American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-9, Issue-4, pp-241-249
www.ajer.org
Research Paper Open Access

Simplified Uniaxial Column Interaction Charts

Mohammed S. Al-Ansari[1], Muhammad S. Afzal[2*]
1(Professor, Department of Civil and Architectural Engineering, Qatar University, Doha, Qatar)
2(Teaching Assistant, Department of Civil and Architectural Engineering, Qatar University, Doha, Qatar)
**Corresponding Author : Mohammed S. Al-Ansari*

ABSTRACT :*This paper presents analytical method for generating the interaction diagrams for design of reinforced concrete (RC) columns. Due to the introduction of new classes in concrete compressive strength ($\boldsymbol{f'_c}$)with somewhat different parameters for the steel grades (f_y), it has become necessary to develop new interaction diagrams. These proposed interaction diagrams take into consideration the different values of gamma ($\boldsymbol{\gamma}$), concrete compressive strength ($\boldsymbol{f'_c}$) and different steel reinforcement ratios (**ρ**). The interaction diagram of any desire level of gamma (γ) can be generated to find the required axial load capacity (**Pc**) and moment capacity (**Mc**) of the columns with different reinforcement ratios. This study also analyzed some numerical examples using the proposed interaction charts to find the values of **Pc** and **Mc** for the uniaxial columns and their results obtained are later compared with the computer software (SP-Column). The data obtained from Interaction charts showed a promising result as the values are quite close to the ones obtained from the computer software.*
KEYWORDS: *Uniaxial columns, Interaction charts, Axial load capacity, Moment capacity.*

Date of Submission:30-03-2019 Date of acceptance: 13-04-2019

I. INTRODUCTION

Columns are the vertical compression members, which transmit loads from the upper floors to the lower levels and to the soil through the foundations [1]. Based on the position of the load on the cross section, columns are classified as concentrically loaded, Figure 1, or eccentrically loaded, Figure 2.

Eccentrically loaded columns are subjected to moments, in addition to axial force. The moments can be converted to a load ***P*** and eccentricity $\boldsymbol{e_X}$ and $\boldsymbol{e_Y}$. The moments can be uniaxial, as in the case when two adjacent panels are not similarly loaded, such as columns ***A*** and ***B*** in Figure 3.

The strength of reinforced concrete columns is determined using the following principles:

1. A linear strain distribution exists across the thickness of the column
2. There is no slippage between the concrete and the steel
3. The concrete strain at failure for strength calculations is set equal to 0.003mm/mm.
4. The tensile resistance of the concrete is negligible and disregarded

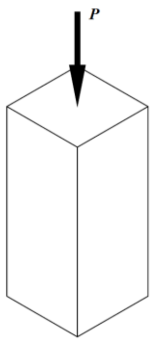

Figure 1: Concentrically Loaded columns

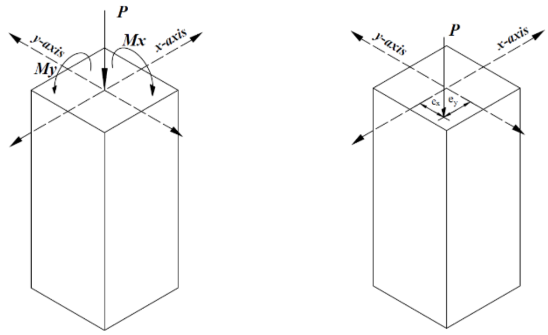

Figure 2: Eccentrically Loaded Column

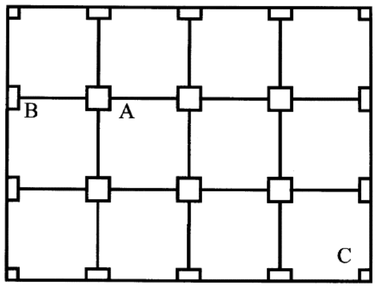

Figure 3: Uniaxially and Biaxially Loaded column

The strength of reinforced concrete columns is usually expressed using interaction diagrams [2] to relate the design axial load $\emptyset \boldsymbol{P_n}$ to the design bending moment $\emptyset \boldsymbol{M_n}$. Figure 4 explains the control points for the column interaction curve *(*$\emptyset P_n - \emptyset M_n$*)*. Each point on the curve represents one combination of design axial load $\emptyset P_n$ and design bending moment $\emptyset M_n$ corresponding to a neutral-axis location. The interaction diagram is separated into a tension control region and a compression control region. The balanced condition occurs when the failure develops simultaneously in tension (i.e., steel yielding) and in compression (concrete crushing).

