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Application of The Wavelet Transform Modulus Maxima And Scale Normalization To Determine Geomagnetic Anomalies of Adjacent Sources

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ABSTRACT: *In the interpretation of potential field data, accurate determining of the position for the anomaly sources and their properties played a really important role. For geomagnetic anomalies of adjacent sources, they always superimpose upon each other not only in the spatial domain but also in the frequency domain, making the identification of these sources considerably problematic. In this paper, a new mother wavelet for effective analysis the properties of the close potential field sources has been used. By theoretical modeling, using the wavelet transform modulus maxima (WTMM) method, we set up a correlative function between the scale parameter and geomagnetic source depth. Moreover, a scale normalization on the wavelet coefficients is introduced to enhance resolution for the separation of these sources in the scalograms, getting easy detection of their depth. After verifying the reliability of the proposed method on the modeling data, a process for the location of the adjacent geomagnetic sources using the wavelet transform is indicateted, and then application for analyzing the geomagnetic data in the Mekong Delta, South Vietnam. The results of this interpretation are consistency with previously published results, but the level of resolution for this technique is quite coincidental with other methods using different geological data.*

KEYWORDS: *WTMM method, scale normalization, potential field data, adjacent sources, correlative function.*

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I. INTRODUCTION

Wavelet transforms originated in geophysics in the early 1980s for the analysis of seismic signals (Kumar *et al*, 1997). Since then, considerable mathematical advances in wavelet theory have enabled a suite of applications in numerous fields. In geophysics, wavelet has becoming a very useful tool because it demonstrated its outstanding capabilities in interpreting nonstationary processes that contains multiscale features, detection of singularities, explanation of transient phenomena, fractal and multifractal processes, signal compression, and some others (Kumar *et al*, 1997; Ouadfeul, 2006; Ouadfeul, 2007; Ouadfeul *et al*, 2010). It is anticipated that in the near future, significant further advances in understanding and modeling geophysical processes will result from the use of wavelet analyzing (Kumar *et al*, 1997). A sizable area of geophysics has been inherited the achievements of wavelet analysis that is interpretation of potential fields data. In this section, it was applied to denoising, separating of local or regional anomalies from the measurement field, determining the location of homogeneous sources and their properties (Fedi *et al*, 1998). Recently, Yang and his colleagues (2010) used the continuous wavelet transform based on complex Morlet wavelet, which had been developed to estimate the source distribution of potential fields. The research group built an approximate linear relationship between the pseudo – wavenumber and the depth of anomaly source, and then they established this method on the aeromagnetic data of the Huanghua depression in central China to define the distribution of volcanic rocks. However, moving from wavelet coefficient domain to pseudo – wavenumber field is quite complicated and takes a lot of time for calculation as well as analysis. In this paper, for a better delineation of source depths, a correlative function between the magnetic anomaly source depth and the scale parameter has been developed by our synthetic example. After discussing the performance of our technique on various source types, we adopt this method on geomagnetic data in the Mekong Delta, Southern Viet Nam to define their source distribution.

II. THEORETICAL BACKGROUND

2.1. The continuous wavelet transform and Farshad – Sailhac wavelet function

The continuous wavelet transform (CWT) of 1-D signal $f(x) \in L^2(R)$ can be given by:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x)\overline{\psi}\left(\frac{b-x}{a}\right)dx = \frac{1}{\sqrt{a}}\left(f * \overline{\psi}\right) \quad (1)$$

Where, a, $b \in R^+$ are scale and translation (shift) parameters, respectively; $L^2(R)$ is the Hilbert space of 1-D wave functions having finite energy; $\overline{\psi}(x)$ is the complex conjugate function of $\psi(x)$, an analyzing function inside the integral (1), $f * \overline{\psi}$ expresses convolution integral of $f(x)$ and $\overline{\psi}(x)$. In particularly, CWT can operate with various complex wavelet functions, if the wavelet function curve looks like the same form of the original signal.

To determine horizontal location and the depth of the magnetic anomaly sources, the complex wavelet function called Farshad-Sailhac (Tin and Dau, 2016) was used. It is given by:

$$\psi^{(FS)}(x) = \psi^{(F)}(x) + i\,\psi^{(S)}(x) \quad (2)$$

where, $$\psi^{(F)}(x) = \frac{4-2x^2}{\left(x^2+2^2\right)^{\frac{5}{2}}} - \frac{1-2x^2}{\left(x^2+1^2\right)^{\frac{5}{2}}} \quad (3)$$

$$\psi^{(S)}(x) = Hilbert(\psi^{(F)}(x)) \quad (4)$$

2.2. The wavelet transform modulus maxima (WTMM) method

Edge detection technique depended on the CWT was proposed by Mallat and Hwang (1992) correlated to constructing the module contours of the CWT coefficients on the signals to be analysed. To apply this technique, the implemented wavelet functions should be produced from the first or second derivative of a feature function which related to transfer field in the of potential field problems. Farshad - Sailhac wavelet function was proven to satisfy the requirements of the Mallat and Hwang method, so the calculation, analysis and interpretation for horizontal position as well as the depth of the regions having a strong magnetic anomalies are counted on the module component of the wavelet transform. The edge detection technique bases on the locations of the maximum points of the CWT coefficients in the scalogram. Accordingly, the edge detection technique using CWT is also called the "wavelet transform modulus maxima" method.

