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Elaboration of kinematic and calculation of pressure and stress components of a fluid flow between two parallel plates

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Abstract:
In this work,we set ourselves the objective to study a flowing of a liquid between two parallel plats with a upper plat animate by a mouvement of translation parallel on itself with nown velocity. we elaborating kinematic of fowing components, calculating gradient and Cauchy-Green tensors, velocity gradient and its adjoint, elementaries invariants. Navier-Stockes equations gives us the internal pressure and stress components. The simulation shows that: X_2 *has a big influence in the behaviour of the kinematic. the velocity follows a dependence on* X_2*. The pressure is linear according to all the variables. The stress components have linear dependence of the variables.*

Keywords:
Parallel plates, Kinematic of flowing, flowing gradient, velocity gradient, stress tensor, internal pressure.

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I. Introduction

The notion of fluid refers to the absence of an organized structure of matter at the microscopic scale, thus allowing large-amplitude movements of atoms. It therefore that fluids group together the liquid and gaseous states [1].
The study of fluids plays a very important role nowadays because it allows the development of models for improving the performance of machines in the maritime, land or space fields. A fluid can be viscous, compressible or incompressible.
When we focus about viscosity, an exact analysis of radiative effects on the magnetohydrodynamic (MHD) free convection flow of an electrically conducting incompressible viscous fluid over a vertical plate is studied where the non-dimensional continuity, momentum, and energy equations are solved using appropriate transformation [2]. A solution in the explicit form of the equation of the momentum diffusion for a viscous fluid flowing around a plate taking into account deceleration with three characteristic regions of a viscous flow have been given in [3].
In the case of incompressible flows, a new scalar projection method presented for simulating incompressible flows with variable density is proposed where the

first phase of the projection is purely kinematics. The predicted velocity field is subjected to a discrete Hodge-Helmholtz decomposition [4]. A solution of an incompressible fluid flow is also studied in [5].
However the study of kinematics plays a very important role because it constitutes for most of the time, the starting point of a study in mechanics of continuous medium, in particulary in mechanic of fluid [6,7,8,9,10].
In this paper, we will set ourselves as an objective to study a fluid flow precisely a liquid between two parallel plats with a upper plat animate by a mouvement of translation parallel on itself. Initially the spatial components of the velocity components will be given. From this velocity, we will elaborate the kinematic spatial components. We will do a mechanical by calculating flowing gradient, Cauchy-Green tensor, velocity gradient and its adjoint, and elementaries invariants. Navier-Stockes equations will be set solve to obtain the expression of the internal pressure and then the expressions of the stress components. we will simulate and interpret the kinematic, the velocity, the internal pressure and the stress components..

II. Mathematical Expressions

In mechanics of continuous mediums as in the particular case of fluid mechanics, a kinematics of transformation is always given from one of the two following configurations that are: the Lagrangian configuration and the Eulerian configuration. The Lagrangian configuration observe the fluid particle passed in a fixed point of the space while the Eulerian configuration follows the particle continuous mediums in its movement.

$$x_i = f_i(X); \tag{1}$$

where $f_{i(1\leq i\leq 3)}$ are one to one applications and $X = (X_1, X_2, X_3)$ in the case of a spatial transformation.
It is possible that a kinematic of transformation becomes defined from the velocity components as:

$$v_i = g_i(X); \tag{2}$$

2.1 Tensors and invariants

Let's consider a fluid flowing between two parallel plates with the upper plat animate by a mouvement of translation of velocity V parallel on itself with nown flowing velocity determine by the following profil of speed:

$$\begin{cases} v_1 = \frac{C_1}{2\mu} X_2 \left(H - X_2\right) + \frac{V}{H} X_2; \\ v_2 = 0; \\ v_3 = 0. \end{cases} \tag{3}$$

where H is the altitude of the upper plate, C is the lineic fall and μ is the first modulus of viscosity.
When we integrate these velocity components, we have:

$$\begin{cases} x_1 = C_1 + \left(\frac{C}{2\mu} X_2 \left(H - X_2\right) + \frac{V}{H} X_2\right) t; \\ x_2 = C_2; \\ x_3 = C_3. \end{cases} \tag{4}$$

where C_1, C_2 and C_3 are constants of integration of time. So the mechanical conditions will be set by $C_1 = X_1$, $C_2 = X_2$ and $C_3 = X_3$, so we obtain:

$$\begin{cases} x_1 = X_1 + \left(\frac{C}{2\mu} X_2 \left(H - X_2\right) + \frac{V}{H} X_2\right) t; \\ x_2 = X_2; \\ x_3 = X_3. \end{cases} \tag{5}$$

