

# Optimizing The Size of An Open Derivation Channel in Mountainous Hydropower Projects

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## Abstract

The optimal design of open derivation channels is a fundamental problem in hydropower engineering, particularly in mountainous regions where complex topography, difficult geological conditions, and high construction costs significantly influence design decisions. Classical hydraulic theory identifies channel cross-sections that minimize wetted perimeter for a given discharge as hydraulically optimal. However, such approaches do not account for construction constraints and economic factors, which often dominate real-world projects.

This paper presents a comprehensive technical-economic methodology for optimizing the cross-sectional dimensions of open derivation channels by minimizing total construction cost per linear meter while satisfying hydraulic performance requirements. The proposed approach integrates uniform flow theory, channel geometry, terrain slope, excavation geometry, reinforced concrete quantities, and market-dependent unit costs. Two hydropower projects in Albania—HPP Krastë and HPP Plani i Bardhë—are analyzed as representative case studies.

A parametric analysis is conducted to investigate the influence of terrain angle, excavation cost, and concrete cost on the optimal width-to-depth ratio of the channel. The results demonstrate that terrain slope is the governing parameter in determining the optimal cross-section, while construction costs exert secondary but measurable effects, particularly for low slope angles. The findings provide practical guidance for hydropower designers working in mountainous terrain and highlight the necessity of moving beyond purely hydraulic optimization.

**Keywords:** Open channels, hydropower derivation systems, techno-economic optimization, mountainous terrain, channel design

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## I. Introduction and Objectives

Hydropower remains one of the most important renewable energy sources worldwide, especially in regions with mountainous topography and abundant water resources. In such regions, derivation-type hydropower plants are commonly adopted, where water is diverted from a river intake and conveyed through a system of open channels, tunnels, or pipelines to a downstream powerhouse.

Among these components, open derivation channels often constitute a significant portion of the total system length and construction cost. Their design therefore has a decisive impact on the technical feasibility and economic performance of the entire hydropower scheme.

Classical open-channel design is based on uniform flow theory and focuses on hydraulic efficiency. Standard textbooks define the optimal channel section as the one that maximizes discharge for a given wetted area, or equivalently, minimizes wetted perimeter. While this criterion is valid from a purely hydraulic perspective, it neglects several critical aspects:

- Excavation geometry in steep terrain,
- Structural requirements of concrete lining,
- Stability of side slopes,

- Access roads and construction logistics,
- Market-dependent costs of materials and labor.

As a result, a hydraulically optimal section may lead to excessive excavation volumes or unnecessary concrete quantities, making it economically suboptimal or even impractical.

The primary objective of this study is to develop and apply a technical-economic optimization framework for the design of open derivation channels in mountainous regions. Specifically, the paper aims to:

- Compare hydraulically optimal and economically optimal channel sections;
- Quantify the influence of terrain slope on optimal channel geometry;
- Assess the sensitivity of optimal dimensions to excavation and concrete costs;
- Provide practical recommendations for hydropower channel design.

## II. Case Studies: HC Krasta and HC Plani i Bardhe

### HC Krasta

The Krasta hydropower plant is located in the upper Mat River basin in northern Albania, a region characterized by steep mountainous terrain and heterogeneous geological formations. The intake is situated at an elevation of approximately 703 m a.s.l., while the powerhouse is located at about 580 m a.s.l.

- The derivation system has a total length of approximately 2.3 km and includes:
- A low intake dam (5 m height),
- A desander with dimensions  $6 \times 24$  m,
- An open derivation channel of approximately 2.0 km,
- A non-pressure tunnel of 300 m length.

The channel alignment crosses slopes with varying inclinations, requiring several adaptations of the typical cross-section during construction.