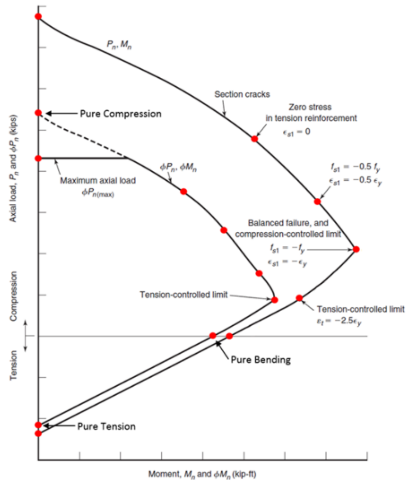

Figure 4: Control Points for Colum Interaction Curve ($\emptyset P_n - \emptyset M_n$) [3]

In this study, the proposed expressions for generating the interaction diagram for RC column are discussed. These interaction diagrams will also take into consideration the different values of gamma $(\boldsymbol{\gamma})$, concrete compressive strength $(\boldsymbol{f'_c})$ and different steel reinforcement ratio *(ρ)*. Numerical examples will also be analyzed using the interaction charts to find the values of Pc and Mc and their results will later be compared with the computer software (SP- Column) [9].

II. INTERACTION CHARTS FORMULATION – ACI CODE DESIGN

The stress and strain distribution of a rectangular column section for the calculation of Pu and Mu is given in Figure 5, [10-11].

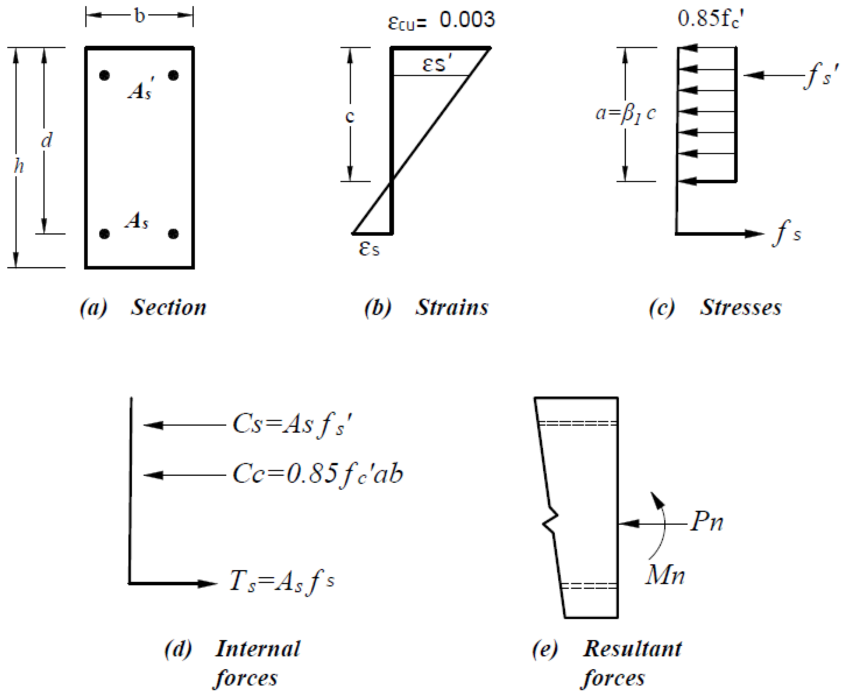

Figure 5: Calculation of Pu and Mu for given strain distribution

The resultant force $\mathbf{P_N}$ is equal to the summation of all internal forces.

$$P_N = C_{Con} - T_s + C_S$$

Similarly, the resultant Moment $\mathbf{M_N}$ is equal to the summation of all internal moments.

$$M_N = M_{conc} + M_T + M_{C_S}$$

Following steps revealed the calculation of the required internal forces and internal moments.

