Modeling and Simulation of V-Shaped Thermal Actuator

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ABSTRACT: This paper presents a dynamic model of V-shaped thermal actuator with an assumption that all beams are supposed to be homogeneous and the heat source is evenly distributed on those beams. The model of actuator is simulated by using the S-Function tool in MatlabSimulink software. The subject is studied using square pulse voltages, with amplitude of 20V and frequency from 1 to 30Hz. The simulation outcomes show that the outgoing parameters reflect exactly the physical nature of the subject and are suitable with the simulation outcomes in the software Ansys. The study outcome would be a foundation for the model development and design of the controller for micro motors using thermal expansion effects.

KEYWORDS - V-shaped thermal actuator, heat expansion effect, modeling and simulation micro motor...

I. INTRODUCTION

Electrothermal microactuators in microelectromechanical systems (MEMS) are gaining increasing attention over the past several decades. Some kind of popular microactuators such as: U-shaped [1], X-shaped [2], Z-shaped [3,4], V-shaped [7-16] and two-hot-arm horizontal thermal actuator [5,6]... Among these, the V-shaped electrothermal micro actuator was published by Michael J.Sinclair 2000[7]. This is considered to be a breakthrough in the structure because it can provide more transposition and impact force to other structures. The V-shaped electrothermal micro actuators have been improved and applied in several MEMS devices, such as: micro gripper[11], micro linear motor [8,9,15], micro rotary motor [12-14, 16]... Nowadays, one of the most critical problems is modeling and control of the V-shaped electrothermal actuators. Quite a few researches have been modeled and simulated by Finite Element Method (FEM) [7-8, 16-18]. Mechanical parameters (temperature, displacement, force...) are analyzed very well. However, it is very difficult to design a remote control for them. In this paper, we established a dynamic model of V-shaped electrothermal actuators, when we consider heat transfer processes similar to electrical problems, with an assumption that, all beams are supposed to be homogeneous and the heat source is evenly distributed on those beams. The model is become ordinary differential equations, therefore the remote controller is designed more easily.

II. STRUCTURE AND WORKING PRINCIPLE

The V-shaped thermal Actuator has n parallel beams, with a slope angle of \( \frac{\pi}{2} - \alpha \) defining movement direction of the peak of the beams. The voltage of the beams is in the form of square pulses or half cycle of sine waves. In the actuation phase (half of the first cycle), the voltage is applied on bond pads (1), the current runs through the beams (2) thus heating the beams and causing expansion and creating vertical displacement. In the after-displacement phase (the other half of the first cycle), the voltage becomes 0, heat emits through the base and air; the beams cool down, the pitch B transfers back to its original position preparing for the next cycle. With this structure, we need to calculate the minimum voltage for the thermal force to over pass the elastic force.
III. MODELING V-SHAPED THERMAL ACTUATOR

3.1 Modeling a single V-shaped Thermal Actuator

A single V-shaped Thermal Actuator is shown in Fig. 2. It is constructed by suspending a shuttle off of the substrate with two symmetric arrays of thin beams. (The beam has length \( L \), width \( w_b \) and thickness \( t_b \); the shuttle has length \( l_s \), width \( w_s \) and thickness \( t_s \)).

The beams are attached to bond pads which are anchored to the substrate as shown in Fig. 2. A voltage is applied across the two bond pads, which induces a current through the thin beams. The current generates ohmic heating, and as the temperature of the beams rises they expand.

The beams are supposed to be uniform and the heat source is evenly distributed on the beams, the differential equations that govern the heat conduction process is given by:

\[
\frac{dT_b}{dt} = \frac{1}{C_b} \left[ 1 - \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) T_b + \frac{1}{R_{bs}} T_s + q_b \right]
\]

(1)

\[
\frac{dT_s}{dt} = \frac{1}{C_s} \left[ \frac{1}{R_{bs}} T_b - \left( \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) T_s + q_s \right]
\]

(2)

Where: \( T_b \) is the temperature of the beam, \( T_s \) is the temperature of the shuttle. \( R_{bp} \) is the thermal resistance between the beams and bond pads, \( R_{bs} \) is the thermal resistance between the beams and the shuttle.

\[
R_{bp} = \frac{l_b}{k_b w_b t_b} = \frac{l_b}{2k_b w_b l_b}
\]

(3)

\( R_{bw} \) is the thermal resistance between the beams and the substrate, \( R_{sw} \) is the thermal resistance between the shuttle and the substrate:

\[
R_{bw} = \frac{8a}{k_b w_b t_b s^2}; R_{sw} = \frac{8a}{k_s w_s l_s s}
\]

(4)

The thermal resistance between the control volumes and the substrate were modeled using a shape factor that approximates the effect of the heat transferring from the sides and bottoms of the control volumes to their effective projected area on the substrate [15]. The shape factors \( S_b \) and \( S_s \) are defined in Equation (5).