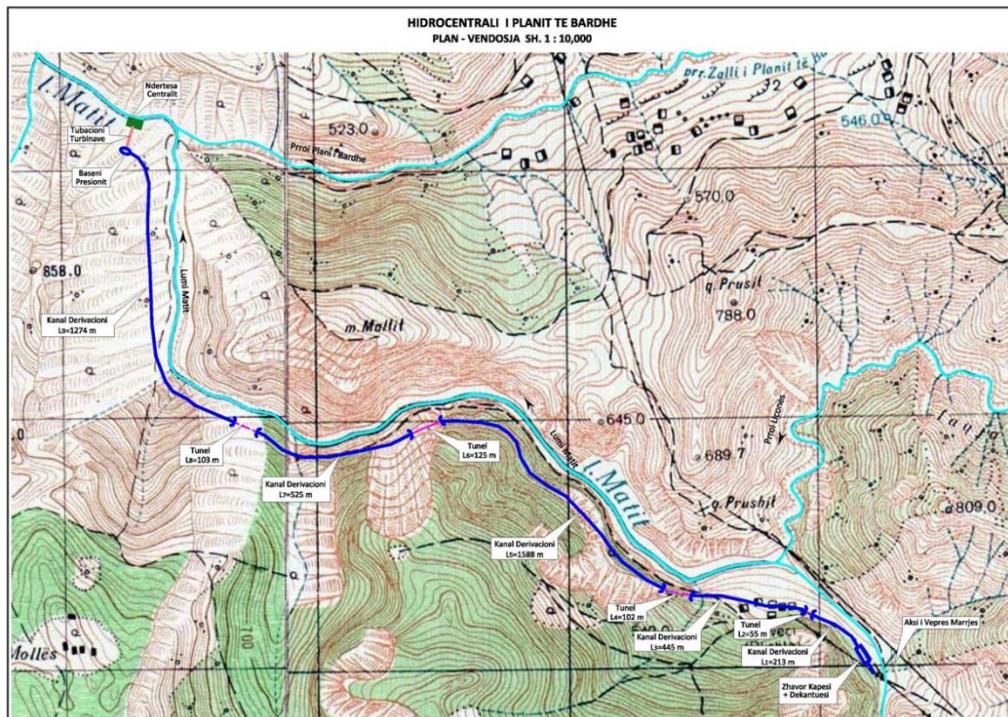
HPP Krasta map

### HC Plani i Bardhe

The Plani i Bardhë hydropower plant is located in the Bulqizë district at lower elevations, between 437 m and 350 m a.s.l. The hydropower scheme includes:

- A 12 m high intake dam,
- A 4.5 km long derivation system,
- 4.1 km open channel,
- 0.4 km tunnel.

The first section of the derivation channel passes through soft clayey formations, while the downstream sections encounter steep rocky slopes, making it an ideal case for investigating the influence of geology and terrain slope on channel optimization.



HPP Plani I Bardhe map

### III. Methodology: Hydraulic Calculation

#### Classification of Flow

Flow in open channels can be classified as:

- Steady or unsteady,
- Uniform or non-uniform.

In this study, steady uniform flow is assumed, which is justified for long derivation channels with constant cross-section, slope, and roughness.

#### Uniform Flow Assumptions

- Under uniform flow conditions:

- Velocity  $V$ , discharge  $Q$ , and wetted area  $S$  are constant along the channel, The energy slope, hydraulic gradient, and channel bed slope coincide. The depth corresponding to uniform flow is referred to as the normal depth  $h_0$ .

### Flow Resistance Formulations

The mean velocity is computed using the Chezy equation:

$$V = C\sqrt{RI}$$

where  $R$  is the hydraulic radius and  $I$  is the channel slope.

The Chezy coefficient  $C$  is commonly expressed using the Manning formulation:

$$C = \frac{1}{n} R^{1/6}$$

where  $n$  is the Manning roughness coefficient, which depends on the channel lining material.

The discharge is then given by:

$$Q = S \cdot \frac{1}{n} R^{1/6} \sqrt{RI}$$

### Channel Geometry and Hydraulic Optimization

#### General Channel Shapes

Open channels may have rectangular, trapezoidal, triangular, circular, or parabolic cross-sections. In hydropower derivation systems, trapezoidal and rectangular sections lined with reinforced concrete are most commonly used due to their structural stability and ease of construction.