1. Plain Concrete Section:

$$C_{conc} = 0.8 \times (0.85\, f_c' b\, a)$$

$$C_{conc} = 0.68\, f_c'\, c\, b\, \beta c \qquad (1)$$

where;

C_{conc} = Internal concrete compression force

f_c'= Compressive concrete strength

b = Column width

a = Depth of the compression stress block

β = **0.85-0.008 (f_c' – 30) ≥ 0.65**

c = Distance from extreme compression fiber to neutral axis

Referring to the Figure 5, the moment about the midpoint of the section ($\mathbf{M_{conc}}$) can be computed as;

$$M_{conc} = C_c\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_{conc} = 0.68\, f_c'\, b\, a\left(\frac{h}{2} - \frac{a}{2}\right) \qquad (2)$$

The $\boldsymbol{\alpha_1}$ and $\boldsymbol{\beta_1}$ values for the plain concrete section are calculated as;

$$Setting\ \alpha_{1-conc} = \alpha_1 = \frac{C_{conc}}{f_c' bh} = 0.68 \times \frac{a}{h} \qquad (3)$$

$$Setting\ \beta_{1-conc} = \beta_1 = \frac{M_{conc}}{f_c' bh^2} = 0.68\left(\frac{h}{2} - \frac{a}{2}\right) \times \frac{1}{h} \times \frac{a}{h} \qquad (4)$$

2. Tension Steel Section:

The Internal Tensile force **Ts** is computed as;

$\boldsymbol{T_s = 0.68\, A_s f_s}$ (5)

where;

$\boldsymbol{A_s} =$ Area of tensile steel reinforcement

$\boldsymbol{f_s} =$ Computed steel stress in tensile steel

The value of the internal moment $\boldsymbol{M_T}$ is;

$\boldsymbol{M_T = 0.68\, A_s f_s \left(\frac{h}{2} - d'\right)}$ (6)

The $\boldsymbol{\alpha_2}$ and $\boldsymbol{\beta_2}$ values for the tension steel section are calculated as;

$$Setting\ \boldsymbol{\alpha_2} = \frac{T_s}{f_c' bh} = \frac{0.68\, A_s f_s}{f_c' b\, h} = \rho_2 \frac{0.68\, fs}{f_c'}$$

$$\boldsymbol{\alpha_2 = \rho_2 \frac{0.68\, f_y}{f_c'}} \tag{7}$$

$$Setting\ \boldsymbol{\beta_2} = \frac{M_T}{\boldsymbol{f_c' bh^2}} = \boldsymbol{0.68\, \frac{A_s f_s \left(\frac{h}{2} - d'\right)}{f_c' bh^2}} \tag{8}$$

Substituting the value of α_2 in the Eqn-8;

$$\boldsymbol{\beta_2 = \left(\frac{1}{2} - \frac{d'}{h}\right)\alpha_2} \tag{9}$$

where;

fy = Yield stress of reinforcing steel

$\boldsymbol{d'}$ = Distance from extreme compression fiber to centroid of reinforcing steel

3. Compression Steel Section:

The Internal compressive force **Cs** is computed as;

$$\boldsymbol{C_s = 0.8\, A_s' f_s'} \tag{10}$$

where;

$\boldsymbol{A_s'} =$ Area of compression steel reinforcement

$\boldsymbol{f_s'} =$ Computed compressive stress in compression steel

The value of the internal moment $\boldsymbol{M_T}$ is;

$\boldsymbol{M_T = 0.8\, A_s' f_s' \left(\frac{h}{2} - d'\right)}$ (11)

The $\boldsymbol{\alpha_3}$ and $\boldsymbol{\beta_3}$ values for the compression steel section are calculated as;

$$Setting\ \boldsymbol{\alpha_3} = \frac{C_s}{f_c' bh} = \frac{0.8\, A_s' f_s'}{f_c' b\, h} = \frac{0.8\, A_s'}{b\, h} \times \frac{f_s'}{f_c'} = \rho_3 \frac{0.8\, fs'}{f_c'}$$

$$\boldsymbol{\alpha_3 = \rho_3 \frac{0.8\, f_y}{f_c'}} \tag{12}$$

$$Setting\ \boldsymbol{\beta_3} = \frac{M_{Cs}}{\boldsymbol{f_c' bh^2}} = \boldsymbol{0.8\, \frac{A_s' f_s' \left(\frac{h}{2} - d'\right)}{f_c' bh^2}} \tag{13}$$

Substituting the value of α_3 in Eqn-13;

$$\boldsymbol{\beta_3 = \left(\frac{1}{2} - \frac{d'}{h}\right)\alpha_3} \tag{14}$$

