Dynamic Modeling Of A Dual Winding Induction Motor Using Rotor Reference Frame

Sule Amachree 1, E.S.Obe.2, D.C Idonibuyeobu,3 S.L.Braide 4

Doctorate Student, Professor, Dr. Depart. of Electrical Engineering, Rivers State University, Port Harcourt, Nigeria)
1,3,4. (Professor, Depart. of Electrical Engineering, University of Nigeria Nsukka, Enugu, Nigeria.
Corresponding author: Sule Amachree

ABSTRACT: A dynamic modelling method is needed for a better understanding of the transient behavior of dual winding induction motor. A good mathematical model using Park’s Transformation equation to describe the behavior of the motor is presented by a system of differential equations. An equivalent circuit of the motor is obtained through a standard induction motor analysis method. This paper presents the dynamic modeling of a dual winding induction motor using rotor reference frame.


I. INTRODUCTION

Induction motors are by far the most used electromechanical devices in the industry today. In-fact, it is crowned as the workhorse of the electric power industry. Induction motor holds many advantages over the other types of motors. They are known to be cheap, rugged, low cost, low maintenance and can be used anywhere even in hazardous location. Despite, it’s numerous advantages, it has one major disadvantage of draw large power from source to be able to operate, thereby causing voltage drop and operates with a lagging poor power factor, especially under starting condition and under light load. Poor power factor adversely affects the economy of distribution and transmission system and this may lead to higher electricity charges. [1-2]. In recent years, multiphase induction motor has attracted more focused attention from various researchers to date. Various research in the field of dual winding induction motors have been one of the major interest of recent in machines analysis and different method have been proposed on its performances characteristics compared to that of a standard three-phase induction motor of similar ratings. [3]. A comprehensive literature review has been taken from Ogunjuyigbe, A.S.O., Ayodede, T.R. & Adetokun, B.B (2018). “modeling and analysis of a dual stator winding induction machine, and others not mentioned revealed that the machine either suffer from overheating due to excessive copper losses in the windings, drawing large power to start and operates with low lagging power factor, [4-6].

II. MATHEMATICAL MODEL OF DUAL WINDING AND THREE-PHASE WINDING INDUCTION MOTOR

The dynamic model of the dual winding and three-phase winding induction motor arrangement is mathematically derived and then, analyzed in an embedded MATLAB/Simulink environment to ascertain the level of performances.
In order to obtain a simplified mathematical model for the analysis of the dual winding and three-phase induction motors, certain assumptions are considered as follows:

* The air-gap is uniform
* Eddy-current, friction, windage losses, are neglected
* The windings are distributed sinusoidal around the air-gap
* The windings are identical and have same resistance

III. VOLTAGE EQUATIONS OF THE DUAL WINDING IN THE ARBITRARY REFERENCE FRAME.

By using the qd0 reference frame method, a poly-phase machine is transformed to a two-phase with their magnetic axis in quadrature form. This method is also commonly referred to as the d-q-0 method in a balance system with 0 relating the zero sequence components. Therefore, the transformation equation from abc to d-q-0 for the main (Winding-1) and rotor Winding and including the flux linkage is stated as follows;

\[
K_{si} = \frac{2}{3} \begin{bmatrix}
\cos \theta_r & \cos \left(\theta_r - \frac{2\pi}{3}\right) & \cos \left(\theta_r + \frac{2\pi}{3}\right) \\
\sin \theta_r & \sin \left(\theta_r - \frac{2\pi}{3}\right) & \sin \left(\theta_r + \frac{2\pi}{3}\right)
\end{bmatrix}
\]

Note: Also, the rotor transformation matrix is stated as follows;

\[
K_r = \frac{2}{3} \begin{bmatrix}
\cos \beta & \cos \left(\beta - \frac{2\pi}{3}\right) & \cos \left(\beta + \frac{2\pi}{3}\right) \\
\sin \beta & \sin \left(\beta - \frac{2\pi}{3}\right) & \sin \left(\beta + \frac{2\pi}{3}\right)
\end{bmatrix}
\]

Where, \( \beta = \theta - \theta_r \), and \( K_{si}, K_r \) are the transformation matrix for the stator and rotor parameters respectively, and \( K_{si}^{-1} \) and \( K_r^{-1} \) are the inverse transformation-matrix for the stator and rotor parameter respectively not stated here. The voltage equations for each winding on the stator and rotor are stated as follows in the natural reference frame;

\[
V_{abc1} = r_{s1} i_{abc1} + \frac{d}{dt} \lambda_{abc1}
\]

\[
V_{abc2} = r_{s2} i_{abc2} + \frac{d}{dt} \lambda_{abc2} + V_{eabc}
\]

\[
V_{abcr} = r_{r} i_{abcr} + \frac{d}{dt} \lambda_{abcr}
\]