#### Trapezoidal Channel Geometry

For a trapezoidal channel with bottom width  $b$ , normal depth  $h$ , and side slope coefficient  $m$ :

Wetted area:

$$S = (b + mh)h$$

Wetted perimeter:

$$P = b + 2h\sqrt{1 + m^2}$$

Hydraulic radius:

$$R = \frac{S}{P}$$

### Optimization Analysis: Hydraulic vs. Economic

#### The Hydraulically optimum Section

The most hydraulically optimum section of the channel is called the one that, for a given wetted area, for a given roughness coefficient and for a given slope, has the greatest carrying capacity. This analysis is done using the Shezi formula:

$$Q = S \cdot C \cdot \sqrt{R \cdot i} = S \cdot \left(\frac{1}{n} \cdot R^{\frac{1}{6}}\right) \cdot \sqrt{R \cdot i} = S \cdot \frac{1}{n} \cdot \left(\frac{S}{P}\right)^{\frac{1}{6}} \cdot \sqrt{\frac{S}{P} \cdot i}$$

The following table gives the value of the b/h ratio depending on the coefficient m:

m	0	0.5	1	1.5	2	2.5	3
b/h	2	1.2	0.8	0.6	0.5	0.4	0.3

$m = \text{ctg}\alpha$  is the slope coefficient

#### The "Technical-Economic" Optimal Section

In practice, the cross-section of the canal, after being calculated hydraulically, also adapts to the terrain where it will be built. This is because the canal is mostly built on steep hilly or mountainous terrain. Horizontal terrain can be considered as a special case of steep terrain (when the slope angle is taken to be zero). Under these conditions, the choice of the most hydraulically advantageous b/ho ratio, is not necessarily the most technically optimal. This is because in the analysis that was done above, none of the construction conditions of the section were taken into account. A cross-section calculated in this way will be called the optimal technical-economic section.

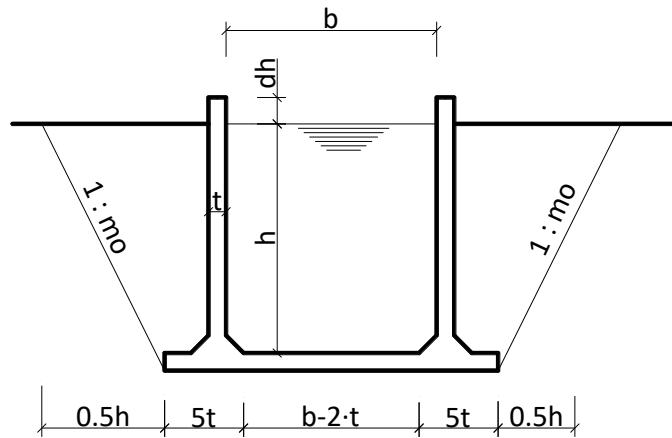
Of the specific conditions for the construction of a canal cross-section, only three have been addressed:

- Cost of Excavation
- Cost of Concrete (including steel)
- Cost of Backfilling with Compacted Soil

The following address the two cases when the canal is constructed on flat terrain and the general case when the terrain represents a line that forms an angle with the horizon .

#### Flat Terrain Analysis

In drawing below a typical cross-section of the channel on horizontal terrain is given, where the section is assumed to be rectangular ( $m=0$ ).



Assuming a rectangular section and its construction with reinforced concrete walls, there are four main dimensions

- Internal width
- Water depth
- Wall thickness
- Slope of the excavated part

For this typical section, the following are also assumed:

The reinforced concrete base will extend on both sides by three times the wall thickness. The excavation base is equal to the concrete base.

The excavation depth is equal to the water depth.

The water depth will be a function of the internal width, since the discharge, slope, and roughness coefficient are assumed constant for all variants that will be tested in order to determine the techno-economic cross-section.

The calculations were carried out by varying the width  $b$  from a given value and computing the normal depth using the Chezy formula. In this way, the ratio  $b/h_0$ , the excavation volume, the concrete volume, and the backfill volume were calculated using the formulas derived from the figure above.

$$G = \frac{(b + 8t) + (b + 8t) + m_0 h^2}{2} \cdot (h + t)$$

$$Bet = (b + 2(h + d_h)) \cdot t + 8t^2$$

$$Mb = m_0 h^2$$

By denoting the corresponding unit costs of the three quantities above as  $C_g$ ,  $C_b$ , and  $C_{mb}$ , the cost per linear meter of the channel is:

$$K_0 = G \cdot C_g + Bet \cdot C_b + Mb \cdot C_{mb}$$

### Sloped Terrain Analysis

In drawing below, a typical cross-section of a channel in sloping terrain is shown, where the section is assumed to be rectangular ( $m = 0$ ). Assuming a rectangular cross-section and its construction with reinforced concrete