4. Construction of Interaction Chart:

The column axial load capacity **Pc** is summation of all internal forces $\boldsymbol{P_N}$

$$\boldsymbol{P_c = \emptyset P_N}$$

where;

$\boldsymbol{P_N = C_{Con} - T + C_S}$ (15)

Therefore, $\boldsymbol{\alpha} = \alpha_1 - \alpha_2 + \alpha_3$

$$\alpha = 0.68 \times \frac{\boldsymbol{a}}{\boldsymbol{h}} - \rho_2 \frac{0.68\, f_y}{f_c'} + \rho_3 \frac{0.8\, f_y}{f_c'} \tag{16}$$

$$\boldsymbol{P_c = \emptyset\, \alpha\, b\, h} \tag{17}$$

For the moment capacity, the column moment capacity **Mc** is summation of all internal moments $\boldsymbol{M_N}$.

$$\boldsymbol{M_c = \emptyset M_N}$$

where;

$$\boldsymbol{M_N = M_{Con} + M_T + M_{C_S}} \qquad (18)$$

Therefore $\boldsymbol{\beta} = \beta_1 + \beta_2 + \beta_3$

$$\beta = 0.68\left(\frac{h}{2} - \frac{a}{2}\right) \times \frac{1}{h} \times \frac{\boldsymbol{a}}{\boldsymbol{h}} + \left(\frac{1}{2} - \frac{d'}{h}\right)\alpha_2 + \left(\frac{1}{2} - \frac{d'}{h}\right)\alpha_3 \qquad (19)$$

$$\boldsymbol{M_C} = \emptyset\, \boldsymbol{\beta b\, h^2} \qquad (20)$$

Computing the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ from the above equations (17) and (20).

$$\boldsymbol{\alpha = \frac{P_c}{bh} = \frac{P_N}{A_g}} \quad , \quad \boldsymbol{\beta = \frac{M_c}{bh^2} = \frac{M_N}{A_g h}}$$

The value of Gamma ($\boldsymbol{\gamma}$) for the column interaction chart is computed as;

$$\boldsymbol{\gamma = \frac{h - 2\,d'}{h}} \qquad (21)$$

Table 1 describes the values of $\frac{d'}{h}$ obtained against different values of $\boldsymbol{\gamma}$ which will be used in Eqn. 19.

Table 1: Values of $\boldsymbol{\gamma}$ vs $\frac{d'}{h}$.

Sr. No	γ	$\frac{d`}{h}$
1	0.6	0.2
2	0.7	0.15
3	0.8	0.1
4	0.9	0.05

Thus, by successfully assigning different values to $\frac{a}{h}$ and substituting in equations (16) and (19), the ($\alpha - \beta$) curve can be constructed (Figure 6).

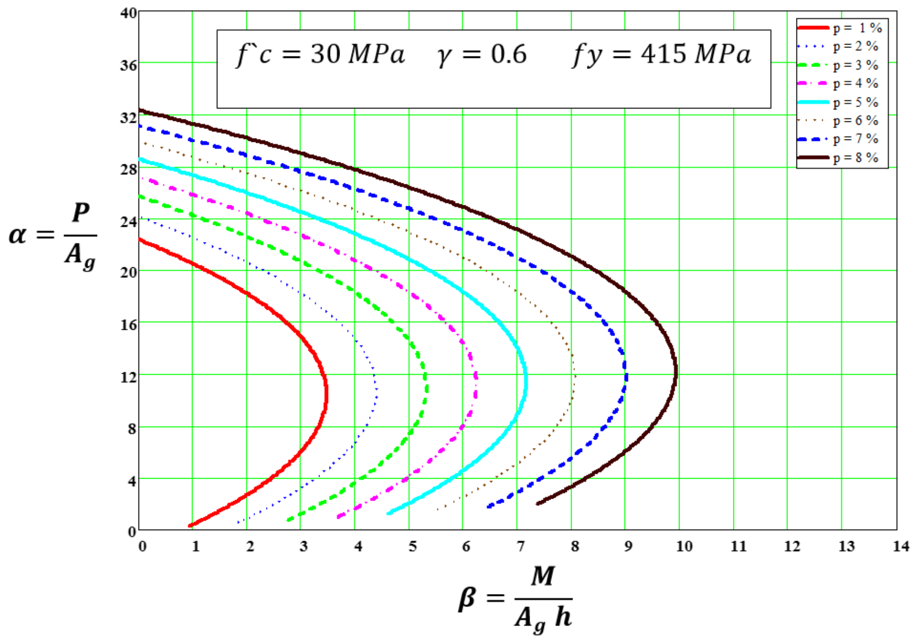

Figure 6: Column Interaction Diagram ($\boldsymbol{\beta - \alpha}$) for $\boldsymbol{\gamma}$=0.6

2.1 Steps to find the Pc and Mc

The following steps need to be followed to compute the values of **Pc** and **Mc** for an economical design.