Yansun *et al* (1994) performed wavelet calculations on the gradient of the data signal to denoise and enhance the contrast in the edges detection method using CWT technique. This helps to better detect the location of small anomalies alongside the large sources because the gradient data has the property of amplifying the instantaneous variations of the signal. Therefore, in the following sections, we apply wavelet transformations on gradient magnetic anomaly instead of applying them on gravity anomaly to analyze the theoretical models and then application for actual data.

2.3. Determination of structural index

We denote $f(x, z=0)$ as measured data in the ground due to a homogeneous source located at $x = 0$ and $z = z_0$ with the structural index N. When we carry out the continuous wavelet transform on the $f(x, z=0)$ with the wavelet functions that are the horizontal derivative of kernel in the formula up field transformation, we obtain an equation related to the wavelet coefficients at two scale levels a and a':

$$W^{\gamma}_{f(x,z=0)}(x,a) = \left(\frac{a}{a'}\right)^{\gamma}\left(\frac{a'+z_0}{a+z_0}\right)^{-\beta} W^{\gamma}_{f(x,z=0)}(x',a') \quad (5)$$

Where, x and a are translation and scale parameters, respectively; β indicates the uniform level of the singulary sources; γ illustrates the order of derivatives of analysing wavelet functions.

According to Sailhac and his colleagues (2000), with the magnetic objects, the relationship between N, β, and γ is given by following formula:

$$N = -\beta - \gamma - 1 \quad (6)$$

For different positions x and x', the connection of scale parameters a and a' is given as follows:

$$\frac{a'+z_0}{x'} = \frac{a+z_0}{x} = const \quad (7)$$

In this paper, the structural index N of anomaly sources are determined by Farshard - Sailhac wavelet function with γ =2, thus the equation (5) can be rewritten as follows:

$$\left(\frac{1}{a}\right)^2 W^2_{f(x,z=0)}(x,a)(a+z_0)^{-\beta} = \left(\frac{1}{a'}\right)^2 W^2_{f(x,z=0)}(x',a')(a'+z_0)^{-\beta} = const \quad (8)$$

Using short notation $W^2_{f(x,z=0)}(x,a) = W_2(x,a)$ and taking the logarithm for both sides of (8) we derive a new expression:

$$\log\left(\frac{W_2(x,a)}{a^2}\right) = \beta \log(a+z_0) + c \quad (9)$$

Where, c is constant related to the $const$ in the right side of equation (8). Therefore, the structural index determination will be done by the estimation for the slope of a straight line:

$$Y = \beta.X + c \quad (10)$$

Where, $Y = \log\left(\frac{W_2(x,a)}{a^2}\right)$, and $X = \log(a+z_0)$

By determining the structural index, we can estimate the relative shapes of the magnetic anomaly sources (table 2).

2.4. The scale normalization

In real cases geomagnetic anomalies of adjacent sources, the superposition of total intensitiy magnetic field, related to different factors such as: position, depth, and the size of component sources. In this case, the maxima associated with bigger anomalies in the scalograms of wavelet coefficients modulus are often dominating those associated with smaller anomalies, making the identification of magnetic sources problematic. To overcome the aforementioned problems, the scale normalization scheme was applied to shorten the gap of wavelet transform coefficient modulus in the scalograms between the large anomalies and small anomalies. Thus, facilitating location of adjacent sources is easier, especially for baby sources.
To separate potential field of adjacent sources from the scalogram, we have introduced a scale normalization a^{-n} on the 1D continuous wavelet transform (equation (1)). Then the normalized 1-D CWT can be expressed as

$$W'(a,b) = a^{-n} \int_{-\infty}^{+\infty} f(x) \frac{1}{\sqrt{a}} \overline{\psi}\left(\frac{b-x}{a}\right) dx \quad (11)$$

Where n is a positive constant. When khi $n = 0$, there is no scale normalization, and the equation (11) returns equation (1). As analyzing some simple magnetic anomalies, we found that with the Farshad - Sailhac wavelet function, n can be changed from 0 to 1.5. When n increases, wavelet transform coefficients $W'(a,b)$ in expression (11) decrease and the gap of wavelet transform coefficient modulus in the scalograms between the large anomalies and small anomalies is shortened. So the resolution of the image is also more improved. In this paper, our team selected $n = 1{,}5$ (highest resolution) to interpretation potential field of adjacent sources from modeling data as well as actual data.

