

Quantum Communication Using Orthogonal Quantum Vectors For Multiple Users Over Gaussian Bosonic Channel

H.R.Sumathi¹ C.Vidya Raj²

¹Dept of Information Science and Engineering ,JSS Academy of Technical Education Bangalore 560060,Karnataka,India.

²Professor, NIE college of Engineering Mysore 570008 , Karnataka, India.

Affiliated to Visvesvaraya Technological University, Belagavi 590014, Karnataka, India.

Corresponding author: H.R.Sumathi

ABSTRACT: *Quantum* communication which transmits the information much faster than light in free space or over fibre optic cable using Light amplification by stimulated emission of Radiation is the important facts of Quantum channels. Taking the advantages of both optical and digital communication to transfer the information in the form of Photons over Gaussian Bosonic channel. Multiple user transmit the qubits over the channel using code division multiple access technique where each user is assigned a unique codes which are orthogonal to one another. At the receiver side using the orthogonal property that is by taking the normalized inner product they can be distinguished. Multiple accesses to the channel can use the principles of spread spectrum communication with different wave length.

KEYWORDS: Bosonic Gaussian channel, Entanglement, Quantum memory, Qubits, Teleportation

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I. INTRODUCTION

Bosonic Gaussian channels (BGCs) are used in theoretical physics, they arise when a harmonic system interacts linearly with a number of bosonic modes which are inaccessible in principle or in practice. They provide realistic noise models for a variety of quantum optical and solid-state systems when treated as open quantum systems, including models for wave guides and quantum condensates. They play a fundamental role in characterizing the efficiency of a variety of tasks in continuous-variables quantum information processing, including quantum communication and cryptography. Most importantly, communication channels such as optical fibers can, to a good approximation, be described by Gaussian quantum channels.

Entanglement plays an important role in quantum computing and communications. The noise in quantum communication channels might be traced back to the entanglement between the channel and its nearby environment. If a photon traversing on an optical fibre is entangled with another photon outside the fibre, and this latter photon has been subjected to some effects, these effects will influence the state of the first photon in the fibre [5]. Therefore, either we keep the photons from becoming entangled, or suffer all the consequences such as channel noise. Fortunately, to entangle two photons, they must be located very close to each other at some point in time; that is, entanglement cannot be created between distant particles. Also, whereas naturally occurring entanglements cause undesirable noise in quantum communications channels, manmade deliberate entanglements may be exploited to improve communications and computing capabilities.

Coherent Quantum information

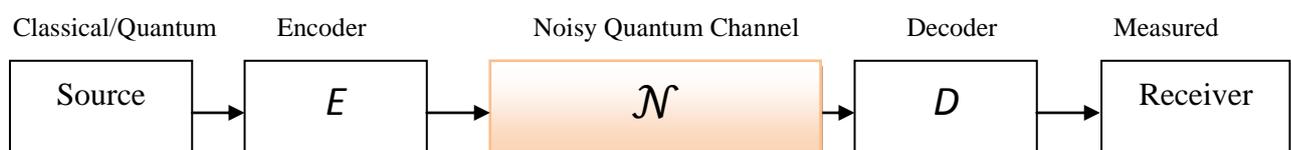


Figure 1. Noisy quantum channel

Photons are encoded in the form of unique quantum states and then transmitted over noisy quantum channel. To describe the information transmission capability of the quantum channel, the distinction between the various capacities of a quantum channel is required. The encoded quantum states can carry classical messages or quantum messages. In case of classical messages, the quantum states encode the output from a *classical information source*, while in the latter case the source is a *quantum information source*. In general, the optimized coherent information is maximized over all possible code words, here restricted to Gaussian states [2]. Unlike in the calculation of the classical capacity [3], the limit of infinite input power $N \rightarrow \infty$ does not usually lead to a diverging entropic quantity. Quantum information transmission can be considered in terms of two at first different communication setups that, however, turn out to be equivalent. The most straightforward formulation is depicted in Fig. 1 where a sender prepares an arbitrary quantum state ρ and sends it through a noisy quantum channel, in the hope that the message can be recovered by the receiver via error correction using the decoder. It turns out that this formulation is hard to analyze, but a crucial reformulation of this process in terms of the "purified" system allows much progress. In the process depicted in Fig. 2 part of an entangled state is sent through the channel, but such a scenario is in fact equivalent given the insight that such an entangled state will allow the transmission of arbitrary quantum states via quantum teleportation.

