Influence of Foundation’s Modulus of Elasticity on Stress Analysis in a Gravity Dam

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ABSTRACT: The foundation in concrete gravity dams is a prominent place in the stability analysis of these structures, since foundation defects or their influence are considered as one of the main mechanisms of deterioration and damage along the structure. This study consists of evaluating the influence of the foundation elasticity on a concrete gravity dam profile with typical dimensions for a Brazilian dam, in which this structure is subjected to gravitational loading (self-weight), hydrostatic pressure and under pressure. Thus, the main stresses and displacements at some points in the structure were evaluated by variation of the elasticity conditions of the foundation. The modeling of this dam-foundation system was analyzed using ANSYS APDL software whose idealization of the foundation soil and dam concrete were considered as materials elastic, homogeneous and linear properties. The results show that the modification of the elasticity of the foundation causes changes in the behavior of tensions and displacements in the structure.

KEYWORDS: Concrete Gravity Dam, Foundation, ANSYS, Stability.

I. INTRODUCTION

The foundation of concrete dams is a prominent region in assessing the stability of these structures. There are many cases of deterioration in this location, mainly due to the reservoir filling and emptying factors that cause a decrease and increase in the values of stress magnitude, besides the variation of the geomechanical behavior of the foundation over the years.

The variation of the efforts increases the stresses in the dam and may cause the instability of the structure and damage to the dam body and dam-foundation set. According to the Brazilian Committee of Large Dams - CBDB (2001) the global safety analysis should be made for all major structures, structural elements and interaction systems between foundations and structures subjected to various loading cases and will include the stability analysis in lower planes than the foundation, the definition of the safety coefficients and the verification between the acting stresses and the permissible stresses of the materials.

Understanding the deformability of the foundation is a very important subject in the design analysis of concrete gravity dams, making it more susceptible to different foundation stresses for more flexible soils. The change in the deformability of the foundation causes changes in the stress state, especially in the region close to the contact between the structure and the ground.

According to Silveira (2018), regarding the study of soil-structure interaction (ISE) in gravity dams, an important factor in the calculation of these structures, which demands special attention in their construction, is the knowledge of the deformability of its foundation that lists the applied requests and the resulting deformations, besides being a region that has frequent problems, due to the different repressions.

In light of the above, the authors of this work aim to evaluate the influence of the relationship between the elastic modulus of the foundation and the concrete by analyzing the stress behavior through this parametric study over points along the structure, besides comparing the results obtained by numerical methods with APDL software to analytical solutions from the gravity method at these points.
A. Main studies

Many researchers and engineers over the years have studied the influence of the foundation on the stability of concrete gravity dam structures. The first reported studies began with French and British engineers such as Sazilly (1853), Delocre (1866), Rankine (1881) and Levy (1895) who published milestones in the designs of these structures. For the most recent studies, the United States Bureau of Reclamation (USBR) in 1976 proposed the use of the Gravity Method in preliminary studies and its use in the calculation of projects in small dams. This idea was reinforced by Grishin (1982) and Jansen (1988). In sequence Lo et al. (1991) evaluated the stability of concrete dams on a rocky foundation by classifying dam-foundation contact into three categories: connected, weakly connected, and unconnected. Another prominent study refers to Ram Kumar and GC Nayak (1994) who presented a stress analysis through the Linear Elastic Fracture Mechanics, considering the variation of the following parameters: dam height, elastic modulus ratio and crest width.

In Brazil due to the fact that it is a country with numerous concrete dams, many researchers have recently studied the influence of the foundation, with emphasis on Gutseind (2003) and (2011) who basically analyzed the tensions and stability of these structures considering the influence of rock-dam contact deformability; In 2006 Ribeiro presented the entire analytical formulation for the gravity method initially developed by USBR; Nascimento Junior (2016) proposed a study related to the evaluation of the influence of the relationship between the modulus of elasticity of the foundation and the concrete of the dam, as well as studied and presented the stress concentration points in the structure and its modification relationship according to the foundation type. Still at the Brazilian research level, especially those developed by the Dynamics and Fluid-Structure Group (GDFE) of the University of Brasilia (UnB), the methodologies adopted to deal with the problems related to dam engineering have been highlighted, as observed in Oliveira (2002), Silva and Pedroso (2005), Coelho (2016) and Farias et al. (2017), Mendes (2018).