The deformation gradient is:

$$\mathbf{F} = \begin{pmatrix} 1 & \left(\frac{C}{2\mu}\left(H - 2X_2\right) + \frac{V}{H}\right) t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

The right Cauchy-Green tensor is $\mathbf{C} = \mathbf{F^T F}$, then:

$$\mathbf{C} = \begin{pmatrix} 1 & \left(\frac{C}{2\mu}\left(H - 2X_2\right) + \frac{V}{H}\right) t & 0 \\ \left(\frac{C}{2\mu}\left(H - 2X_2\right) + \frac{V}{H}\right) t & \left(\frac{C}{2\mu}\left(H - 2X_2\right) + \frac{V}{H}\right)^2 t^2 + 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{7}$$

From this previous tensor, we find:

$$det\left(\mathbf{C}\right) = 1. \tag{8}$$

That previous result shows that we are in an incompressible flowing.
And the velocity gradient is:

$$\mathbf{D} = \begin{pmatrix} 0 & \frac{C}{2\mu}\left(H - 2X_2\right) + \frac{V}{H} & 0 \\ \frac{C}{2\mu}\left(H - 2X_2\right) + \frac{V}{H} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

The adjoint of the velocity gradient is defined by:

$$\mathbf{D}^* = det\left(\mathbf{D}\right) \mathbf{D}^{-1}. \tag{10}$$

Without calculus, we can see that $det\left(\mathbf{D}\right) = 0$. So then:

$$\mathbf{D}^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{11}$$

The calculation of isotropic invariants gives:

$$\begin{cases} I_1 = trace\,(\mathbf{D}) = 0; \\ I_2 = trace\,(\mathbf{D}^*) = 0; \\ I_3 = det\,(\mathbf{D}) = 0. \end{cases} \tag{12}$$

The condition $I_1 = 0$ translate also the incompressibility of the fluid flow.
We can remind that an other conditon of the incompressibility of continuous medium is given by:

$$div\,(\mathbf{v}) = \frac{\partial v_1}{\partial X_1} + \frac{\partial v_2}{\partial X_2} + \frac{\partial v_3}{\partial X_3} = 0. \tag{13}$$

This previous result is true and easy to verify from the velocity components.
The stress tensor is difined by $\Sigma = -P\mathbf{I} + 2\mu\mathbf{D}$, then:

$$\Sigma = \begin{pmatrix} -P & C\,(H - 2X_2) + 2\mu\frac{V}{H} & 0 \\ C\,(H - 2X_2) + 2\mu\frac{V}{H} & -P & 0 \\ 0 & 0 & -P \end{pmatrix}. \tag{14}$$

where P is the internal pressure.

2.2 Navier Stockes equations and pressure

The Navier-Stockes equations are given by:

$$\begin{cases} \varrho\left(\frac{\partial v_1}{\partial t} + v_1\frac{\partial v_1}{\partial X_1} + v_2\frac{\partial v_1}{\partial X_2} + v_3\frac{\partial v_1}{\partial X_3}\right) = -\frac{\partial P}{\partial X_1} + \varrho g_{x_1} + \mu\left(\frac{\partial^2 v_1}{\partial X_1{}^2} + \frac{\partial^2 v_1}{\partial X_2{}^2} + \frac{\partial^2 v_1}{\partial X_3{}^2}\right); \\ \varrho\left(\frac{\partial v_2}{\partial t} + v_1\frac{\partial v_2}{\partial X_1} + v_2\frac{\partial v_2}{\partial X_2} + v_3\frac{\partial v_2}{\partial X_3}\right) = -\frac{\partial P}{\partial X_2} + \varrho g_{x_2} + \mu\left(\frac{\partial^2 v_2}{\partial X_1{}^2} + \frac{\partial^2 v_2}{\partial X_2{}^2} + \frac{\partial^2 v_2}{\partial X_3{}^2}\right); \\ \varrho\left(\frac{\partial v_3}{\partial t} + v_1\frac{\partial v_3}{\partial X_1} + v_2\frac{\partial v_3}{\partial X_2} + v_3\frac{\partial v_3}{\partial X_3}\right) = -\frac{\partial P}{\partial X_3} + \varrho g_{x_3} + \mu\left(\frac{\partial^2 v_3}{\partial X_1{}^2} + \frac{\partial^2 v_3}{\partial X_2{}^2} + \frac{\partial^2 v_3}{\partial X_3{}^2}\right). \end{cases} \tag{15}$$