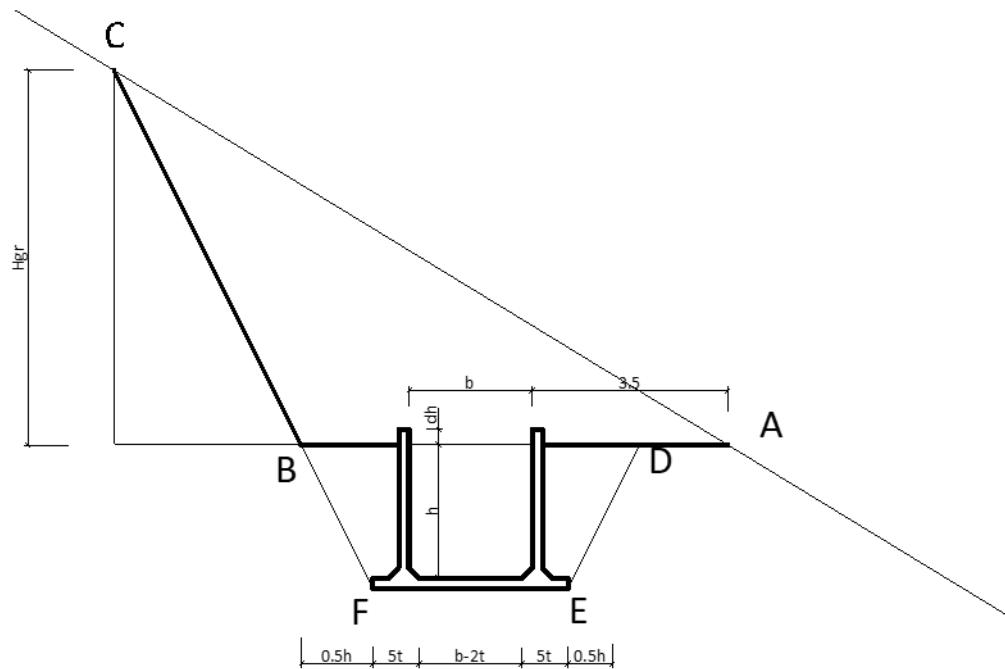
walls, there are six main dimensions:

- Internal width
- Water depth
- Wall thickness
- Slope of the excavated side
- Angle of the terrain relative to the horizontal
- Height of the slope excavation

For this typical cross-section, the following assumptions are also made:

- The reinforced concrete base extends on both sides by three times the wall thickness. The excavation base is equal to the concrete base.
- The excavation depth is equal to the water depth.
- On one side of the channel, the construction of a service road with a width of 3.5 m is assumed.

The water depth will be a function of the internal width, since the discharge, slope, and roughness coefficient are assumed constant for all variants that will be tested in order to determine the optimal techno-economic cross-section.



The calculations were carried out by varying the width  $b$  from a given value and calculating the normal depth using the Chezy formula. In this way, the ratio  $b/h_0$ , the excavation volume, the concrete volume, and the backfill volume were determined using formulas derived from the figure above.

$$G = \text{Area } ABC + \text{Area } DEFB = f(b/h_0, \alpha)$$

$$Bet = (b + 2(h + d_h)) \cdot t + 8t^2$$

$$Mb = m_0 h^2$$

As can be seen, in this case the excavation volume depends on the angle of the terrain relative to the horizontal, whereas the concrete and backfill volumes depend only on the dimensions  $b$ ,  $h_0$ , and  $t$ .

By denoting the corresponding unit costs of the three quantities above as  $C_g$ ,  $C_b$ , and  $C_{mb}$ , the cost per linear meter of the channel is:

$$K_0 = G \cdot C_g + Bet \cdot C_b + Mb \cdot C_{mb}$$

For this case for the derivation channel, the calculations were performed using the above methodology,. These calculations were carried out for different terrain angles ranging from  $20^\circ$  to  $60^\circ$ .

#### IV. Influence of Market Costs

Based on the model developed for calculating the optimal dimensions of a channel constructed in sloping terrain, an analysis was carried out of the influence that excavation and concrete costs have on determining these dimensions (the ratio  $b/h_0$ ).

To perform this analysis, calculations were carried out by varying:

- The terrain angle  $\alpha$  from  $0^\circ$  to  $60^\circ$
- The excavation cost from 400 ALL/m<sup>3</sup> to 1,200 ALL/m<sup>3</sup>. These are costs commonly encountered in such works, for formations ranging from soft soil to rock.
- The concrete cost from 8,000 ALL/m<sup>3</sup> to 12,000 ALL/m<sup>3</sup>. These are costs typically encountered in this type of work for both short and long transport distances.