Recall and transform equations (5-7), using the transformation equations given in equations (3-4) will yield;

\[
V_{q1s1} = r_{s1} i_{q1s1} + \omega_1 \lambda_{ds1} + \frac{d}{dt} \lambda_{qs1}
\]

\[
V_{d1s1} = r_{s1} i_{d1s1} - \omega_1 \lambda_{qs1} + \frac{d}{dt} \lambda_{ds1}
\]

\[
V_{q2s2} = r_{s2} i_{q2s2} + \omega_2 \lambda_{ds2} + P \lambda_{qs2} + V_{Cq}
\]

\[
V_{d2s2} = r_{s2} i_{d2s2} - \omega_2 \lambda_{qs2} + P \lambda_{ds2} + V_{Cd}
\]

For the auxiliary winding, the voltage equations have two additional terms \( V_{Cq} \) and \( V_{Cd} \) added to them to account for the capacitor connected across it. But in the d-q-0 reference frame, the \( V_{Cq} \) and \( V_{Cd} \) are given as;

\[
\frac{dV_{Cq}}{dt} = \frac{i_{ds2}}{C} \omega V_{Cd}
\]

\[
\frac{dV_{Cd}}{dt} = \frac{i_{ds2}}{C} + \omega V_{Cq}
\]

Thus

\[
V_{Cqs2} = \frac{1}{\omega} \left[ \frac{i_{ds2}}{C} - PV_{Cds2} \right]
\]
Substituting the values of $V_{qs2}$ and $V_{ds2}$ into the voltage equation for the auxiliary winding in equation (10) and (11) yields

\[ V_{qs2} = r_{s2} i_{qs2} + \omega_2 \lambda_{ds2} + P \lambda_{qs2} - \frac{1}{\omega} \left[ \frac{i_{qs2}}{c} - PV_{cs2} \right] \]  

(16)  

\[ V_{ds2} = r_{s2} i_{ds2} - \omega_2 \lambda_{qs2} + P \lambda_{ds2} + \frac{1}{\omega} \left[ \frac{i_{qs2}}{c} + PV_{cs2} \right] \]  

(17)  

\[ V_{qr} = r_{q1} i_{qr} + (\omega - \omega_r) \lambda_{dr} + P \lambda_{qr} \]  

(18)  

\[ V_{dr} = r_{d1} i_{dr} + (\omega - \omega_r) \lambda_{qr} + P \lambda_{dr} \]  

(19)  

Similarly, the flux linkage equations for the stator windings respectively and for the rotor are written in the expanded form as given:

\[ \lambda_{qs1} = i_{qs1} L_{l_{s1}} + L_m (i_{qs1} + i_{qs2} + i_{qr}) \]  

(20)  

\[ \lambda_{ds1} = i_{ds1} L_{l_{s1}} + L_m (i_{ds1} + i_{ds2} + i_{dr}) \]  

(21)  

\[ \lambda_{qs2} = i_{qs2} L_{l_{s2}} + L_m (i_{qs1} + i_{qs2} + i_{qr}) \]  

(22)  

\[ \lambda_{ds2} = i_{ds2} L_{l_{s2}} + L_m (i_{ds1} + i_{ds2} + i_{dr}) \]  

(23)  

\[ \lambda_{qr} = i_{qr} L_{l_{r}} + L_m (i_{qs1} + i_{qs2} + i_{qr}) \]  

(24)  

\[ \lambda_{dr} = i_{dr} L_{l_{r}} + L_m (i_{ds1} + i_{ds2} + i_{dr}) \]  

(25)  

IV. DYNAMIC EQUIVALENT CIRCUIT OF D-Q AXIS.

For ease of analysis, the d-q voltage equations and the flux linkage equations suggest the equivalent circuit diagram as shown below respectively.