Formal Model of a Quantum Channel

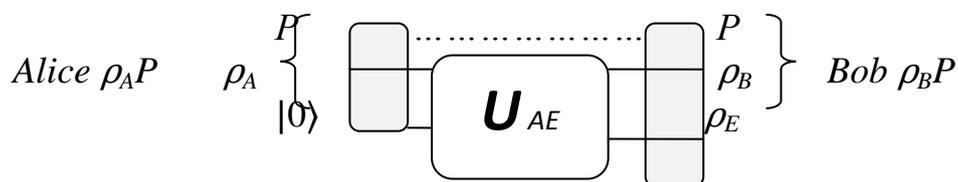


Figure 2. Formal model, The output is mixed state

- Classical Quantity: A measure of the *classical transmission capabilities* of a quantum channel. (For example, the Holevo information, quantum mutual information, etc)
- Quantum Quantity: A measure of the *quantum transmission capabilities* of a quantum channel. (For example, the quantum coherent information)

Each quantum channel can be represented as a CPTP map, hence the process of information transmission through a quantum communication channel can be described as a quantum operation. The most general model of a quantum channel describes the transmission of an input quantum bit and its interaction with the environment. Assuming Alice sends quantum state ρ_A into the channel this state becomes entangled with the environment ρ_E , which is initially in a pure state $|0\rangle$. For a mixed input state a so-called *purification state* P can be defined, from which the original mixed state can be restored by a partial trace operation. Hence, Alice's state ρ_A can be expressed as the partial trace of the pure system $\rho_A P$. The basic unitary operation U_{AE} of a quantum channel N entangles $\rho_A P$ with the environment and outputs Bob's mixed state as ρ_B and the purification state as P . The purification state is a reference system, it cannot be accessed, and it remains the same after the transmission. The unitary transformation U_{AE} affects both the system ρ_A and the entanglement between ρ_A and the reference system P . The reference state P is not affected by the quantum channel. The output of the noisy quantum channel is denoted by ρ_B , the post state of the environment by ρ_E .

Theorem 1: Let $|\phi_0\rangle, |\phi_1\rangle, \dots, |\phi_{n-r}\rangle$ be a set of orthogonal n qubit quantum entangled states. If $r = 1$, these orthogonal entangled states cannot be perfectly distinguished by less than n cooperating participants under LOCC.

Proof: Since $r = 1$, Rewrite these states as $|\phi_0\rangle, |\phi_1\rangle, \dots, |\phi_{n-1}\rangle$ for convenience. Without loss of generality, consider that the number of the cooperating participants is $n - 1$. Arbitrarily $n - 1$ cooperating participants measure their own particles in the basis $\{|0\rangle, |1\rangle, \dots, |n - 1\rangle\}$ locally, and they can obtain at most $n - 1$ measurement outcomes, that is, after the measurement the participants know that m ($m = 1, 2, 3, \dots, n - 1$) particles are the same. Since there exist n quantum entangled states, then there are at least two identical measurement outcomes. Hence, the two states cannot be distinguished. For example, suppose that the first $n - 1$ players cooperate to measure these n quantum states, then they have the same measurement outcomes for $|\phi_{n-2}\rangle$ and $|\phi_{n-1}\rangle$, that is, the measurement outcomes obtained by each participant are different. Hence, they cannot perfectly distinguish the states $|\phi_{n-2}\rangle$ and $|\phi_{n-1}\rangle$ under LOCC. For the case of less than $n - 1$, it can be analyzed similarly.

