Simplified Uniaxial Column Interaction Charts

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ABSTRACT: This paper presents analytical method for generating the interaction diagrams for design of reinforced concrete (RC) columns. Due to the introduction of new classes in concrete compressive strength ($f'_c$) with somewhat different parameters for the steel grades ($f_y$), it has become necessary to develop new interaction diagrams. These proposed interaction diagrams take into consideration the different values of gamma ($\gamma$), concrete compressive strength ($f'_c$) and different steel reinforcement ratios ($\rho$). The interaction diagram of any desire level of gamma ($\gamma$) can be generated to find the required axial load capacity ($P_c$) and moment capacity ($M_c$) of the columns with different reinforcement ratios. This study also analyzed some numerical examples using the proposed interaction charts to find the values of $P_c$ and $M_c$ for the uniaxial columns and their results obtained are later compared with the computer software (SP-Column). The data obtained from Interaction charts showed a promising result as the values are quite close to the ones obtained from the computer software.

KEYWORDS: Uniaxial columns, Interaction charts, Axial load capacity, Moment capacity.

I. INTRODUCTION

Columns are the vertical compression members, which transmit loads from the upper floors to the lower levels and to the soil through the foundations [1]. Based on the position of the load on the cross section, columns are classified as concentrically loaded, Figure 1, or eccentrically loaded, Figure 2.

Eccentrically loaded columns are subjected to moments, in addition to axial force. The moments can be converted to a load $P$ and eccentricity $e_x$ and $e_y$. The moments can be uniaxial, as in the case when two adjacent panels are not similarly loaded, such as columns A and B in Figure 3.

The strength of reinforced concrete columns is determined using the following principles:

1. A linear strain distribution exists across the thickness of the column
2. There is no slippage between the concrete and the steel
3. The concrete strain at failure for strength calculations is set equal to 0.003mm/mm.
4. The tensile resistance of the concrete is negligible and disregarded
The strength of reinforced concrete columns is usually expressed using interaction diagrams [2] to relate the design axial load $\Phi P_n$ to the design bending moment $\Phi M_n$. Figure 4 explains the control points for the column interaction curve $(\Phi P_n - \Phi M_n)$. Each point on the curve represents one combination of design axial load $\Phi P_n$ and design bending moment $\Phi M_n$ corresponding to a neutral-axis location. The interaction diagram is separated into a tension control region and a compression control region. The balanced condition occurs when the failure develops simultaneously in tension (i.e., steel yielding) and in compression (concrete crushing).

In this study, the proposed expressions for generating the interaction diagram for RC column are discussed. These interaction diagrams will also take into consideration the different values of gamma ($\gamma$), concrete compressive strength ($f'_c$) and different steel reinforcement ratio ($\rho$). Numerical examples will also be analyzed using the interaction charts to find the values of $P_c$ and $M_c$ and their results will later be compared with the computer software (SP-Column) [9].
II. INTERACTION CHARTS FORMULATION – ACI CODE DESIGN

The stress and strain distribution of a rectangular column section for the calculation of Pu and Mu is given in Figure 5, [10-11].

The resultant force $P_N$ is equal to the summation of all internal forces.

$$ P_N = C_{\text{con}} - T_s + C_s $$

Similarly, the resultant Moment $M_N$ is equal to the summation of all internal moments.

$$ M_N = M_{\text{conc}} + M_T + M_{\text{s}} $$

Following steps revealed the calculation of the required internal forces and internal moments.

1. Plain Concrete Section:

$$ C_{\text{con}} = 0.8 \times (0.85 f'_c b a) $$

$$ C_{\text{con}} = 0.68 f'_c c b \beta_c $$

where;

- $C_{\text{con}} = $ Internal concrete compression force
- $f'_c = $ Compressive concrete strength
- $b = $ Column width
- $a = $ Depth of the compression stress block
- $\beta = 0.85 \cdot 0.008 (f'_c - 30) \geq 0.65$
- $c = $ Distance from extreme compression fiber to neutral axis