In the tables below, the calculations and corresponding graphs prepared for three concrete cost levels and three excavation cost levels are presented. These were produced during the design of the “Plani i Bardhë” hydropower plant project in the Bulqizë district.

The tables below show the influence of excavation cost for a fixed concrete cost of  $c_b = 8000$  ALL/m<sup>3</sup>.

$\alpha$	$C_g = 400$ leke/m <sup>3</sup>			$C_g = 800$ leke/m <sup>3</sup>			$C_g = 1200$ leke/m <sup>3</sup>		
	$b$	$h$	$\beta=b/h$	$b$	$h$	$\beta=b/h$	$b$	$h$	$\beta=b/h$
<b>0</b>	3.83	1.50	2.56	4.12	1.40	2.95	4.33	1.34	3.24
<b>10</b>	3.46	1.65	2.09	3.49	1.64	2.13	3.50	1.63	2.15
<b>20</b>	3.18	1.80	1.77	3.12	1.84	1.70	3.09	1.86	1.66
<b>30</b>	2.95	1.95	1.52	2.84	2.03	1.40	2.79	2.06	1.35
<b>40</b>	2.73	2.11	1.29	2.61	2.23	1.17	2.55	2.29	1.12
<b>50</b>	2.51	2.33	1.08	2.38	2.48	0.96	2.28	2.62	0.87
<b>60</b>	2.21	2.72	0.81	2.08	2.93	0.71	1.87	3.36	0.56

The tables below show the influence of excavation cost for a fixed concrete cost of  $c_b = 10,000$  ALL/m<sup>3</sup>.

$\alpha$	Cg= 400 leke/m <sup>3</sup>			Cg= 800 leke/m <sup>3</sup>			Cg= 1 200 leke/m <sup>3</sup>		
	b	h	$\beta=b/h$	b	h	$\beta=b/h$	b	h	$\beta=b/h$
<b>0</b>	3.76	1.52	2.47	4.02	1.43	2.81	4.21	1.37	3.07
<b>10</b>	3.45	1.66	2.08	3.48	1.64	2.12	3.50	1.64	2.14
<b>20</b>	3.20	1.79	1.79	3.14	1.82	1.72	3.10	1.85	1.68
<b>30</b>	2.99	1.92	1.56	2.88	2.00	1.44	2.82	2.04	1.38
<b>40</b>	2.78	2.07	1.34	2.64	2.20	1.20	2.58	2.26	1.14
<b>50</b>	2.56	2.28	1.13	2.42	2.44	0.99	2.35	2.52	0.94
<b>60</b>	2.24	2.67	0.84	2.12	2.85	0.74	2.04	2.99	0.68

The tables below show the influence of excavation cost for a fixed concrete cost of  $cb = 12\ 000$  ALL/m<sup>3</sup>.

$\alpha$	Cg= 400 leke/m <sup>3</sup>			Cg= 800 leke/m <sup>3</sup>			Cg= 1 200 leke/m <sup>3</sup>		
	b	h	$\beta=b/h$	b	h	$\beta=b/h$	b	h	$\beta=b/h$
<b>0</b>	3.71	1.54	2.40	3.94	1.46	2.70	4.12	1.40	2.95
<b>10</b>	3.44	1.66	2.07	3.47	1.65	2.11	3.49	1.64	2.13
<b>20</b>	3.22	1.78	1.81	3.15	1.81	1.74	3.12	1.84	1.70
<b>30</b>	3.02	1.90	1.59	2.90	1.98	1.47	2.84	2.03	1.40
<b>40</b>	2.82	2.04	1.38	2.68	2.17	1.24	2.61	2.23	1.17
<b>50</b>	2.61	2.23	1.17	2.45	2.40	1.02	2.38	2.48	0.96
<b>60</b>	2.35	2.53	0.93	2.12	2.85	0.75	2.08	2.93	0.71

The tables below show the influence of concrete cost for a fixed excavation cost of  $cg = 800$  ALL/m<sup>3</sup>.