What yields:

$$\begin{cases} -\frac{\partial P}{\partial X_1} + \varrho g_{x_1} - 2\mu = 0; \\ -\frac{\partial P}{\partial X_2} + \varrho g_{x_2} = 0; \\ -\frac{\partial P}{\partial X_3} + \varrho g_{x_3} = 0. \end{cases} \tag{16}$$

According to our flowing, the gravity is $(g_{x_1}, g_{x_2}, g_{x_3}) = (0, -g, 0)$ and then:

$$\begin{cases} -\frac{\partial P}{\partial X_1} - 2\mu = 0; \\ -\frac{\partial P}{\partial X_2} - \varrho g = 0; \\ -\frac{\partial P}{\partial X_3} = 0. \end{cases} \tag{17}$$

When we integrate the two first equations of the previous system, we obtain:

$$\begin{cases} P = -2\mu X_1 + C_4; \\ P = -\varrho g X_2 + C_5, \end{cases} \tag{18}$$

where C_4 and C_5 are respectively constants of X_1 and X_2. By identification according to the previous system, we find:

$$P(X_1, X_2) = -2\mu X_1 - \varrho g X_2 + P_0, \tag{19}$$

where P_0 is the initial pressure.
From this pressure, we obtain:

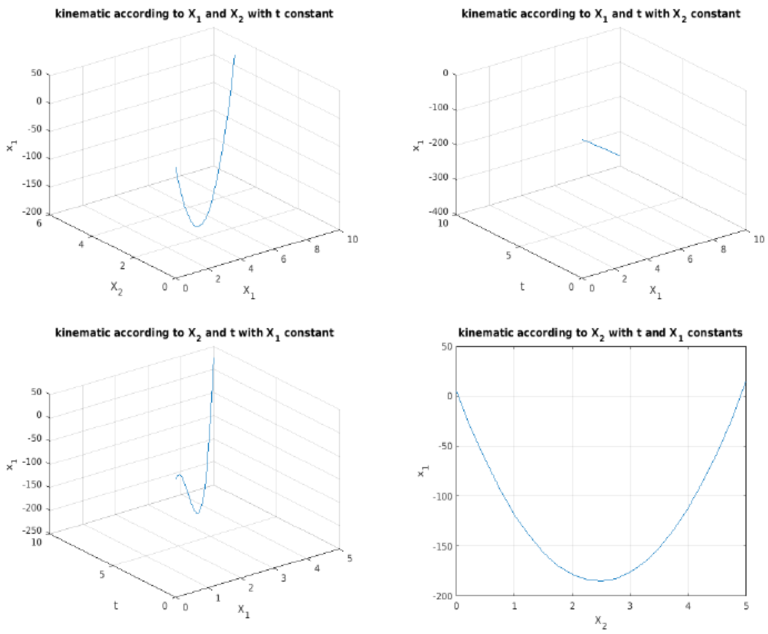
$$\begin{cases} \Sigma_{11} = \Sigma_{22} = \Sigma_{33} = 2\mu X_1 + \varrho g X_2 - P_0; \\ \Sigma_{12} = \Sigma_{21} = C(H - 2X_2) + 2\mu \frac{V}{H}; \\ \Sigma_{13} = \Sigma_{23} = \Sigma_{31} = \Sigma_{32} = 0. \end{cases} \tag{20}$$

3 simulation and interpretation

In all this part of similation, we will consider $X_1 = 0 : 0.5 : 10$; $X2 = 0 : 0.25 : 5$; $X3 = 0 : 0.1 : 2$; $h = 5$; $t = 0 : 0.5 : 10$; $t = 5$; $\mu = 2.5$; $C = -2\mu$; $V = 2$; $\varrho = 1$; $g = 9.8$; and $P_0 = 10$.

3.1 Kinematic

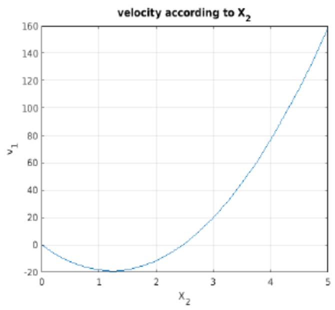
In the simulation of kinematic, we consider one or two variables of this last according to the case.



the previous graphics show a big dependance of the kinematic of flowing to the variable X_2 with a polynomial behaviour. In the case of $X_2 = constant$, we observe a linear behaviour. So then X_2 variable has a big influence in the behaviour of the kinematic.