Step-1: Find the value of $\boldsymbol{\beta_u}$ from the moments and the given cross-section.

$$\boldsymbol{\beta_u = \frac{M_u}{A_g h}}$$

Step-2: Find the value of α_u from the axial load and the given cross-section.

$$\alpha_u = \frac{Pu}{A_g}$$

Step-3: Extend a line through point (β_u , α_u) from the origin **(0,0)** to the desired **ρ** line.
Step-4: Determine the new points **(β , α)** on the desired **ρ** line.
Step-5: Compute $P_C = \emptyset \alpha b\, h$ and $M_C = \emptyset \beta b\, h^2$
Step-6: Check if the value of $P_C > Pu$ and $M_C > Mu$ for the design to beacceptable.
Step-7: For economical section, the value of ***Mc*** and ***Pc*** should be closer to ***Mu*** and ***Pu*** respectively

III. NUMERICAL EXAMPLES

An example is illustrated to compare the results obtained using the $(\beta - \alpha)$ column interaction charts with the finite element software.

A square column section of 400 mm x 400 mm with $\gamma = 0.6$, ϕ = 0.7 is loaded externally with an axial load of Pu =900 kN and with an External moment of Mu = 150 kN-m. The concrete compressive strength and steel yield strength are $f_C{}' = 30\ MPa$ and $f_y = 415\ MPa$ respectively.

Determine the column strength **Pc** and **Mc** for different reinforcement ratios (**ρ** =0.01 and **ρ**=0.04).

Solution:

The values of the **Pc** and **Mc** are determined by following the steps **1** to **7** and the values **α**and **β** for (**ρ** =0.01 and **ρ**=0.04) are reflected in column interaction diagram Figure 7.

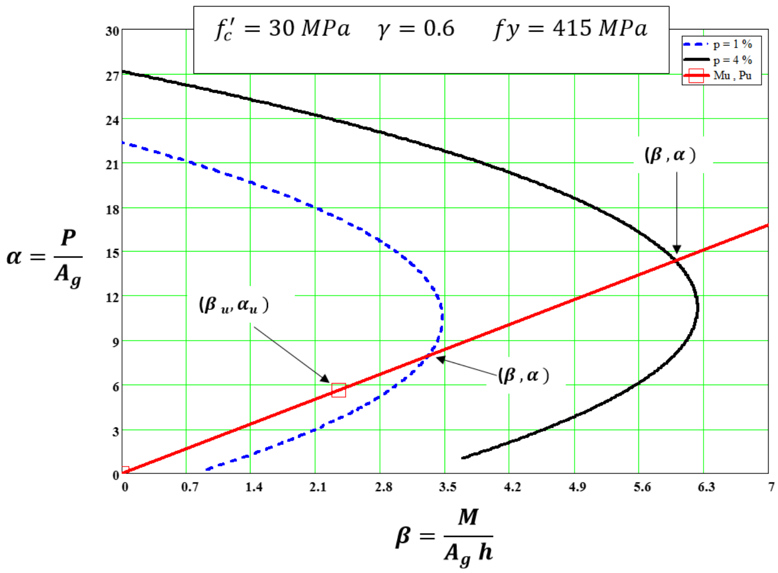

Figure 7: Column Interaction Diagram of square section (400 mm x 400 mm)

The results obtained are also compared with the Finite Element software **SP column** and are shown in Table 2.

Table 2: Square Column Design

Column Size (mm x mm)	Steel Reinforcement ratio (ρ)	β	α	Column Interaction Chart Pc (kN)	Column Interaction Chart Mc (kN-m)	SP Column Finite Element Pc (kN)	SP Column Finite Element Mc (kN.m)
400 x 400	0.01	3.231	7.9241	888	149	1099	183.31
400 x 400	0.04	5.9816	14.286	1710	285	1708	284

The column interaction diagram for other remaining values of **γ**= **0.7**, **0.8** and **0.9** with $f_C' = 30$ Mpa and f_y =415 MPa are displayed in Figures 8 to 10 respectively.

