections is predominant, we put the corresponding coefficient is equal to unity and the other is zero. But if the two convections are of equal importance, we put the two coefficients equal to unity, and we note by $\mathrm{C}_{\mathrm{T}}$ their sum.
- Next, the reference quantities are as follows:
$x^{+}=\frac{x}{L} ; y^{+}=C_{1} \frac{y}{L} ; \varphi^{+}=\varphi ; r^{+}=\frac{r}{L} ; V_{x}^{+}=C_{2} \frac{V_{x}}{U_{\infty}} ; V_{y}^{+}=C_{3} \frac{V_{y}}{U_{\infty}} ; V_{\varphi}^{+}=C_{2} \frac{V_{\varphi}}{U_{\infty}} ;$
$U e^{+}=C_{4} \frac{U e}{U_{\infty}} ; U e_{x}^{+}=C_{4} \frac{U e_{x}}{U_{\infty}} ; U e_{\varphi}^{+}=C_{4} \frac{U e_{\varphi}}{U_{\infty}} ; U_{0}=C_{5} U_{\infty} ; T^{+}=\frac{C_{6} \frac{T}{\Delta T}-C_{\text {Nat }} \frac{T_{\infty}}{\Delta T}}{C_{\text {For }}+C_{\text {Nat }}}$
With Ci , the barycentric convection coefficients that manage the mixed convection:
$C_{1}=\left(\frac{C_{f o r} \operatorname{Re}_{\infty}^{\frac{1}{2}}+C_{n a t} G r^{\frac{1}{4}}}{C_{f o r}+C_{n a t}}\right) ; C_{2}=\left(\frac{C_{\text {for }}+C_{n a t} \operatorname{Re}_{\infty} G r^{-\frac{1}{2}}}{C_{\text {for }}+C_{n a t}}\right), C_{3}=\left(\frac{C_{f o r} \mathrm{Re}_{\infty}^{\frac{1}{2}}+C_{n a t} \operatorname{Re}_{\infty} G r^{-\frac{1}{4}}}{C_{\text {for }}+C_{n a t}}\right) ;$
$C_{4}=\left(\frac{C_{f o r}+C_{\text {nat }} \operatorname{Re}_{\infty} G r^{-\frac{1}{4}}}{C_{\text {for }}+C_{\text {nat }}}\right) ; C_{5}=\left(\frac{C_{\text {for }}+C_{n a t} \frac{G r^{\frac{1}{2}}}{\operatorname{Re}_{\infty}}}{C_{\text {for }}+C_{n a t}}\right) ; C_{6}=2 C_{\text {for }} E_{K}^{-1}+C_{n a t}$
Then, the dimensionless equations in the boundary layer are written:
- Equation of continuity
$\frac{\partial V_{x}^{+}}{\partial x^{+}}+\left(\frac{C_{1} C_{2}}{C_{3}}\right) \frac{\partial V_{y}^{+}}{\partial y^{+}}+\frac{1}{r^{+}} \frac{\partial V_{\varphi}^{+}}{\partial \varphi^{+}}+\frac{V_{x}^{+}}{r^{+}} \frac{d r^{+}}{d x^{+}}=0$


## - Momentum equation

$V_{x}^{+} \frac{\partial V_{x}^{+}}{\partial x^{+}}+\left(\frac{C_{1} C_{2}}{C_{3}}\right) V_{y}^{+} \frac{\partial V_{x}^{+}}{\partial y^{+}}+\frac{V_{\varphi}^{+}}{r^{+}} \frac{\partial V_{x}^{+}}{\partial \varphi^{+}}-\frac{\left(V_{\varphi}^{+}\right)^{2}}{r^{+}} \frac{d r^{+}}{d x^{+}}=\left(\frac{C_{2}^{2}}{C_{4}^{2}}\right) U e^{+} \frac{\partial U e^{+}}{\partial x^{+}}+\left(\frac{C_{1}^{2} C_{2}}{\operatorname{Re}_{\infty}}\right) \frac{\partial^{2} V_{x}^{+}}{\left.\partial y^{+}\right)^{2}}+$
$\left(\frac{C_{2}^{2}}{C_{6}}\right) C_{T} \Omega T^{+} S_{x}$
$V_{x}^{+} \frac{\partial V_{\varphi}^{+}}{\partial x^{+}}+\left(\frac{C_{1} C_{2}}{C_{3}}\right) V_{y}^{+} \frac{\partial V_{\varphi}^{+}}{\partial y^{+}}+\frac{V_{\varphi}^{+}}{r^{+}} \frac{\partial V_{\varphi}^{+}}{\partial \varphi^{+}}-\frac{\left(V_{x}^{+} V_{\varphi}^{+}\right)}{r^{+}} \frac{d r^{+}}{d x^{+}}=\left(\frac{C_{2}^{2}}{C_{4}^{2}}\right) \frac{U e^{+}}{r^{+}} \frac{\partial U e^{+}}{\partial \varphi^{+}}+\left(\frac{C_{1}^{2} C_{2}}{\operatorname{Re}_{\infty}}\right) \frac{\partial^{2} V_{\varphi}^{+}}{\left(\partial y^{+}\right)^{2}}+$ $\left(\frac{C_{2}^{2}}{C_{6}}\right) C_{T} \Omega T^{+} S_{\varphi}$