Referring to the Figure 5, the moment about the midpoint of the section ($M_{\text{conc}}$) can be computed as;

$$ M_{\text{conc}} = C_c \left( h^2 - \frac{a^2}{2} \right) $$

$$ M_{\text{conc}} = 0.68 f'_c b a \left( \frac{h}{2} - \frac{a}{2} \right) $$

The $\alpha_1$ and $\beta_1$ values for the plain concrete section are calculated as;

$$ Setting \; \alpha_{1-\text{conc}} = \alpha_1 = \frac{C_{\text{con}}}{f'_c b h} = 0.68 \times \frac{a}{h} $$

$$ Setting \; \beta_{1-\text{conc}} = \beta_1 = \frac{M_{\text{conc}}}{f'_c b h^2} = 0.68 \left( \frac{h}{2} - \frac{a}{2} \right) \times \frac{1}{h} \times \frac{a}{h} $$
2. Tension Steel Section:

The Internal Tensile force $T_s$ is computed as;

$$ T_s = 0.68 A_s f_s $$

where;

$A_s = \text{Area of tensile steel reinforcement}$

$f_s = \text{Computed steel stress in tensile steel}$

The value of the internal moment $M_T$ is;

$$ M_T = 0.68 A_s f_s \left( \frac{h}{2} - d \right) $$

(6)

The $\alpha_2$ and $\beta_2$ values for the tension steel section are calculated as;

$$ \alpha_2 = \rho_2 \frac{0.68 f_y}{f_c} $$

$$ \beta_2 = \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_2 $$

(9)

Setting $\alpha_2 = \frac{T_s}{f_c bh} = \frac{0.68 A_s f_s}{f_c b h} = \rho_2 \frac{0.68 f_y}{f_c}$

(7)

$$ \alpha_2 = \rho_2 \frac{0.68 f_y}{f_c} $$

(7)

$$ \beta_2 = \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_2 $$

(9)

3. Compression Steel Section:

The Internal compressive force $C_s$ is computed as;

$$ C_s = 0.8 A_s' f_s' $$

(10)

where;

$A_s' = \text{Area of compression steel reinforcement}$

$f_s' = \text{Computed compressive stress in compression steel}$

The value of the internal moment $M_T$ is;

$$ M_T = 0.8 A_s' f_s' \left( \frac{h}{2} - d' \right) $$

(11)

The $\alpha_3$ and $\beta_3$ values for the compression steel section are calculated as;

$$ \alpha_3 = \rho_3 \frac{0.8 f_y}{f_c} $$

$$ \beta_3 = \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_3 $$

(14)

Setting $\alpha_3 = \frac{C_s}{f_c bh} = \frac{0.8 A_s' f_s'}{f_c b h} = \rho_3 \frac{0.8 f_y}{f_c}$

(12)

$$ \alpha_3 = \rho_3 \frac{0.8 f_y}{f_c} $$

(12)

$$ \beta_3 = \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_3 $$

(14)

4. Construction of Interaction Chart:

The column axial load capacity $P_c$ is summation of all internal forces $P_N$

$$ P_c = \emptyset P_N $$

(15)

where;

$$ P_N = C_{con} - T + C_s $$

Therefore $\alpha = \alpha_1 - \alpha_2 + \alpha_3$

$$ \alpha = 0.68 \frac{a}{h} - \rho_2 \frac{0.68 f_y}{f_c} + \rho_3 \frac{0.8 f_y}{f_c} $$

(16)

$$ P_c = \emptyset \alpha b h $$

(17)

For the moment capacity, the column moment capacity $M_c$ is summation of all internal moments $M_N$.
where,

\[ M_N = M_{\text{con}} + M_T + M_{\text{cs}} \]  

Therefore

\[ \beta = \beta_1 + \beta_2 + \beta_3 \]

\[ \beta = 0.68 \left( \frac{h}{2} - \frac{a}{2} \right) + \frac{1}{h} \times \frac{a}{h} + \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_2 + \left( \frac{1}{2} - \frac{d'}{h} \right) \alpha_3 \]  

\[ M_{C} = \beta b h^2 \]  

Computing the values of \( \alpha \) and \( \beta \) from the above equations (17) and (20).

\[ \alpha = \frac{P_C}{bh} = \frac{P_N}{A_g} \quad \beta = \frac{M_{C}}{bh^2} = \frac{M_{N}}{A_g h} \]

The value of Gamma (\( \gamma \)) for the column interaction chart is computed as;

\[ \gamma = \frac{h - 2 d'}{h} \]  

Table 1 describes the values of \( \frac{d'}{h} \) obtained against different values of \( \gamma \) which will be used in Eqn. 19.