2.5. The relationship between scale and source depth

We note that a scale in the wavelet transform relates to the depth of anomaly sources. However, it is not the depth and does not provide a direct intuitive interpretation of depth. To interpret the scalogram through the theoretical models with the sources are built from the distinct shaped magnetic objects, the authors have shown a close linear correlation between the source depth z and the product of scale a and measured step Δ by the scaling factor k:

$$z = k.(a.\Delta) \quad (12)$$

The scaling factor k in the equation (12) leans on the structural index N of the source. The next, in the results and discussions, the component k will be determined and applied to estimate the depth of the singulary sources as analysing actual data.

III. RESULTS AND DISCUSSIONS

3.1. Theoretical models

3.1.1. Model 1: Simple anomaly sources

In this model, the magnetic source is homogeneous sphere with the radius of 1 km. It is magnetized vertically with magnetization intensity of 6 mA/m. The center of the sphere is located at horizontal coordination x = 35 km and vertical coordination z = 3 km. The measurement on the ground goes through the sphere and the length of the profile is 70 km, with the step size of $\Delta = 0.1$ km.

Fig. 1a and Fig. 1b are the total intensity magnetic anomaly and the gradient of the total intensity magnetic anomaly caused by the sphere in turn.

According to the results plotting of module in the Fig. 1c or Fig. 1d, the maximum point of the wavelet transform coefficients is easily found and it located at ($b = 350.0; a = 27.0$) or ($b = 350.0; a' = 6.1$). To multiply the value of b with measured step $\Delta = 0.1$ km, the horizontal location of the center anomaly source will be identified: $x = 350.0 \times 0.1 = 35$ km. This value of x is accordant with the parameter of the model. Therefore, the modulus maxima in the wavelet scalogram are capable of identifying the source horizontal position.

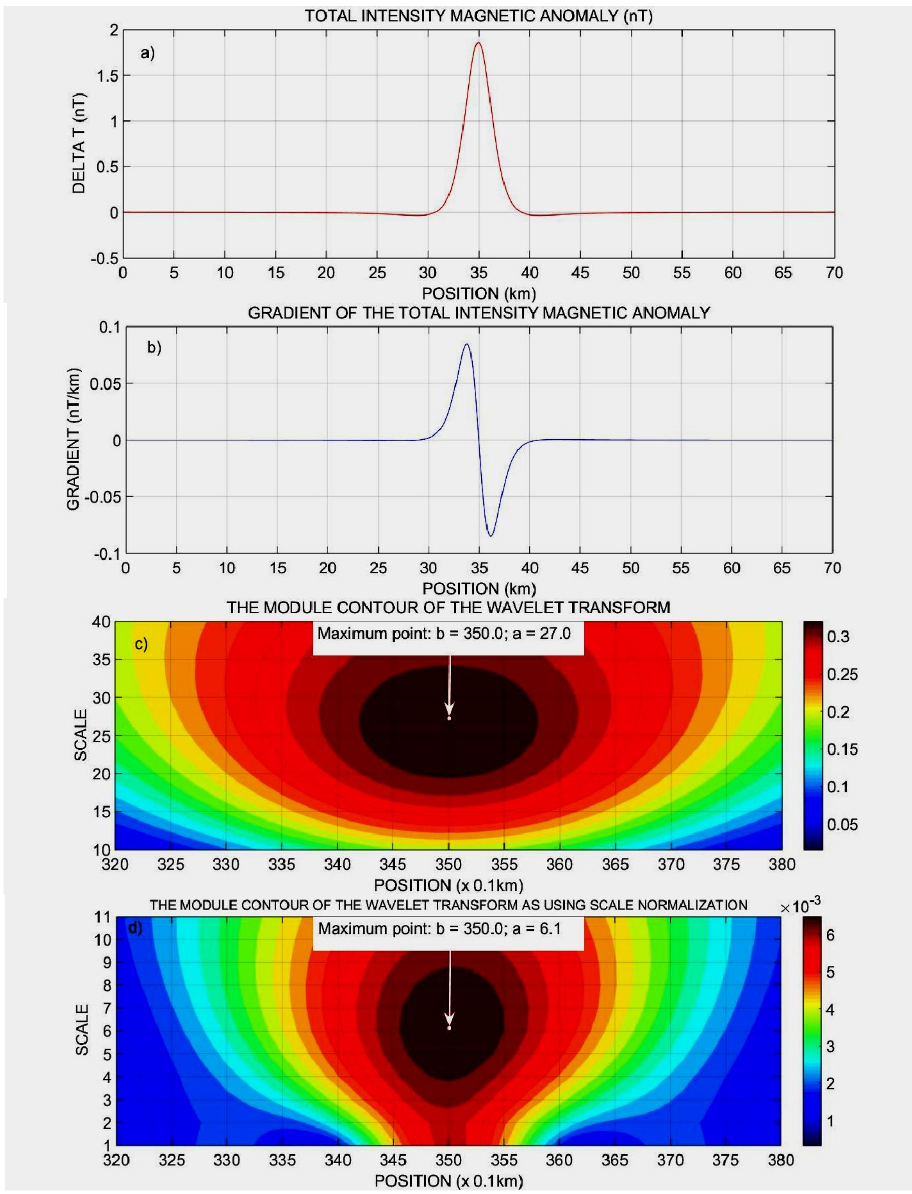


Fig. 1. The graphs of the model 1. a) The total intensity magnetic anomaly, b) The gradient of the total intensity magnetic anomaly, c) The module contours of the wavelet transform, d) The module contours of the wavelet transform as using scale normalization