II. THEORETICAL FORMULATION

B. Study of the influence of the foundation

According to Tyelk (1938) apud Chen (2011) the gravity method assumes the validity of the "trapezoidal law", which assumes a linear variation of vertical tensions between upstream and downstream faces in all horizontal planes. This assumption is based on the theory of flat sections, where any horizontal plane remains flat after deformation. However, if a thin section (e.g. triangular profile dam) is placed on the foundation as a thin half-space, stresses are redistributed due to dam-foundation interaction (Figure 1).

![Fig.1. Stress redistribution due to dam-foundation interaction](image)

The solution of this gravity dam contact problem has been studied in many countries by rigorous theoretical and experimental tools. These analyzes indicate that the assumption "trapezoidal law" is not valid around the dam base (about 1/3 - 1/4 of the dam height) due to the concentration of heel and foot stresses of the profile, which is attributable to the deformation adjustment for consistency between the body and the dam foundation. Tensions in this case mainly depend on the elastic properties of the dam and foundation: the Young's modulus and the Poisson's ratio $\nu_c$ and $\nu_r$.

The Figure 2 shows the distribution of vertical stresses at different levels of $E_c/E_r$ for empty (a) and full (b) reservoir situations. For the empty reservoir, the $E_c/E_r$ relation gives higher stress concentrations for $\sigma_y$ and $\tau$. For the full
reservoir, if \( E_c/E_r \) is very small (ie, a flexible foundation), \( \sigma_y \) has traction on both upstream and downstream edge; when \( E_c/E_r \approx 1 \), there are higher concentrations of stresses at the dam heel; if \( E_c/E_r \) relation is very large (ie, rigid foundation), stresses are increasingly concentrated on both the downstream and upstream edge.

![Fig. 2. Vertical, horizontal and shear stresses influenced by modification of foundation elasticity considering (a) Self Weight and (b) Self Weight + Hydrostatic Pressure](image)

**C. Gravity Method**

The Gravity Method was developed in 1976 by the United States Bureau of Reclamation (USBR) responsible for water resources in the United States, and according to Ribeiro (2006), the method is used for preliminary studies of gravity dams, depending on the phase of design and the necessary information and can be used for the definitive design of straight gravity dams where retraction joints are not locked or grouted (USBR, 1976).

The main hypotheses considered in this method are:
- Concrete used in the dam is a homogeneous, isotropic and uniformly elastic material;
- There are no differential movements at the dam site due to hydrostatic loads acting on the reservoir;
All loads presented are transferred to the foundation by the “beam action”, i.e., the dam is analyzed in a unit width cut without lateral restriction;

- Normal stresses in horizontal planes vary linearly from upstream to downstream faces (Figure 3);
- Shear stresses in horizontal planes vary parabolically from the upstream face to the downstream face (Figure 4).

![Normal Tensions to the Horizontal Plane - Adapted Ribeiro (2006)](image)

![Vertical or vertical shear stresses - Adapted Ribeiro (2006)](image)

The symbology, loading, and profile for the analysis for the gravity method are shown in Figure 5. Loading includes the dam's own weight, upstream and downstream hydrostatic pressure and underpressure.