## - Heat equation

$V_{x}^{+} \frac{\partial T^{+}}{\partial x^{+}}+\left(\frac{C_{1} C_{2}}{C_{3}}\right) V_{y}^{+} \frac{\partial T^{+}}{\partial y^{+}}+\frac{V_{\varphi}^{+}}{r^{+}} \frac{\partial T^{+}}{\partial \varphi^{+}}=\left(\frac{C_{1}^{2} C_{2}}{\operatorname{Re}_{\infty} \operatorname{Pr}}\right) \frac{\partial^{2} T^{+}}{\left(\partial y^{+}\right)^{2}}$

## - The dimensionless boundary conditions

On the wall $y^{+}=0 ; T^{+}=1 ; V_{x}^{+}=0 ; V_{y}^{+}=0$;
Away from the wall $y^{+}=\infty ; T^{+}=0 ; V_{x}^{+}=\left(\frac{C_{2}}{C_{4}}\right) U e_{x}^{+} ; V_{\varphi}^{+}=\left(\frac{C_{2}}{C_{4}}\right) U e_{\varphi}^{+}$

- Nusselt number:

$$
N u=-\left(\frac{C_{T} C_{1}}{C_{6}}\right)\left(\frac{\partial T^{+}}{\partial y^{+}}\right)_{y^{+}=0} \text { or } N u\left(\frac{C_{6}}{C_{T} C_{1}}\right)=-\left(\frac{\partial T^{+}}{\partial y^{+}}\right)_{y^{+}=0}
$$

- Friction coefficients :
$C f_{U}=L_{C f}\left(\frac{\partial V_{x}^{+}}{\partial y^{+}}\right)_{y^{+}=0} ; C f_{W}=L_{C f}\left(\frac{\partial V_{\varphi}^{+}}{\partial y^{+}}\right)_{y^{+}=0} ; L_{C f}=\frac{2}{\operatorname{Re}_{\infty}}\left(\frac{C_{1}}{C_{2} C_{5}^{2}}\right)$


## III. DIGITAL RESOLUTION

The field of study is subdivided NxMxL curvilinear parallelepipeds attached to the body of the cone and defined by dimensionless steps $\Delta x_{+}, \Delta y_{+}, \Delta \varphi_{+}$, so that it is described by: $(L-1) \Delta x_{+},(M-1) \Delta y_{+},(N-1) \Delta \varphi_{+}, \mathrm{N}$ and L are the numbers of meridians and parallels.
The equations of continuity, momentum and heat associated with boundary conditions are discretized using an implicit finite difference method.To simplify the quantities, let $\mathrm{U}, \mathrm{V}, \mathrm{W}$ be the components
of the velocity $\left(V_{x}^{+}, V_{y}^{+}, V_{\varphi}^{+}\right)$and by T the dimensionless temperature $T^{+}$. Likewise for the dimensionless size of the modulus of the external speed by $\mathrm{Ue}=U e^{+}, x_{P}=x^{+}$and by $y_{P}=y^{+}$. The equations of momentum and heat are in the form of:
$A_{j} X_{j-1}+B_{j} X_{j}+C_{j} X_{j+1}=D_{j} \quad 2 \leq j \leq J \max -1$
Where X represents the quantities $\mathrm{U}, \mathrm{W}$ and T, and Jmax characterizes the thickness of the boundary layer. The system of equations associated with the discretized boundary conditions are solved by the Thomas algorithm. The normal component V of velocity is deduced from the continuity equation:

$$
\begin{equation*}
V_{i+1 ; j+1}^{k}=V_{i+1 ; j}^{k}-\left(\frac{C_{3}}{C_{1} C_{2}}\right) \Delta y^{+}\left[\frac{\left(U_{i+1 ; j}^{k}-U_{i ; j}^{k}\right)}{\Delta x^{+}}+\frac{1}{r_{i+1}^{+}} \frac{\left(W_{i+1 ; j}^{k+1}-W_{i+1 ; j}^{k-1}\right)}{2 \Delta \varphi^{+}}+\frac{U_{i+1 ; j}^{k}}{\Delta x^{+}}\left(1-\frac{r_{i}^{+}}{r_{i+1}^{+}}\right)\right] \tag{27}
\end{equation*}
$$