The value of the scaling factor $a = 27.0$ or $a' = 6.1$ are related to the depth of the source, but it is not the depth. To find the correlative function between the depth z and scaling factor a or a' we take the value of z from 1 km to 6 km and repeated the survey process as well as $z = 3$ km. The survey results are represented in table 1 and figure 2.

Table1 1. Analytical results with Farshard-Sailhac wavelet function

z (km)	Δ (km)	a (n = 0)	(a. Δ)	a' (n = 1,5)	(a'.Δ)
1.5	0.1	13.6	1.36	2.8	0.28
2.0	0.1	18.2	1.82	4.0	0.40
2.5	0.1	22.6	2.26	5.0	0.50
3.0	0.1	27.0	2.70	6.1	0.61
3.5	0.1	31.6	3.16	7.2	0.72
4.0	0.1	35.8	3.58	8.2	0.82
4.5	0.1	40.2	4.02	9.2	0.92
5.0	0.1	44.8	4.48	10.0	1.00
5.5	0.1	49.2	4.92	11.0	1.10
6.0	0.1	53.6	5.36	12.0	1.20
6.5	0.1	58.2	5.82	13.2	1.32
7.0	0.1	62.6	6.26	14.0	1.40
7.5	0.1	67.0	6.70	15.2	1.52
8.0	0.1	71.6	7.16	15.8	1.58
8.5	0.1	76.0	7.60	17.0	1.70
9.0	0.1	80.2	8.02	18.0	1.80

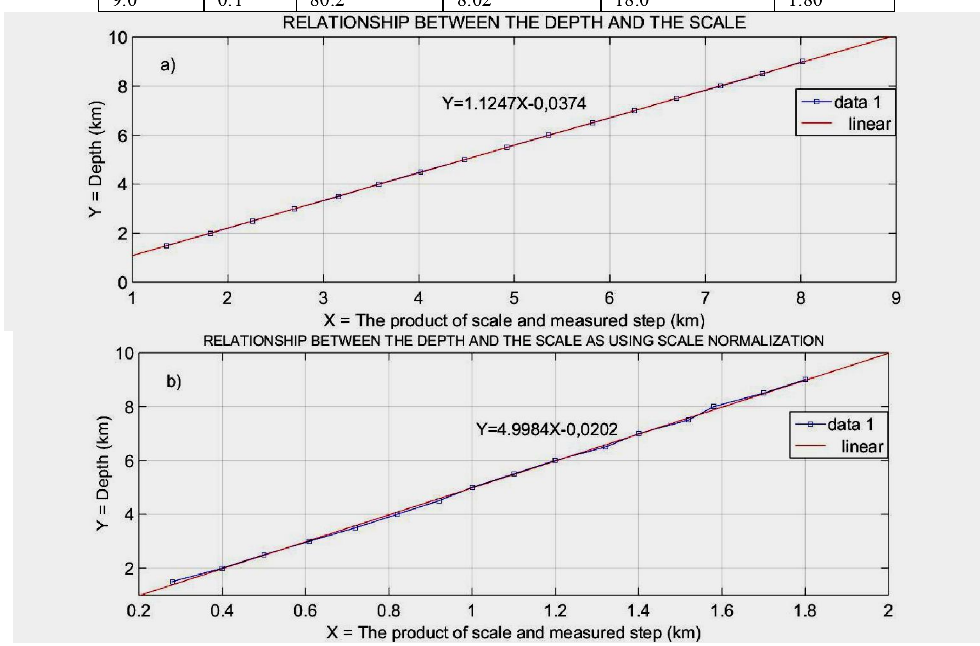


Fig. 2. The relationship between the depth and the product of scale and measured step
a) no scale normalization, b) using scale normalization

As can be seen in the Fig. 2, the approximate linear relationship between the scale parameter and magnetic source depth is determined:

$z \approx 1.1247 \times (a.\Delta)$ (km) as no scale normalization (13)

$z \approx 4.9984 \times (a'.\Delta)$ (km) as using scale normalization with $n = 1.5$ (14)

When magnetic sources are far away from the observe plane, they are usually assumed as spheres (Yang *et al*, 2010). Then the relative source depths can be estimated from the maximum points of the CWT coefficients in the scalogram by equation (13) or (14). However, often other simple sources, such as cylinder, thin bottomless dike, fault or contact, were used widely in the fact. Thus, it is necessary for us to check our technique with different form of sources insteady of spherical form. Explorative results the coefficient k or k' corresponding different shaped sources are presented in table 2.