\[
S_b = \frac{w_b}{w_b} \left( \frac{2a}{l_b} + 1 \right) + 1; S_s = \frac{t_s}{w_s} \left( \frac{2a}{t_s} + 1 \right) + 1
\]

(5)

\( C_b \) and \( C_s \) are the heat capacities of the beams and the shuttle:

\[
C_b = w_b l_b t_b D_c; \quad C_s = w_s l_s t_s D_c
\]

(6)

\( q_b \) and \( q_s \) are ohmic heat generation in the beam and the shuttle. The ohmic heat generation in the control volumes is accurately modeled as a quadratic function of the current - which can be found from the voltage applied to the Thermal Actuator and the calculated resistance of the Thermal Actuator - as shown in Equation (7) with \( V \) as the input voltage.

\[
q_b = \frac{2l_b^2 R_b}{C_b}; \quad q_s = \frac{l_s^2 R_s}{C_s}
\]

(7)
Where, the current is determined: \( i = \frac{u}{r_b + r_s} \) \hspace{1cm} (8)

\[ r_b = \frac{\rho L}{w_b t_b} \text{ and } r_s = \frac{\rho L}{w_s t_s} \] are the beam resistance and the shuttle resistance; \( l_b \) is length of the beams \((l_b=2L)\); \( g_a \) is distance between the control volumes and the substrate; \( w_b \) and \( w_s \) are in-plane thickness of the beams and shuttle; \( t_b, t_s \) are out-of-plane thickness of the control volume; \( k_s \) is thermal conductivity of polysilicon and \( k_a \) is thermal conductivity of air; \( C_p \) is specific heat of polysilicon; \( D \) is density of polysilicon.

2.3. Modeling V-shaped Thermal Actuator (n parallel beams)

Similar to a single V-shaped Thermal Actuator, the differential equation that govern the heat conduction process in V-shaped Thermal Actuator is given by:

\[
\begin{align*}
\frac{dT_{b1}}{dt} &= \frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) T_{b1} + \frac{1}{R_{bs}} T_s + q_b \\
\frac{dT_{b2}}{dt} &= \frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) T_{b2} + \frac{1}{R_{bs}} T_s + q_b \\
&\quad \vdots \\
\frac{dT_{bn}}{dt} &= \frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) T_{bn} + \frac{1}{R_{bs}} T_s + q_b \\
\frac{dT_s}{dt} &= \frac{1}{C_s} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) T_{b1} + \frac{1}{R_{bs}} T_{b2} + \cdots + \frac{1}{R_{bs}} T_{bn} - \frac{n}{R_{bs}} T_s + q_s 
\end{align*}
\] (9)

These equations in matrix form are:

\[
\frac{d\mathbf{T}}{dt} = \mathbf{\bar{A}} \mathbf{T} + \mathbf{\bar{B}} \mathbf{Q}
\] (10)

\[
\text{Where: } \frac{d\mathbf{T}}{dt} = \begin{bmatrix} \frac{dT_{b1}}{dt} \\ \frac{dT_{b2}}{dt} \\ \vdots \\ \frac{dT_{bn}}{dt} \\ \frac{dT_s}{dt} \end{bmatrix}, \mathbf{T} = \begin{bmatrix} T_{b1} \\ T_{b2} \\ \vdots \\ T_{bn} \\ T_s \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} q_b \\ q_b \\ \vdots \\ q_b \\ q_s \end{bmatrix}, \mathbf{\bar{B}} = \begin{bmatrix} \frac{1}{C_b} & 0 & \cdots & 0 \\ 0 & \frac{1}{C_b} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{C_b} \\ 0 & 0 & \cdots & 0 \end{bmatrix}
\]

\[
\mathbf{\bar{A}} = \begin{bmatrix} -\frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) & 0 & 0 & \frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) \\ 0 & -\frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & -\frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) & 0 \\ \frac{1}{C_s} R_{bs} & \frac{1}{C_s} R_{bs} & \frac{1}{C_s} R_{bs} & \frac{1}{C_s} R_{bs} \\ \frac{1}{C_s} R_{bs} & \frac{1}{C_s} R_{bs} & \frac{1}{C_s} R_{bs} & \frac{1}{C_s} R_{bs} \\ \frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) & \frac{1}{C_b} \left( \frac{1}{R_{bp}} + \frac{1}{R_{bs}} + \frac{1}{R_{bw}} \right) & -\frac{n}{R_{bs}} & -\frac{n}{R_{bs}} 
\end{bmatrix}
\] (12)

Where: \( T_b \) (i=1÷n) is the temperature of the \( i^{th} \) beam;