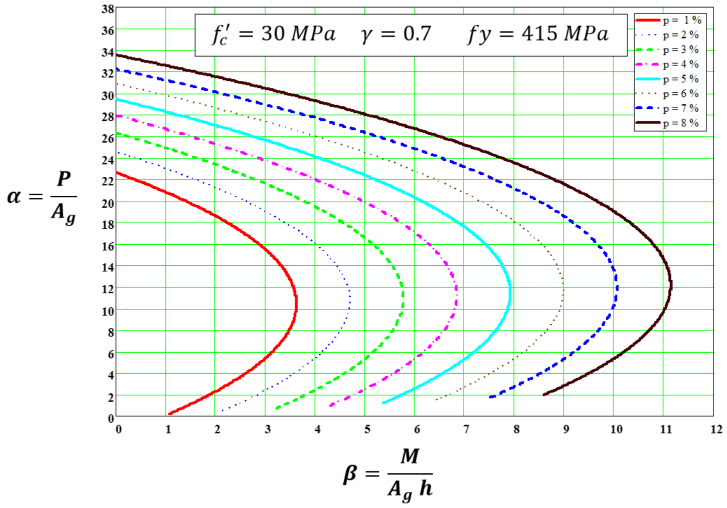

Figure 8: Column Interaction Diagram ($\beta - \alpha$) for γ=0.7

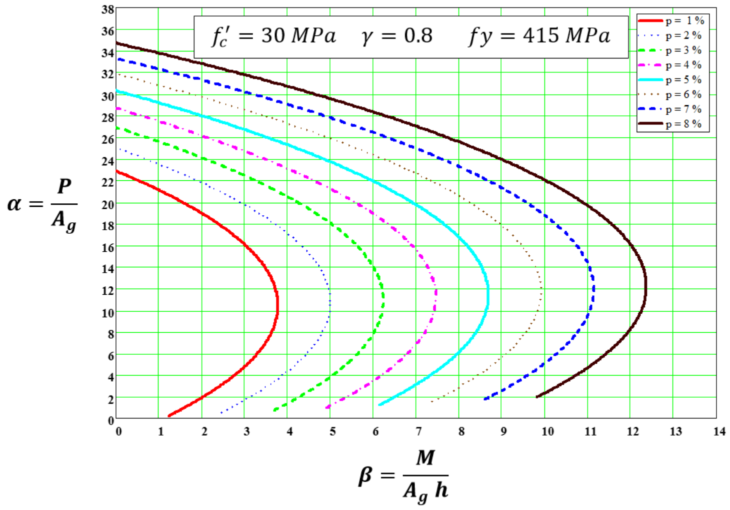

Figure 9: Column Interaction Diagram ($\beta - \alpha$) for γ=0.8

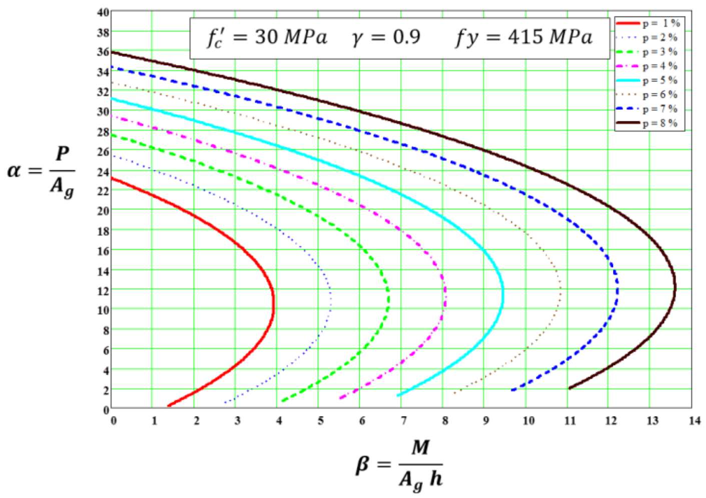

Figure 10: Column Interaction Diagram ($\beta - \alpha$) for γ=0.9

Some more examples for the uniaxial columns with different column sizes were also solved using the **($\beta - \alpha$)** chart and the results obtained were later compared with the Computer Software SP Column. These columns are having different reinforcement ratios (ρ)with different values of gamma (γ). The input data for these columns are given in Table 3.