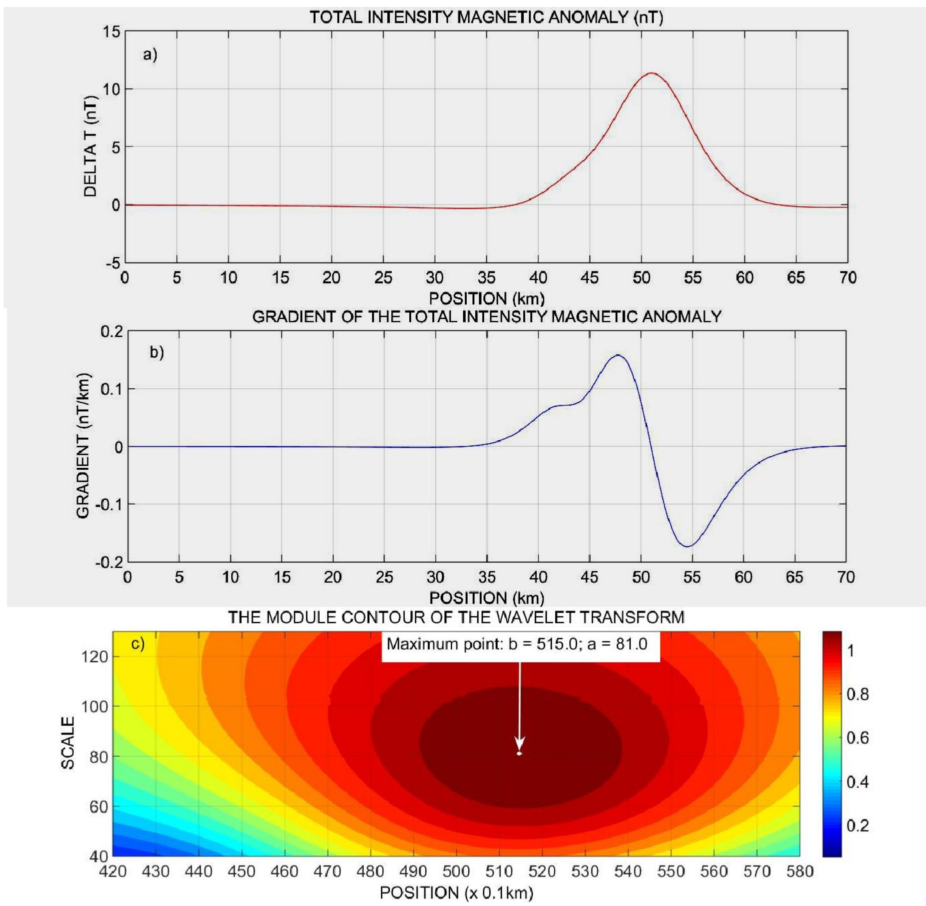
Table 2. Structural index *N* and equivalent parameter k or *k'*

Shaped source	Structural index *N*	k	*k'*
Sphere	3	1.1247	4.9984
Cylinder	2	1.0991	4.4214
Thin bottomless dike	1	0.5981	3.6475
Fault or contact	0	0.2026	2.0474

3.1.2. Model 2: Adjacent anomaly sources

We consider the magnetic total field anomaly (Fig. 3a) produced by a homogeneous sphere and a unified cylinder. They are magnetized vertically with the same magnetization intensity of 6 mA/m. The sphere has a radius of 5.5 km and is located at horizontal coordination x = 51 km and vertical coordination z = 9 km, while the cylinder is situated at horizontal coordination x = 43 km and vertical coordination z = 5.5 km with a radius of 1 km. The measurement on the ground goes through those anomaly objects and the length of the profile is 70 km, with the step size of Δ = 0.1 km.

As can be seen in the Fig. 3c, only one maximum point of the wavelet transform coefficients has been found, and it situated at ($b = 515.0; a = 81.0$) corresponding position of the sphere (large anomaly). Therefore, in this model, if applying only the method as model 1, we have considerably problematic to identify position of the cylinder (small anomaly) since the significantly strong effect of the magnectic field caused by the sphere.