With, $1 \leq i \leq N-1 ; 1 \leq k \leq L-1 e t 2 \leq j \leq j \max -1$
The convergence within the boundary layer is achieved when the following criteria are simultaneously checked for $\mathrm{U}, \mathrm{W}$ and T :
$\left|\frac{\left|X^{(p+1)}\right|-\left|X^{(p)}\right|}{\operatorname{Sup}\left(\left|X^{(p+1)}\right|,\left|X^{(p)}\right|\right.}\right| \leq \varepsilon$
Where $\mathrm{X}=(\mathrm{U}, \mathrm{W}, \mathrm{T}), X^{(p)}$ and $X^{(p+1)}$ are respectively the values of the quantity X of the iterations p and $\mathrm{p}+1$.
Partial derivatives of local Nusselt number expressions and parietal friction coefficients are approximated by a three-point discretization.

## IV. RESULTS AND DISCUSSIONS

The results thus presented are obtained for a cone of revolution of length $L=1 \mathrm{~m}$, of demiangle of opening $\theta_{0}=20^{\circ}$, of temperature $\mathrm{T}_{\mathrm{P}}=373.15 \mathrm{~K}$ and one will take the Richardson number $\Omega$ $=1$, the Reynolds number $\operatorname{Re}=1$, the Prandtl number $\operatorname{Pr}=0.72$, the Grashoft number $\mathrm{Gr}=1$, the Eker number $\mathrm{E}_{\mathrm{K}}=1$ and the temperature of the fluids very far from the cone wall $\mathrm{T} \infty=273.15 \mathrm{~K}$.

First, we validated the numerical code by comparing the results of our calculations with those of Rakotomanga [1]. The Figure 2, illustrating the evolution of the dimensionless temperature T and the meridian component $U$ of the velocity of the fluid particles as a function of the dimensionless normal coordinate $\mathrm{y}_{\mathrm{p}}$ close to the stopping point. This figure shows that our results are in good agreement; the relative differences are almost zero. Then, the curves of Figure 3.a, illustrating the variations of $U$ and shows that this dimensionless meridian component is slightly increasing of $x_{P}$ on the totality of the surface of the cone and abruptly increasing just at the upper end where $x_{P}=1$ fault of the disturbance. These results corroborate with those of the evolution of the dimensionless meridian coefficient of friction $\mathrm{Cf}_{\mathrm{U}}$, represented by the quantity $\left(\frac{C_{1} \cdot \operatorname{Re}}{C_{2} \cdot C_{5}^{2}}\right) C f_{U}$ of Figure 8.a. The positive values of $\mathrm{Cf}_{\mathrm{U}}$ explain the adherence phenomenon of the fluid particles on the surface of the cone by the boundary layer. Figure $3 . b$ shows that this meridian component $U$ evolves in a sinusoidal way as a function of. The growth of the angle of inclination, increases the amplitude of the sinusoid, it is constant there is a privileged point located in the vicinity of the corresponding meridian $\varphi=90^{\circ}$ where at this point, the component $U$ does not depend on the inclination angle. These results thus corroborate with the evolution of the dimensionless normal component V of the particle velocity (Figure 5.b) and those of the dimensionless meridian coefficient of friction $\mathrm{Cf}_{\mathrm{U}}$ as shown in Figure 8.b. The curves in Figure 3.c show us the dimensionless meridian component $U$ is a linearly increasing function of $y_{p}$. In general, the influence of the angle of inclination of the solid body is reflected by the variation of the angle of inclination, the increase of $\alpha$ slightly increases the thickness of the boundary layer and decreases the values of U. Similarly to U, the Figure $4 . a$ shows for the
dimensionless azimuthal component W , the fluid particles evolve slightly decreasing as a function of $\mathrm{x}_{\mathrm{P}}$. These results corroborate with those of the $\mathrm{Cf}_{\mathrm{w}}$ azimuthal wall friction coefficient, which is represented by the dimensionless quantity $\left(\frac{C_{1} \cdot \operatorname{Re}}{C_{2} \cdot C_{5}^{2}}\right) C f_{W}$ of the Figure 9.a. The positive values of $\mathrm{Cf}_{\mathrm{W}}$ explain the adherence phenomenon of the fluid particles on the boundary layer. Similarly for the Figure 4.b, the component W is a sinusoidal function of, its value is maximum at the meridian $\varphi=90$ ${ }^{\circ}$ and zero at the meridians $\varphi=0^{\circ}$ and $\varphi=180^{\circ}$. Note that increasing the angle of inclination increases the amplitude of the sinusoid and is constant, there is an axis of symmetry of the sinusoid located near the meridian corresponding to $\varphi=90^{\circ}$ where in this meridian, the component W reaches its maximum value. These results corroborate with those of the $\mathrm{Cf}_{\mathrm{w}}$ azimuthal wall friction coefficient of the Figure 9.b. The positive and zero values of $\mathrm{Cf}_{\mathrm{w}}$ explain the phenomena adhesion $\left(\varphi=90^{\circ}\right)$ and delamination ( $\varphi=0^{\circ}$ and $\varphi=180^{\circ}$ ).We represent in Figure 4.c the variations of the dimensionless azimuthal component W , the curves show that W is a linearly increasing function of yp . The increase of $\alpha$ slightly increases the thickness of the boundary layer and value of the component W. The Figure 5.a shows that the V component is more or less constant as a function of $\mathrm{x}_{\mathrm{P}}$ but very noticeable for the disturbance zones abrupt increase near the leading edge and abrupt decrease at the upper end of the cone. The negative values of the normal component of the velocity characterize a movement of the fluid particles towards the wall, that is to say, the wall of the cone aspirates the fluid particles. The positive values of V obtained especially for the large angles of inclination ( $\alpha \geq 45^{\circ}$ ), we speak of the phenomenon of fluid discharge by the cone. In general, the influence of the angle of inclination of the solid results in a slight increase in the thickness of the boundary layer and the phenomenon of change of direction of the fluid particles (suction or discharge) as shown in Figure 5.c.The Figures 6.a and 6.b show that the temperature remains constant as a function of xp and $\varphi$ except near the lower ends (stopping point) and higher where there are disturbances. Increasing the angle of inclination increases the temperature. These results corroborate with the evolution of the Nusselt number which is represented by the dimensionless quantity $\left(\frac{C_{6}}{C_{T} \cdot C_{1}}\right)$ Nus of Figures 7.a and 7.b but in reverse phenomenon. The results obtained agree that the intensity of heat exchange between the wall and the fluid is practically uniform along the surface of the cone, with the exception of the leading edge and at the upper end of the cone where the disturbances of the flow slightly decrease the heat exchange on the less exposed face. The influence of the angle of inclination is not negligible, the increase of the angle $\alpha$ causes a decrease in the intensity of heat exchange and this decrease in intensity is remarkable for the large inclinations of the body solid ( $\alpha \geq 45^{\circ}$ ).
Finally, the figure 6 .c shows the fluid temperature decreases linearly as a function of yp and this result explains the phenomenon of cooling of the temperature of the fluids in the boundary layer. In addition, it can be considered that increasing the inclination angle causes a slight increase in the thickness of the boundary layer.