![Fig:4: d-axis equivalent circuit of the dual winding induction motor.](image)

![Fig:5: q-axis equivalent circuit of the dual winding induction motor.](image)

V. ELECTROMAGNETIC TORQUE ($T_{em}$) AND SPEED.

The electromagnetic torque is derived from the sum of the input power supplied to all the windings of the main winding, auxiliary winding, including the rotor winding of the dual winding induction motor in the d-q reference frame is given as;
Note that $w$ can take any arbitrary value such that if it is stationary, $w=0$; if it is referred to the rotor, $w = w_r$ and for synchronously rotating reference frame, $w = w_s$. And the speed is given as:

$$
\omega_r = \frac{P}{2j} \int (T_{em} - T_L) \, dt
$$

Voltage conditions for the main winding supply

$$
\begin{align*}
V_a &= v_m \cos(\omega t) \\
V_a &= v_m \cos(\omega t + 2\pi f) \\
V_a &= v_m \cos(\omega t - 2\pi f)
\end{align*}
$$

### VI. MODELING IN DIFFERENT FREQUENCY FRAME

In the rotor reference frame, the speed of the reference frame is given as $\omega = \omega_r$ and the angular position $\theta = \theta_r$. By substituting $\omega = \omega_r$ in the following equations, (8),(9),(16),(17),(18) and (19). And $\theta = \theta_r$ in equations (3-4) yields:

$$
\begin{align*}
V_{qs1} &= r_{s1}i_{qs1} + \omega_r \lambda_{ds1} + P \lambda_{qs1} \\
V_{ds1} &= r_{s1}i_{ds1} - \omega_r \lambda_{qs1} + P \lambda_{ds1} \\
V_{qs2} &= r_{s1}i_{qs2} + \omega_r \lambda_{ds2} + P \lambda_{qs2} - \frac{1}{\omega_r} \left[ \frac{dV_{ds2}}{dt} - PV_{Cds2} \right] \\
V_{ds2} &= r_{s1}i_{ds2} - \omega_r \lambda_{qs2} + P \lambda_{ds2} + \frac{1}{\omega_r} \left[ \frac{dV_{qs2}}{dt} + PV_{Cqs2} \right] \\
V_{qr} &= r_{t}i_{qr} + P \lambda_{qr} \\
V_{qr} &= r_{t}i_{dr} + P \lambda_{dr}
\end{align*}
$$

In the synchronously rotating reference frame, the speed of the reference frame is given as $\omega = \omega_s$ and the angular position $\theta = \theta_s = \omega_s t$. Similarly, in the stationary reference frame, the speed of the reference is given as $\omega = 0$, and the angular position $\theta = 0$.

### MACHINE DATA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Resistance</td>
<td>3.72Ω</td>
</tr>
<tr>
<td>Auxiliary Resistance</td>
<td>3.72Ω</td>
</tr>
<tr>
<td>Rotor Resistance</td>
<td>2.12Ω</td>
</tr>
<tr>
<td>Mutual Inductance</td>
<td>0.032H</td>
</tr>
<tr>
<td>Stator Inductance</td>
<td>0.022H</td>
</tr>
<tr>
<td>Rotor Inductance</td>
<td>0.005H</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>0.066kgm$^2$</td>
</tr>
<tr>
<td>Rated Voltage</td>
<td>415V</td>
</tr>
<tr>
<td>No of Pole</td>
<td>4</td>
</tr>
<tr>
<td>Capacitance</td>
<td>10.100µF</td>
</tr>
<tr>
<td>Load Torque</td>
<td>15Nm</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>50Hz</td>
</tr>
</tbody>
</table>

### VII. SIMULATION RESULTS AND DISCUSSION
The following graphs shows the rotor speed, electromagnetic torque, currents and torque-speed curve of a dual winding induction motor.

**Fig: 6:** Main Winding Current against Time

**Fig: 7:** Auxiliary Winding Current against Time.

**Fig: 8** Rotor Speed against Time.
Fig: 9: Electromagnetic Torque against Time

Fig: 10: Electromagnetic Torque against Mechanical Rotor Speed

The introduction of the capacitor at the auxiliary end of the second winding reduces the in-rush current taken by the capacitor during starting, the introduction of the capacitor also enables the machine to attain steady state fastas it takes seconds for the machine which is modelled in the rotor reference frame to attain dynamic stability, the capacitor inclusion in the torque equation also enables the electromagnetic torque and mechanical rotor speed reaches synchronism at 0.3 secs. as shown in Fig:9 and Fig:10.

VIII. CONCLUSION

From the MATLAB simulation of dynamic analysis, it can be seen that the capacitor excitation in the auxiliary winding gave a significant reduction effect of the high inrush current drawn by the induction machine. From the torque speed characteristic, it can be seen that the high starting torque which is produced by the dual winding induction motor reaches a steady state condition in a very short time interval. Hence the speed of the motor show relative stability from transient to steady state, hence the dual winding induction motor gives a better performance and overall efficiency as compared to a single winding induction motor. This arrangement will reduce the known disadvantages of conventional induction motors by providing a system in which the magnetic flux density in the stator is maintained at a maximum level.
REFERENCE