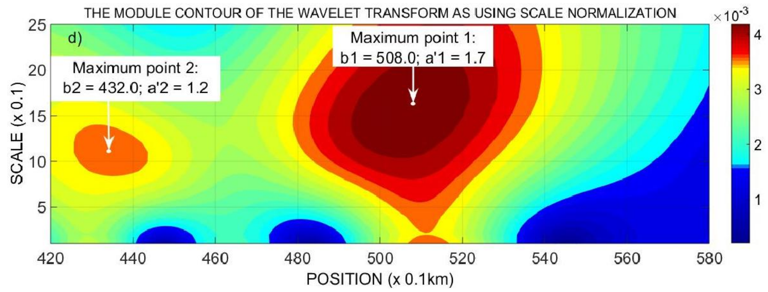


Fig. 3. The graphs of the model 2. a) The total intensity magnetic anomaly, b) The gradient of the total intensity magnetic anomaly, c) The module contours of the wavelet transform, d) The module contours of the wavelet transform as using scale normalization

To solve this problem, the scale normalization in the continuous wavelet transform (equation 11) on the gradient magnetic total field anomaly produced by two objects (Fig. 3b) has been used. The results plotting of module in the figure 3d show that there are two maximum points of the wavelet transform coefficients corresponding two sources caused anomalies, and they are located at: ($b_1 = 508.0$; $a_1^{'} = 1.7$) and ($b_2 = 432.0$; $a_2^{'} = 1.2$). Then, the horizontal and vertical location of the center anomaly sources will be identified: $x_1 = 508.0 \times 0.1 = 50.8$ km; $x_2 = 432.0 \times 0.1 = 43.2$ km; $z_1 = 4.9984 \times 0.1 \times 1.7 = 8.5$ km; $z_2 = 4.4214 \times 0.1 \times 1.2 = 5.3$ km. These values of x and z are accordant with the parameters of the model and having appropriate error. Thus, the modulus maxima in the wavelet scalogram combines with scale normalization are capable of identifying the adjacent geomagnetic sources and their properties.
From good results as analyzing the theoretical models, a process for determining the location of adjacent anomalous sources has been developed, and then application for actual data.

3.2. The process for the location of the magnetic anomaly sources using Farshard - Sailhac wavelet transform

The determination of the horizontal position and depth of the magnetic singulary sources using Farshard - Sailhac wavelet transform can be summarized in the process including the following steps:
Step 1: Taking the horizontal gradient of the magnetic anomaly along the measured profile.
Step 2: Performing Farshad - Sailhac wavelet transform with the horizontal gradient of the magnetic anomaly.
After carry out complex CWT, there are four distinct data sets: real part, virtual component, module factor, and phase ingredient. The module data will be used in the next step.
Step 3: Changing the different scales a and repeating the multiscale CWT.
Step 4: Plotting the module contours by the CWT coefficients with different scales a in the scalogram (a, b).
Step 5: Determining the position of the magnetic anomaly sources.
On the plot of module contours we are going to find the maximum points of the wavelet transform coefficients. The horizontal and vertical coordinate of there points are b_i and a_i, respectively (where i express numerical order of the sources). The position of the sources will be determined by following equation:

$$x_i = b_i \times \Delta \quad (15)$$

Step 6: Detecting the depth of the magnetic anomaly sources.
Calculating the structural index of the anomaly sources identified in the step 5. Next, estimating the relative shape of the sources, and then determining k_i or $k_i^{'}$ factors from table 2. The depth of the sources will be detected by following equation:

$$z_i = k_i .(a_i .\Delta) \quad \text{as no scale normalization} \quad (16)$$

$$z_i = k_i^{'} .(a_i^{'} .\Delta) \quad \text{as using scale normalization} \quad (17)$$

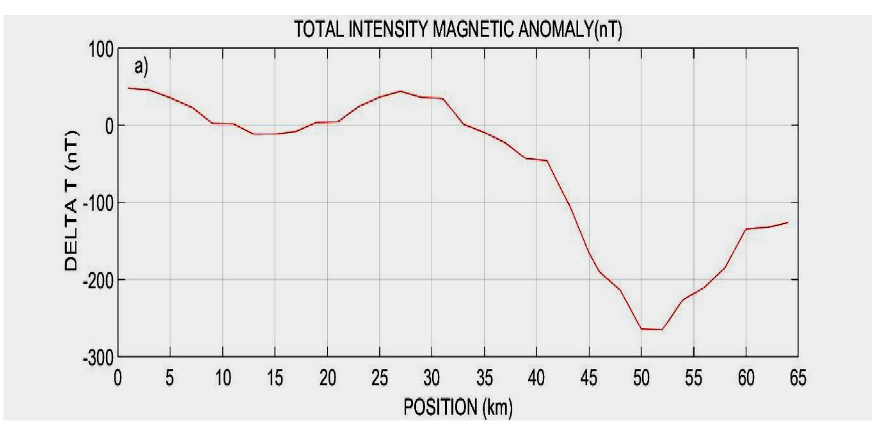
3.3. Analysis the magnetic data from the Mekong Delta

Application the process for the location of the magnetic anomaly sources using Farshard - Sailhac wavelet transform to analyse actual data, we have interpreted some of measured profiles on the map of total intensity magnetic anomaly in the Mekong Delta. The analysis results are good accuracy and fair compliance with the previous publication of the geological data. Nevertheless, in this paper, the research group only shows the interpretation results for Tra Vinh – Soc Trang profile.