![Typical cross section with usual load combination - Adapted Ribeiro (2006)](image)

1) Stresses normal to the horizontal plane ($\sigma_z$)
Based on the assumption of the classical theory of pressure bending beams, which leads to a linear variation of the normal stresses in the z direction ($\sigma_z$) along the section, we have:
\[ \sigma_z(y) = a + by \quad \text{Eq. (1)} \]

\[ a = \left( \frac{\sum W}{T} - 6\frac{\sum M}{T^2} \right) \quad \text{Eq. (2)} \]

\[ b = \left( \frac{12\sum M}{T^3} \right) \quad \text{Eq. (3)} \]

Where:
\( \sigma_z \) = stress normal to the horizontal plane;
\( \sum W \) = resulting from vertical forces in the considered section;
\( T \) = horizontal distance from the upstream face to the downstream face of the considered section;
a and b = constants to be determined
\( y \) = distance between the neutral line and the point where \( \sigma_z \) must be obtained in the section;

2) Horizontal or vertical shear stresses (\( \tau_{xy} = \tau_{yx} \))
Assuming a parabolic variation of shear stresses in a horizontal plane, we have:
\[ \tau_{xy}(y) = a_1 + b_1 y + c_1 y^2 \quad \text{Eq. (4)} \]

\[ a_1 = \tau_{xy}(0) = \tau_{zyd} \quad \text{Eq. (5)} \]

\[ a_1 = (\sigma_{zd} - p')\tan(\phi_d) \quad \text{Eq. (6)} \]

\[ b_1 = \frac{1}{T} \left( \frac{6\Sigma V}{T} + 2\tau_{zyu} + 4\tau_{zyd} \right) \quad \text{Eq. (7)} \]

\[ c_1 = \frac{1}{T^2} \left( \frac{6\Sigma V}{T} + 3\tau_{zyu} + 3\tau_{zyd} \right) \quad \text{Eq. (8)} \]

Where:
\( \tau_{zy}(y) \) = shear stress in a vertical or horizontal plane;
\( \sigma_{zd} \) = normal downstream stress;
\( p' \) = downstream hydrostatic pressure;
\( \phi_d \) = angle formed by the face of the downstream element with the vertical;
\( \Sigma V \) = horizontal resultant of forces above the section, equal to \( V + V' \);
\( \tau_{zyu} \) = upstream shear stress;
\( \tau_{zyd} \) = downstream shear stress;
\( a_1, b_1, c_1 \) = constants to be determined

3) Normal stresses to the vertical plane (\( \sigma_y \))
The distribution of normal stresses along the dam height is linear, but along any horizontal section it is a third degree parabolic:
\[ \sigma_y(y) = a_2 + b_2 y + c_2 y^2 + d_2 y^3 \quad \text{Eq. (9)} \]

\[ a_2 = a_1\tan(\phi_d) + p' \quad \text{Eq. (10)} \]

\[ b_2 = b_1\tan(\phi_d) + \frac{\partial a_1}{\partial z} \quad \text{Eq. (11)} \]
\[ c_2 = c_1 \tan(\phi_0) + \frac{1}{2} \frac{\partial a_1}{\partial z} \]  
Eq. (12)

\[ d_2 = \frac{1}{3} \frac{\partial c_1}{\partial z} \]  
Eq. (13)

Where:

\( a_2, b_2, c_2, d_2 \) = constants to be determined

### III. DATA AND MODEL DEFINITION

The dam dimensions are typical for a Brazilian profile to be studied are presented in Table 1 and Figure 6.

**Table 1. Dimensions of the typical profile of a Brazilian dam**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H )</td>
<td>80</td>
</tr>
<tr>
<td>( h_c )</td>
<td>20</td>
</tr>
<tr>
<td>( h_g )</td>
<td>15</td>
</tr>
<tr>
<td>( B )</td>
<td>70</td>
</tr>
<tr>
<td>( H_r )</td>
<td>72</td>
</tr>
</tbody>
</table>

![Fig. 5. Dimensions of the evaluated system and its discretization.](image)

The dam concrete has the following physical properties: specific mass \( \rho_c = 2400 \text{ kg/m}^3 \), Young's modulus \( E_c = 25000 \text{ MPa} \), Poisson's ratio \( \nu_c = 0.20 \). The foundation rock is assumed to be massless \( (\rho_f = 0 \text{ kg/m}^3) \) and the Young’s modulus \( E_f \) was varied for three cases with a Poisson's ratio \( \nu_f = 0.25 \).