3.2 Velocity

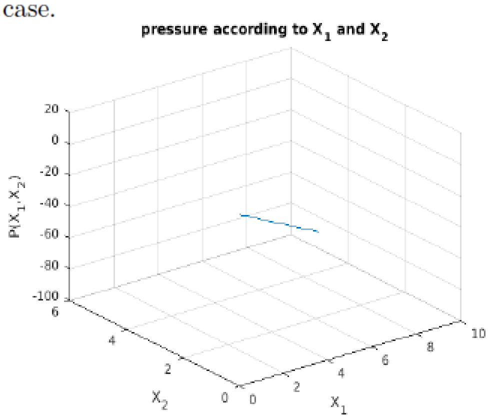
In the simulation of velocity, it depends only on X_2, So then we have:

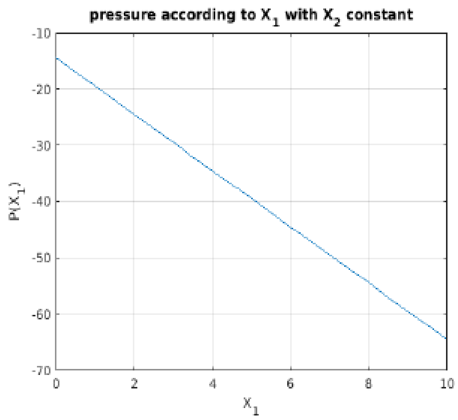


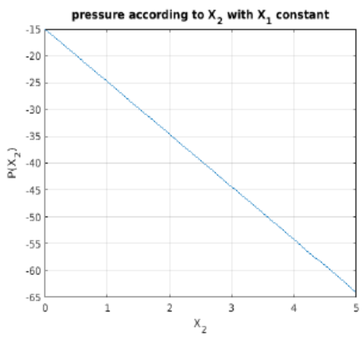
In this previous graphic, we see a polynomial behaviour as we can see it in the expression of the velocity which has a polynomial dependence on X_2.

3.3 Pressure

In the simulation of pressure, one or two variables of this last according to the case.



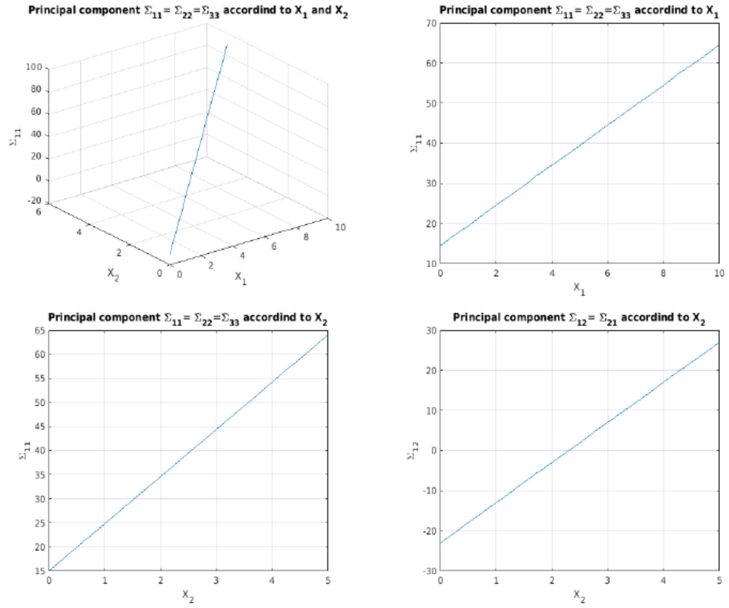




In this previous graphics, we see a linear behaviour of the pressure according to the variables as we can see it in its the expression.

3.4 Stress components

In the simulation of stress components, we consider one or two variables of this last according to the case. So then:



The previous graphics of the stress components show that we have linear dependence of the variables as we can see it in the expressions.

Conclusion

In this work, we set ourselves the objective to study a flowing of a liquid between two parallel plats with a upper plat animate by a mouvement of translation parallel on itself with a velocity components which are nown. A mechanical study is done by elaborating kinematic of deformation components, calculating flowing gradient, Cauchy-Green tensor, velocity gradient and its adjoint, and elementaries invariants. Navier-Stockes equations are resolved to find the internal pressure and then explicit expression of the stress components.

The simulation shows that:

the kinematic graphics has a big dependance of the spacial variable X_2 with a polynomial behaviour but in the case $X_2 = constant$, we observe a linear behaviour. then X_2 has a big influence in the behaviour of the kinematic.

the velocity simulation shows a polynomial behaviour as we can see it in the expression of the velocity which has a polynomial dependence on X_2.

In the simulation of the pressure, graphics show a linear behaviour according to all the variables as we can see in its expression.

The simulation of the stress components show that we have linear dependence of the variables as we can see it in its expression.

As a contribution we elaborate kinematic of flowing of liquide in incompressibity and find the internal pressure and and the stresse components.

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