Fig. 4. The profile survey on the aeromagnetic map at the 1/500.000 scale (Department of Geology and Minerals of Viet Nam)

The total intensity of aeromagnetic map at the 1/500.000 scale (Department of Geology and Minerals of Viet Nam) has been used. In this map, a profile 64 km long from Tra Vinh province (latitude $9^0 48'50''$ N, longitude $106^0 17'56''$ E) to Soc Trang province (latitude $9^0 36'10''$ N, longitude $105^0 58'26''$ E) (Fig. 4) is selected, then the data has been interpolated into regular points (step size 1.0 km) by cubic spline. Using the International Geomagnetic Reference Field (IGRF) from Kyoto University, the total intensity magnetic anomalies of the profile has been calculated. The results are represented in the Fig.5a, where includes a strong anomaly at position 51^{st} km.

As can be seen in the Fig. 5c there is only one the maximum point of the wavelet transform coefficients corresponding the large source caused significantly strong anomaly, and it is situated at: $x_1 = 51$ (km), $a_1 = 7.7$. To determine the small source in the profile, the scale normalization in the continuous wavelet transform (equation 11) on the gradient of the total magnetic field anomaly (Fig 5b) has been applied. The plotting results in the Fig. 5d shows two maximum points of the wavelet transform coefficients corresponding two anomaly sources, and they are located at: ($b_1 = 51.0$; $a_1^{'} = 1.7$) and ($b_2 = 43.0$; $a_2^{'} = 1.2$). Then, the horizontal of the center anomaly sources will be identified by equation (15): $x_1 = 51.0 \times 1 = 51$ km; $x_2 = 43.0 \times 1 = 43$ km.



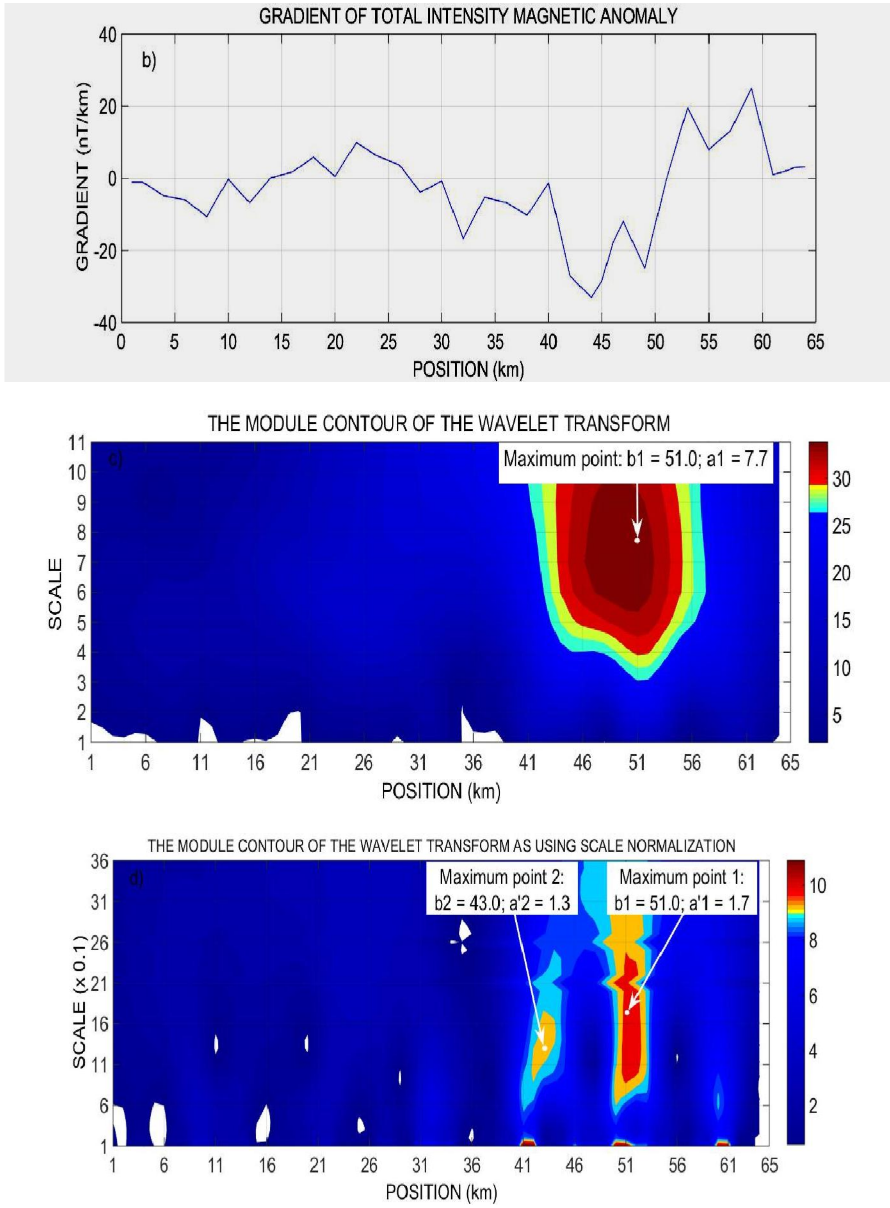
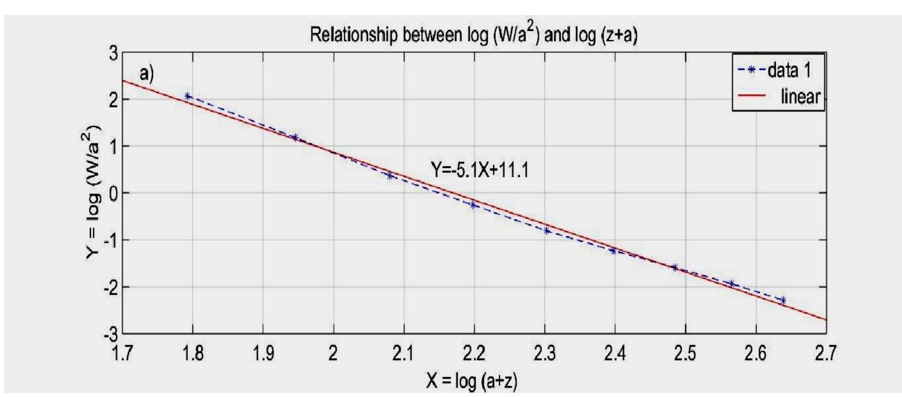