Table 3: Input data for column design

Column Identifier	*Column Size (mm x mm)*	*f'_c (MPa)*	*f_y (MPa)*	*Pu (kN)*	*Mu (kN-m)*
C1	200 x 400	30	415	300	60
C2	200 x 400	20	300	200	50
C3	300 x 500	30	415	600	150
C4	300 x 500	40	300	800	192
C5	300 x 315	30	415	600	100

The above five (5) columns **C1** to **C5** were analyzed using the $(\boldsymbol{\beta} - \boldsymbol{\alpha})$ chart to find the values of ***Pc*** and ***Mc*** and these values were compared with the computer software. The results obtained are depicted in Table 4.

Table 4: Column design results

Column Identifier	*Steel ratio (ρ)*	*γ*	*β*	*α*	*ϕ*	*Column Interaction Chart*		*SP Finite Element*	
						Pc (kN)	*Mc (kN-m)*	*Pc (kN)*	*Mc (kN-m)*
C1	0.01	0.6	3.002	6.0268	0.7	338	67	433	86
C2	0.01	0.7	1.8972	3.01	0.7	106	67	253	63
C3	0.02	0.6	4.2801	8.556	0.65	834	209	1122	280
C4	0.02	0.7	4.6058	9.608	0.7	1009	242	1290	309
C5	0.08	0.6	9.2467	17.435	0.65	1071	180	1171	195

IV. RESULTS AND DISCUSSIONS

The results obtained from the Column interaction charts showed a safe and conservative column design strength when compared with the results obtained from the finite element software. The column (C5) with a higher reinforcement ratio *(ρ=8%)*, also showed promising results with the difference of only *9 %* with the finite element software. The bar charts in Figure 11 and 12 compares the values of Pc and Mc for the selected columns (C1 to C5) respectively.

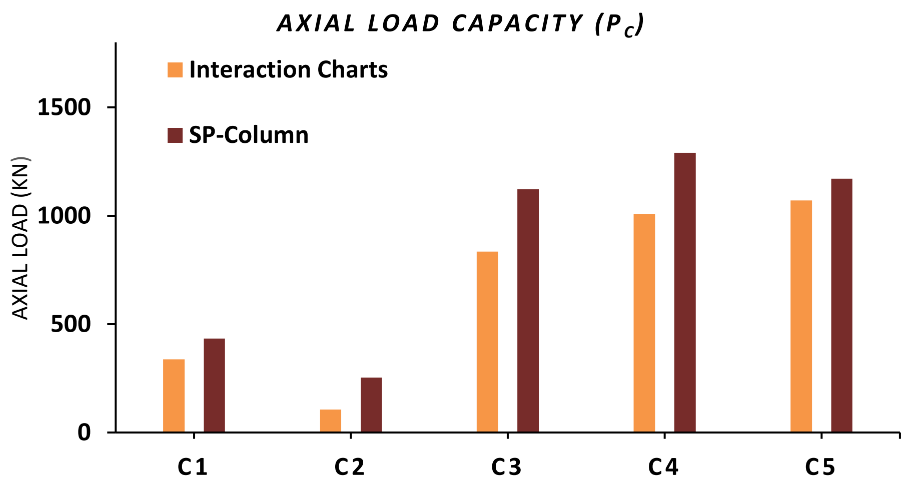

Figure 11: Axial Load capacity comparison (C1-C5)