The Holevo-Schumacher-Westmoreland (HSW) theorem quantifies explicitly the amount of classical information that can be transmitted through the noisy quantum channel, using *product input states*. In this sense it is the quantum counterpart of Shannon's noisy channel coding theorem, but limited to a special encoder/decoder setting. From engineering point of view, to maximize the channel capacity we have to select from a set of these pure states as a possible subset that satisfy orthogonal property, which subset contains all those pure density metrics, which are able to maximize the information transmission. Note that encoding is just one side of the problem. To maximize the capacity of a quantum channel, we have to optimize the measurement process, too. Quantum information transmission requiring is only that the receiver be able to simulate the statistics of certain restricted measurements. In the case of quantum identification, these are for arbitrary rank one projector. They are the measurements which allow the receiver to ask the (quantum) question. "Is the state equal to or orthogonal to it?" Obviously, in quantum theory this question cannot be answered with certainty, but for each test state it yields a characteristic distribution.

The Pegg-Barnett Phase

$$\text{A phase state } |\theta\rangle = \text{Lim}_{s \rightarrow \infty} (s+1)^{-1/2} \sum_{n=0}^s \exp(in\theta) |n\rangle$$

To operate the limit is to perform the calculation in a finite space, and after the physical averages are calculated, one is to take the limit $s \rightarrow \infty$. The parameter θ can take any values between 0 and 2π , there are an infinite number of these states, which are non orthogonal. One can construct a set of orthogonal states if only a specified values of $\theta = \theta_m$.

$$\theta_m = \theta_0 + 2\pi m / (s+1).$$

Thus above equation is simple to verify because,

$$\text{Exp}(iny) |n\rangle = |n + \gamma\rangle.$$

The orthonormal condition can be seen as follows:

$$\langle \theta_p | \theta_m \rangle = \langle \theta_0 | \exp[-in(p2\pi/(s+1))] \exp[-in(m2\pi/(s+1))] | \theta_0 \rangle$$

Thus

$$\langle \theta_p | \theta_m \rangle = \text{Lim}_{s \rightarrow \infty} (s+1)^{-1} \sum_{q=0}^s \langle q | q \rangle \exp [i 2\pi / (s+1) q(m-p)] = \delta_{mp}.$$

The Hermitian phase operator is defined as

$$\Phi_\theta = \sum_{m=0}^s \theta_m | \theta_m \rangle \langle \theta_m | ,$$

or

$$\Phi_\theta = \theta_0 + 2\pi / (s+1) \sum_{m=0}^s m | \theta_m \rangle \langle \theta_m | .$$

Φ_θ depends on the arbitrary reference phase θ_0 . The phase operator is Hermitian and satisfies the eigenvalue equation

$$\Phi_\theta | \theta_m \rangle = \theta_m | \theta_m \rangle$$

In Pegg-Barnett Phase, the trigonometric functions behave in a normal way.

For purposes of assessing quantum or classical information capacities the full knowledge of the channel is not required. Transforming the input or the output with any unitary operation (say, Gaussian unitaries) will not alter any of these quantities. It is then convenient to take advantage of this freedom to simplify the description of the BGCs. To do so, we first notice that the set of Gaussian maps is closed under composition. Consider then Φ' and Φ'' two Bosonic Gaussian channels (BGCs) described respectively by the elements X', Y', v' and X'', Y'', v'' . The composition $\Phi'' \circ \Phi'$ where, in Schrödinger representation, we first operate with Φ' and then with Φ'' , is still a BGC and it is characterized by the parameters.

II. CONCLUSION

Quantum based communications is identified with the case of the Gaussian Bosonic channel itself obeying quantum rules. Whether classical or quantum, computing and communications have many overlapping areas, Quantum communications combines quantum signal processing with classical communication channels. Signal processing is critical for efficient high quality communication between parties where reliable quantum information has to be received. Now a day's Satellite transmission seems to be more promising mode of communication between the transmitter and the receiver, after China launched first Quantum satellite. Quantum memories, which allow to store and retrieve quantum information, in the form of photons in well defined quantum states will also be important to connect the ground stations of satellite links. The presence of the loss explains why long distance classical telecommunication links use repeaters that are optical amplifiers to periodically boost signal strength, and why directly sending single photons over thousands of kilometers of fiber is not an option. Unfortunately, similar amplification is not possible for quantum signals because of the no-cloning theorem, so more elaborate techniques such as quantum repeaters that preserve quantum information are needed.

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