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>( \gamma )</th>
<th>( \frac{d'}{h} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.7</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Thus, by successfully assigning different values to \( \frac{a}{h} \) and substituting in equations (16) and (19), the \( (\alpha - \beta) \) curve can be constructed (Figure 6).

![Figure 6: Column Interaction Diagram (\( \beta - \alpha \)) for \( \gamma = 0.6 \)](attachment)

2.1 Steps to find the \( P_C \) and \( M_C \)

The following steps need to be followed to compute the values of \( P_C \) and \( M_C \) for an economical design.

**Step-1:** Find the value of \( \beta_u \) from the moments and the given cross-section.

\[ \beta_u = \frac{M_u}{A_g h} \]
Step-2: Find the value of $\alpha_u$ from the axial load and the given cross-section.

$$\alpha_u = \frac{P_u}{A_g}$$

Step-3: Extend a line through point $(\beta_u, \alpha_u)$ from the origin $(0,0)$ to the desired $p$ line.

Step-4: Determine the new points $(\beta, \alpha)$ on the desired $p$ line.

Step-5: Compute $P_C = \frac{P}{ab} h$ and $M_C = \frac{M}{bh} h^2$.

Step-6: Check if the value of $P_C > P_u$ and $M_C > M_u$ for the design to be acceptable.

Step-7: For economical section, the value of $M_C$ and $P_C$ should be closer to $M_u$ and $P_u$ respectively.

III. NUMERICAL EXAMPLES

An example is illustrated to compare the results obtained using the $(\beta - \alpha)$ column interaction charts with the finite element software.

A square column section of 400 mm x 400 mm with $\gamma = 0.6$, $\phi = 0.7$ is loaded externally with an axial load of $P_u=900$ kN and with an External moment of $M_u = 150$ kN-m. The concrete compressive strength and steel yield strength are $f'_C = 30$ MPa and $f_y = 415$ MPa respectively.

Determine the column strength $P_C$ and $M_C$ for different reinforcement ratios ($\rho = 0.01$ and $\rho = 0.04$).

Solution:

The values of the $P_C$ and $M_C$ are determined by following the steps 1 to 7 and the values $\alpha$ and $\beta$ for ($\rho = 0.01$ and $\rho = 0.04$) are reflected in column interaction diagram Figure 7.

![Figure 7: Column Interaction Diagram of square section (400 mm x 400 mm)](image)

The results obtained are also compared with the Finite Element software SP column and are shown in Table 2.

<table>
<thead>
<tr>
<th>Column Size (mm x mm)</th>
<th>Steel Reinforcement ratio ($\rho$)</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>Column Interaction Chart</th>
<th>SP Column Finite Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 x 400</td>
<td>0.01</td>
<td>3.231</td>
<td>7.9241</td>
<td>888</td>
<td>149</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>5.9816</td>
<td>14.286</td>
<td>1710</td>
<td>285</td>
</tr>
</tbody>
</table>

The column interaction diagram for other remaining values of $\gamma = 0.7$, $0.8$ and $0.9$ with $f'_C = 30$ Mpa and $f_y = 415$ MPa are displayed in Figures 8 to 10 respectively.
Some more examples for the uniaxial columns with different column sizes were also solved using the $(\beta - \alpha)$ chart and the results obtained were later compared with the Computer Software SP Column. These
columns are having different reinforcement ratios ($\rho$) with different values of gamma ($\gamma$). The input data for these columns are given in Table 3.