Fig. 5. The graphs of the actual data. a) The total intensity magnetic anomaly, b) The gradient of the total intensity magnetic anomaly, c) The module contours of the wavelet transform, d) The module contours of the wavelet transform as using scale normalization



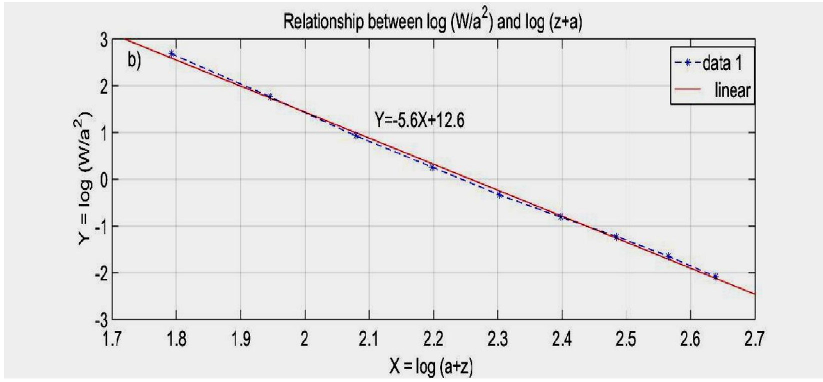


Fig. 6. The graphs of the relation between $\log(W/a^2)$ and $\log(a+z)$.
a) anomaly source 2 at 43rd km, b) anomaly source 1 at 51st km

Next, in order to determine the depth of these anomalous sources, the graphs of the relation between $\log(W/a^2)$ and $\log(a+z)$ are performed, and the results are displayed in the Fig 6.

Fig. 6b is the logarithm curve of wavelet transform $\log(W/a^2)$ with logarithm of $(a+z)$ of the anomaly source located at position of 51 km. Using the least square method to determine the equation of linear line: $Y=-5.6X+12.6$, so $\beta\approx-6$ (equation 10), thus, the structural index is estimated: $N=6-2-1=3$ (equation 6). Consequently, the source may be a sphere and the scaling factor $k=1.1247$ or $k'=4.9984$ (table 2). To multiply the scaling factor k with $(a_1.\Delta)$ or k' with $(a'_1.\Delta)$, the depth of the source at 51st km will be detected, and it is about 8.6 km. Taking a similar analysis for the other anomaly on the profile, we obtained the summarized results in table 3.

Table 3. The results of interpretation of Tra Vinh – Soc Trang profile

Anomaly source No.	Horizontal position (km)	Uniform level β	Structural index N	Relative shape	Depth (km)
1	51	6	3	Sphere	8.6
2	43	5	2	Cylinder	5.7

IV. CONCLUSIONS

In this paper, a new mother wavelet namely Farshard - Sailhac to solve the potential field inverse problems for determination of the horizontal position, depth and structural index of the adjacent geomagnetic sources has been introduced and successfully applied. Through the analysis of theoretical models, using the wavelet transform modulus maxima technique, the correlative function approximate linear between the depth of anomaly sources and the scale parameter have been established. Then, the process for the location of the magnetic anomaly sources using Farshard - Sailhac wavelet transform has been developed and applied. The results of interpretation of Tra Vinh – Soc Trang profile illustrated that there are two magnetic anomaly sources along the profile, including one sphere, and one cylinder, with their position, depth and structural index are quite coincident with the previously published results (Dau, 2013).

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