$\alpha$	Cb= 8 000 leke/m <sup>3</sup>			Cb= 10 000 leke/m <sup>3</sup>			Cb= 12 000 leke/m <sup>3</sup>		
	b	h	$\beta=b/h$	b	h	$\beta=b/h$	b	h	$\beta=b/h$
<b>0</b>	4.12	1.40	2.95	4.02	1.43	2.81	3.94	1.46	2.70
<b>10</b>	3.49	1.64	2.13	3.48	1.64	2.12	3.47	1.65	2.11
<b>20</b>	3.12	1.84	1.70	3.14	1.82	1.72	3.15	1.81	1.74
<b>30</b>	2.84	2.03	1.40	2.88	2.00	1.44	2.90	1.98	1.47
<b>40</b>	2.61	2.23	1.17	2.64	2.20	1.20	2.68	2.17	1.24
<b>50</b>	2.38	2.48	0.96	2.42	2.44	0.99	2.45	2.40	1.02
<b>60</b>	2.08	2.93	0.71	2.12	2.85	0.74	2.12	2.85	0.75

The tables below show the influence of concrete cost for a fixed excavation cost of  $cg = 1 200$  ALL/m<sup>3</sup>.

$\alpha$	$C_b = 8\ 000 \text{ leke/m}^3$			$C_b = 10\ 000 \text{ leke/m}^3$			$C_b = 12\ 000 \text{ leke/m}^3$		
	$b$	$h$	$\beta = b/h$	$b$	$h$	$\beta = b/h$	$b$	$h$	$\beta = b/h$
<b>0</b>	4.33	1.34	3.24	4.21	1.37	3.07	4.12	1.40	2.95
<b>10</b>	3.50	1.63	2.15	3.50	1.64	2.14	3.49	1.64	2.13
<b>20</b>	3.09	1.86	1.66	3.10	1.85	1.68	3.12	1.84	1.70
<b>30</b>	2.79	2.06	1.35	2.82	2.04	1.38	2.84	2.03	1.40
<b>40</b>	2.55	2.29	1.12	2.58	2.26	1.14	2.61	2.23	1.17
<b>50</b>	2.28	2.62	0.87	2.35	2.52	0.94	2.38	2.48	0.96
<b>60</b>	1.87	3.36	0.56	2.04	2.99	0.68	2.08	2.93	0.71

The tables below show the influence of concrete cost for a fixed excavation cost of  $c_g = 1\ 200 \text{ ALL/m}^3$ .

## V. Conclusions

By comparing the above results, the following conclusions are reached:

- **The terrain angle** is the parameter that has the greatest influence on determining the optimal dimensions of the cross-section. The ratio  $b/h_0$  decreases as the terrain angle increases. For angle values from  $0^\circ$  to  $20^\circ$ , the relationship is non-linear, whereas for values greater than  $20^\circ$  this relationship is almost linear.
- **Excavation and concrete costs** influence the determination of the optimal cross-sectional dimensions of a channel to a lesser extent than the terrain angle.
- In determining the cross-sectional dimensions, **excavation cost** has a noticeable influence for terrain angles from  $0^\circ$  to  $10^\circ$ . This is observed in all three groups calculated for concrete prices ranging from 8,000 to 12,000 ALL/m<sup>3</sup>.
- In determining the cross-sectional dimensions, **concrete cost** has a small influence for terrain angles from  $0^\circ$  to  $10^\circ$ . This is likewise observed in all three groups calculated for excavation prices ranging from 800 to 1,200 ALL/m<sup>3</sup>.
- Calculations for determining the optimal cross-section do not need to be carried out for each individual profile. Therefore, the cross-sections obtained from topographic surveys are first analyzed, and a representative typical cross-section is selected for all profiles. In this way, an average slope angle of the mountain side where the channel is constructed is determined, and the calculations described above are performed using this value.
- The dimensions calculated using this methodology are then further adapted to the specific conditions of each section of the channel under consideration, depending on geological conditions and specific circumstances that may arise during construction. Thus:
  - In the derivation channel of the **Krastë** hydropower plant, after opening the derivation alignment, it was found that the channel had to be constructed with three different slopes along the first 1,400 m. As a result, three different types of cross-sections were used (see drawings of typical sections).
  - In the derivation channel of the **Plani i Bardhë** hydropower plant, in the first section with a length of about 1,500 m the formations are soft clayey soils, and therefore a reinforced concrete typical section was used. In the other section, where the slope is steeper and the formation is rocky, the typical section was adapted by

removing the two side anchors.

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