## v. CONCLUSION

We conducted a numerical study of the flow and heat transfer in the boundary layer developed around a cone of revolution in an upward vertical forced flow. The conservation equations were solved by an implicit finite difference scheme associated with Thomas algorithm.
We have reported mainly the study of the influence of inclination on the components of velocity, temperature, Nusselt number and parietal friction coefficients. Thus, we have shown that a strong inclination of the cone generates a movement of the fluid particles in the boundary layer, suction and discharge of the fluid particles. Moreover, we have evidence of the existence of a privileged point for which the velocity component is independent of the angle of inclination and the axis of symmetry for
which the phenomenon of adhesion is maximal. However, the effect of the taper of the cone is relatively very small on the heat exchange and on the thickness of the boundary layer.


Figure 2 : Curves of comparison of the dimensionless component meridian of the velocity $U$ and temperature T against $\mathrm{yp}(\mathrm{j})$


Figure 3.a : Meridian component of the Velocity against $X_{P}$


Figure 3.c: Meridian component of the velocity against $\mathrm{yP}_{\mathrm{P}}$


Figure 3.b : Meridian component of the velocity against $\varphi$

Figure 4.a : Azimuthal component of the velocity against $X_{P}$


Figure 4.b : Azimuthal component of the velocity against $\varphi$


Figure 5.a : Normal component of the of the velocity against $X_{P}$


Figure 5.c : Normal component of the velocity against yp


Figure 4.c : Azimuthal component of the velocity against $\mathrm{yp}_{\mathrm{P}}$


Figure 5.b : Normal component of the velocity against $\varphi$


Figure 6.a: Temperature profile against $x_{P}$


Figure 6.b: Temperature profile against $\varphi$


Figure 7.a: Nusselt number against xp


Figure 6.c : Temperature profile against $y_{P}$


Figure 7.b:Nusselt number against $\varphi$

Figure 8.a : Meridian friction coefficient against xp



Figure 8.b Meridian friction coefficient against $\varphi$


Figure 9.a: Azimuthal friction coefficient against xp


Figure 9.b : Azimuthal friction coefficient against $\varphi$

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