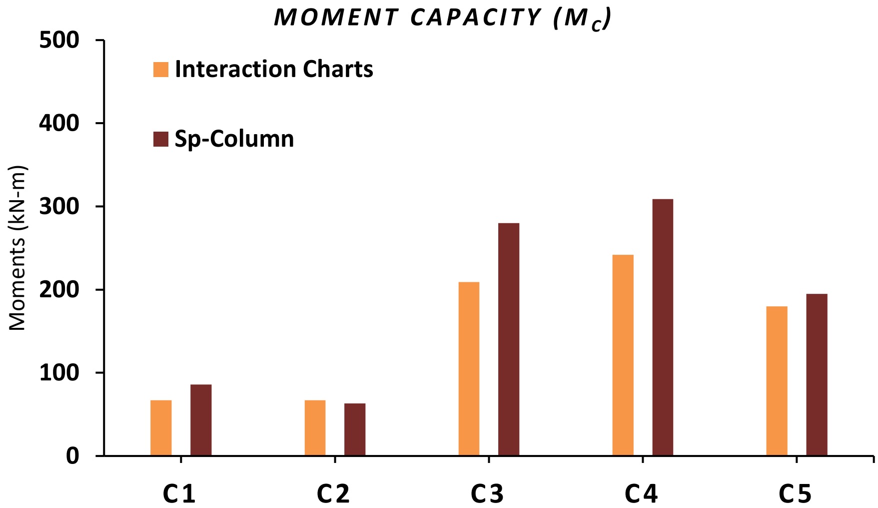

Figure 12: Moment capacity comparison (C1-C5)

V. CONCLUSION:

In the present work, an analytical model is derived for the hand computation of $(\beta - \alpha)$ interaction diagram of reinforced concrete column design. The charts with the different gamma values *(γ = 0.6, 0.7, 0.8 and 0.9)*, having different reinforcement ratios were formulated. Moreover, the charts for any desired value of gamma such as 6.5, 7.2 etc; can also be generated using the steps mentioned in this study to find the required values of *Pc* and *Mc* respectively. This study derives the interaction charts having the f_c'= 30 MPa and *fy* = 415 MPa but is not limited to these parameters. The charts can also be updated based on the required concrete compressive strength (f_c') and steel yield stress (f_y).

Several numerical examples of RC column design were analyzed by using the developed column interaction charts having different gamma values *(γ)* with different symmetrical reinforcement layout that have different reinforcement ratios ***(ρ)***. The analytical results obtained from these interaction charts are compared with the finite element software (SP-Column). The results obtained are in close agreement with the finite element method. The average variation of analytically computed values to the finite element software was not more than 10% which shows relatively satisfactory results.

Therefore, the developed interaction charts can help in finding the required ***Pc*** and ***Mc*** for the preliminary design of reinforced uniaxial concrete columns with symmetrical reinforcement layout.

REFERENCES

[1] M. S. Al-Ansari and Ahmed B. Senouci, "MATHCAD: Teaching and Learning Tool for Reinforced Concrete Design", International journal of Engineering Education, Vol-15,No. 1, pp. 64-71, (1999).
[2] Al-Sherrawi, Mohannad& M Salman, Hamza. "Analytical Model for Construction of Interaction Diagram for RC Columns Strengthened by Steel Jacket.International Journal of Science and Research (IJSR). 6. 324-328.10.21275/ART20177139 (2017).
[3] Interaction Diagram- Tied Reinforced Concrete Columns", [online] 2018,"https://www.structurepoint.org/pdfs/Interaction-Diagram-Tied-Reinforced-Concrete-Column-Symmetrical-ACI318-14.htm", (Accessed: 11 November 2018).
[4] M.Y.Rafiqa and C. Southcombea, "Genetic algorithms in optimal design and detailingof reinforced concrete biaxial columns supported by a declarative approach for capacitychecking", Journal of Computers and Structures, Elsevier, Vol. 69, Issue-4, Nov. (1998).
[5] J. A. Rodriguez and J. Dario Aristizabal-Ochoa, "Biaxial Interaction Diagrams for ShortRC Columns of Any Cross Section", ASCE Journal of Structural Engineering Volume -125 Issue 6 - June (1999).
[6] J. C. McCormac and Russell H. Brown, "Design Of Reinforced Concrete", Wiley; 10thEdition, USA, (2015).
[7] Zhenhai Guo, "Principles of Reinforced Concrete", Butterworth-Heinemann, (2014).
[8] N. Subramanian, "Design of Reinforced Concrete Structures", Oxford University Press, (2014).
[9] SP Column v5.10, "Design and Investigation of Reinforced Concrete Column Sections". 5420 Old Orchard Rd Skokie, IL 60077, USA, (2016).
[10] ACI-318-14, "Building Code Requirements for Structural Concrete", American ConcreteInstitute,ISBN: 978-0-87031-930-3, USA, (2014).
[11] James G. MacGregor and James K. Wight, "Reinforced Concrete: Mechanics andDesign", Prentice Hall, 6th Edition, (2011).

Mohammed S. Al-Ansari" Simplified Uniaxial Column Interaction Charts" American Journal of Engineering Research (AJER), vol.8, no.04, 2019, pp.241-249