The effect of the deformability of the foundation has been studied considering the variation of the interaction between the dam and the foundation by the ratio between \( E_f/E_c \), \( E_f \) being the modulus of elasticity of the foundation and \( E_c \) the modulus of elasticity of the concrete. For this study, 4 cases were analyzed: \( E_f/E_c = 1; 2; 5 \) and 20.

In the finite element modeling it was assumed that the foundation and dam material is linearly elastic, isotropic and homogeneous. For the structure and foundation used the Plane 183 finite element for the plane state of deformation. In the interface of the soil-structure interaction problem the chosen elements are the CONTA 172 and TARGE 169, which make the connection between the nodes and the elements in the contact surfaces.

The loads considered in this study are gravitational weight –dam self-weight- and hydrostatic pressure along the upstream busbar.

### IV. RESULTS

For this work, we analyzed the 4 cases for both types of loading: the state of construction that includes only the dam's self-weight and the state of operation that besides the dam's self-weight includes the hydrostatic pressure exerted on the upstream.
Were calculated vertical, horizontal and shear stresses for both types of loading at the interface between the dam and the foundation, ie at zero height. For vertical stress, the results were compared with the gravity method studied by Ribeiro (2006) and Nascimento Júnior (2016).

Because of a large range of results, it was decided to summarize these data in graphs showing the variation of elasticity and its respective tensions along the base of the structure. In addition, numerical results are presented for the extreme cases with the $E_f/E_c = 1$ (flexible foundation) and $E_f/E_c = 20$ (rigid foundation) ratios in both loading states.

The Figures 6 and 7 present the results for the vertical stresses evaluating the gravitational weight of the dam, the first represents the flexible foundation, while the second represents a rigid foundation. Looking at the results, you notice a reduction in the magnitude of the vertical stress values as the stiffness of foundation increases.

The Figures 8 and 9 present the results for vertical stresses evaluating the combination of gravitational weight of the dam and the hydrostatic pressure changing the elasticity of the foundation from flexible to rigid. Similarly to the one evaluated in the previous item, there is a reduction in the magnitudes of the vertical stress values as the stiffness of foundation increases.
values when the foundation becomes more rigid. In addition, hydrostatic pressure attenuates the upstream stresses calculated when considering only the dam weight and increases the stresses in the downstream.

These results show the influence of the foundation on the stress magnitudes along the dam, but with relevance to the contact region and at the points which are susceptible to stress concentrations: the heel and the foot of the dam.

Thus, the next results in Figures 10; 11; 12 and 13 present in detail the vertical, horizontal, shear and principal stresses along the contact region between the dam and the foundation.
Fig. 10. Vertical Stress for (a) PP and (PP + PH) uploads

Fig. 11. Horizontal Stress for (a) PP and (PP + PH) uploads

Fig. 12. Shear Stress for (a) PP and (PP + PH) uploads
V. CONCLUSION

From the results of this study it can be verified that the vertical stresses are much higher than the horizontal and shear stresses, which is clear when observing the magnitudes for the maximum or principal stresses expressed in Figure 13 with very high values, close to the vertical stresses.

For the vertical and principal stresses, it is observed the occurrence of highest magnitudes for the loading state at the upstream, and by the insertion of hydrostatic pressure in the operating state, the magnitudes in the upstream face are smoothed whereas the values increase in downstream face.

The effect of the deformability of the foundation causes an increase to the magnitudes of these stresses, especially at the critical points, like the heel and foot of the dam, in both loading states. In addition it can be seen that the results for a foundation with a rigidity of 50GPa already return satisfactory results.

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