<table>
<thead>
<tr>
<th>Column Identifier</th>
<th>Column Size (mm x mm)</th>
<th>$f_c$ (MPa)</th>
<th>$f_y$ (MPa)</th>
<th>$P_u$ (kN)</th>
<th>$M_u$ (kN-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>200 x 400</td>
<td>30</td>
<td>415</td>
<td>300</td>
<td>60</td>
</tr>
<tr>
<td>C2</td>
<td>200 x 400</td>
<td>20</td>
<td>300</td>
<td>200</td>
<td>50</td>
</tr>
<tr>
<td>C3</td>
<td>300 x 500</td>
<td>30</td>
<td>415</td>
<td>600</td>
<td>150</td>
</tr>
<tr>
<td>C4</td>
<td>300 x 500</td>
<td>40</td>
<td>300</td>
<td>800</td>
<td>192</td>
</tr>
<tr>
<td>C5</td>
<td>300 x 315</td>
<td>30</td>
<td>415</td>
<td>600</td>
<td>100</td>
</tr>
</tbody>
</table>

The above five (5) columns C1 to C5 were analyzed using the ($\beta - \alpha$) chart to find the values of $P_c$ and $M_c$ and these values were compared with the computer software. The results obtained are depicted in Table 4.

<table>
<thead>
<tr>
<th>Column Identifier</th>
<th>Steel ratio ($\rho$)</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\phi$</th>
<th>Column Interaction Chart</th>
<th>SP Finite Element</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>0.01</td>
<td>0.6</td>
<td>3.002</td>
<td>6.0268</td>
<td>0.7</td>
<td>338</td>
<td>433</td>
</tr>
<tr>
<td>C2</td>
<td>0.01</td>
<td>0.7</td>
<td>1.8972</td>
<td>3.01</td>
<td>0.7</td>
<td>106</td>
<td>253</td>
</tr>
<tr>
<td>C3</td>
<td>0.02</td>
<td>0.6</td>
<td>4.2801</td>
<td>8.556</td>
<td>0.65</td>
<td>834</td>
<td>1122</td>
</tr>
<tr>
<td>C4</td>
<td>0.02</td>
<td>0.7</td>
<td>4.6058</td>
<td>9.608</td>
<td>0.7</td>
<td>1009</td>
<td>1200</td>
</tr>
<tr>
<td>C5</td>
<td>0.08</td>
<td>0.6</td>
<td>9.2467</td>
<td>17.435</td>
<td>0.65</td>
<td>1071</td>
<td>1171</td>
</tr>
</tbody>
</table>

IV. RESULTS AND DISCUSSIONS

The results obtained from the Column interaction charts showed a safe and conservative column design strength when compared with the results obtained from the finite element software. The column (C5) with a higher reinforcement ratio ($\rho = 8\%$) also showed promising results with the difference of only 9% with the finite element software. The bar charts in Figure 11 and 12 compares the values of $P_c$ and $M_c$ for the selected columns (C1 to C5) respectively.

![AXIAL LOAD CAPACITY ($P_c$)](image)

Figure 11: Axial Load capacity comparison (C1-C5)
Figure 12: Moment capacity comparison (C1-C5)

V. CONCLUSION:

In the present work, an analytical model is derived for the hand computation of \((\beta - \alpha)\) interaction diagram of reinforced concrete column design. The charts with the different gamma values \((\gamma = 0.6, 0.7, 0.8\text{ and } 0.9)\), having different reinforcement ratios were formulated. Moreover, the charts for any desired value of gamma such as 6.5, 7.2 etc; can also be generated using the steps mentioned in this study to find the required values of \(P_c\) and \(M_c\) respectively. This study derives the interaction charts having the \(f'_c = 30\) MPa and \(f_y = 415\) MPa but is not limited to these parameters. The charts can also be updated based on the required concrete compressive strength \((f'_c)\) and steel yield stress \((f_y)\).

Several numerical examples of RC column design were analyzed by using the developed column interaction charts having different gamma values \((\gamma)\) with different symmetrical reinforcement layout that have different reinforcement ratios \((\rho)\). The analytical results obtained from these interaction charts are compared with the finite element software (Sp-Column). The results obtained are in close agreement with the finite element method. The average variation of analytically computed values to the finite element software was not more than 10% which shows relatively satisfactory results.

Therefore, the developed interaction charts can help in finding the required \(P_c\) and \(M_c\) for the preliminary design of reinforced uniaxial concrete columns with symmetrical reinforcement layout.

REFERENCES