The ohmic heat generations \((q_b, q_s)\) are determined: \( q_b = \frac{(i/n)^2 r_b}{C_b} \); \( q_s = \frac{i^2 r_s}{C_s} \) \hspace{1cm} (13)

\[
\text{Where: } i = \frac{u}{r_b + r_s}
\] \hspace{1cm} (14)

Heat difference \( \Delta T = T_s - T_b \) \hspace{1cm} (15)

Expansion of beams can be express by: \( \Delta \mathbf{L} = \alpha_i L \Delta T \) \hspace{1cm} (16)

\[
\text{Where: } \Delta \mathbf{L} = \begin{bmatrix} \Delta L_{b1} \\ \Delta L_{b2} \\ \vdots \\ \Delta L_{bn} \\ \Delta L_s \end{bmatrix}, \mathbf{\bar{L}} = \begin{bmatrix} L_{b1} \\ L_{b2} \\ \vdots \\ L_{bn} \\ L_s \end{bmatrix}
\] (17)

The Modeling Actuator is described by equation system:

\[
\frac{d\mathbf{\bar{L}}}{dt} = \mathbf{\bar{A}} \mathbf{\bar{T}} + \mathbf{\bar{B}} \mathbf{\bar{Q}}
\]

\[
\mathbf{\bar{L}} = \alpha_i L (T - T_b)
\] \hspace{1cm} (18)

As showing in Fig. 3, displacement of driving beams can be calculated:
Fig. 3. Displacement calculation

\[ \Delta d_i = \frac{1}{2} \sqrt{\left(l_b + \Delta l_{b1}\right)^2 - \left(l_b \cos \alpha \right)^2} - \frac{1}{2} l_b \sin \alpha \]  

(19)

Because the V-shaped Thermal Actuator has n parallel beams, therefore the displacement of actuator is displacement of driving beams and displacement of shuttle

\[ \Delta d = \frac{1}{2} \sqrt{\left(l_b + \Delta l_{b1}\right)^2 - \left(l_b \cos \alpha \right)^2} - \frac{1}{2} l_b \sin \alpha + \Delta l_s \]  

(20)

IV. SIMULATION THE V-SHAPED THERMAL ACTUATOR BY MATLAB SIMULINK

The modeling actuator is described by equations (9) and (10), it can be simulated by S-Function tool in Matlab Simulink software. The Actuator is fabricated by MEMS standard technology and using SOI (silicon on insulator) wafer with the thickness of the device layer is 30µm and one mask only. The basic parameters of the Actuator are shown by Table 1 and 2.

<table>
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<tr>
<th>Parameters</th>
<th>Value</th>
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<tr>
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<td>µm</td>
</tr>
<tr>
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<td>µm</td>
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</tr>
<tr>
<td>k_a</td>
<td>0.026</td>
<td>(W/m.°C)</td>
</tr>
</tbody>
</table>

The subject is studied in square pulse voltage, with an amplitude of 20V and frequency of 1 Hz, Fig. 4.

Fig. 4. Source voltage

The temperature simulation of single beam and shuttle is shown in Fig. 5 and 6.
Fig. 5. Temperature simulation of single beam

The simulation outcomes show that it reflects exactly physical characteristics of the beam system. Highest temperature focuses on the shuttle beam (about 375 °C), average temperature on single beam is 256 °C, which matches the simulation results on Ansys [14,16]. Due to thermal inertia, the reaction of the subject can be “late”, when the source voltage is in form of square pulse (Fig. 4), the thermal reaction is in form of concave trapezium (Fig. 5 and 6). Transient time of the system is about 0.05s (Fig. 5 and 6), which means that the system will make good responses with a frequency lower than 20Hz. When working at high frequency, due to the “late” feature of the subject, the temperature of the system decreases, Figures 7 and 8.

Fig. 6. Temperature simulation of shuttle

V. CONCLUSION

This paper has studied and proposed fundamental theories to make the subject model simple. Especially, the model is become ordinary differential equations, therefore Simulation and the remote controller are designed more easily.

The subject model is the foundation for future studies on design controller of the subject while building models of other systems using V shaped actuators like: micro gripper, micro motor(linear or rotary).

The simulation results show that the subject model reflects exactly the physical process as studied and analyzed theory. The simulation results on Matlab Simulink match the simulation on Ansys and also match previously published studies.

The study group also dida basic survey on response frequency of the subject. The results are basis for studying and alerting working frequency of the beam system.

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REFERENCES

JOURNAL PAPERS:
[13]. Nguyen Tien Duzng, Pham Hong Phuc, Nguyen Quang Dich; Design and fabrication of a 2.5mm-diameter micro rotational motor based on MEMS technology; Journal of Science & Technology Technical Universities; No.108-2015, pages 26-32.

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