

In Presence of Hydromagnetic Field of an Unsteady Mixed Convective Boundary Layer Flow over a vertical plate

Md.Jashim Uddin

Department of Electrical & Electronic Engineering,
International Islamic University Chittagong, Bangladesh

Abstract: We study the effect of a non-steady hydromagnetic field at the boundary on mixed convective boundary layer flow occurring as the result of natural convection heat transfer. Governing partial differential equations are made by symmetry transformations into ordinary differential equations, and these are tackled over with the Runge-Kutta method jammed right down to ensure more precise results. We characterize the velocity and temperature fields in the boundary layer obtained flow parameter. The flow phenomenon has been suppressed with the help of parameters obtained for controlling mass transfer effects. Image If the mixed convective parameter, the Prandtl numbers and the magnetic field over their corresponding ranges are increased only slightly then a velocity overshoot may be noted near the plate, as will a temperature overshoot. We obtain the non-dimensional velocity and temperature profile by numerical calculation and show them graphically. In the main, these are presented for different values of the relevant parameters.

Keywords: Boundary layer flow, Hydromagnetic Field, Mixed convection, Unsteady

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I. INTRODUCTION

A mixed convection flow, or a combined forced and free convection flow, appears in many engineering facilities and natural processes. These flows are described by the Buoyancy Richardson number (measure of influence of natural convection on fluid flow compared with that of forced convection) as a function of what the flow conditions are and the state of heating at the surface. Convection heat transfer in the fluid flow is a phenomenon from a theoretical and a practical point of view because in its many engineering geophysical fields applications this be worth studying very much. The mixed convection flow due to a heated or cooled vertical flat has been one of the most fundamental problems in heat transfer theory and therefore has been studied extensively by Sparrow et al. [1] Watanabe [2] has considered the mixed convection boundary layer flow across isothermal vertical porous flat plates with uniform injection or suction. Sattar [3] carried out analytical studies on the combined forced and free convection flow in a porous medium. Hassanien [4] has considered combined effect of variable permeability and thermal conductivity on mixed convection flow past an impermeable vertical wedge with non-similar solutions for case of variable surface heat quanta. Any [5] discussed the mixed convection over a horizontal plate off plate along an flat two-dimensional Afzal And Hussain: The research of Yao [6]. Good source consulting YouHossain and Takhar [7] experimentally investigated the effect of radiation on mixed convection along a vertical plate with a uniform free stream and surface temperature. Aydin and Kaya [8] carried out computational analysis of the mixed heat convection flow of a viscous fluid slowly dissipated on a vertical flat plate. Reddy [9] has considered the problem of Magneto-hydrodynamic (MHD) and thermal radiation of a viscous fluid in laminar boundary layer flow over a stretching sheet.]. studied the MHD and thermal radiation boundary layer flow of a nanofluid past a stretching sheet. Parvin et al. [10] examined the Prandtl number effect on forced convection flow and thermal field characteristics inside an open cavity with porous wavy isothermal wall using water-CuO nanofluid. Ellahi et al. [11] analyzed the steady flow of viscous fluid with slip effects, heat/mass transfer. Williams et al., [12] addressed the unsteady free convection flow over a vertical flat plate under the common assumption of different by even a little wall temperatures being related to time and/or distance. They collected semi-similar solutions for a variety of classes in the distribution of wall temperatures. Kumari et al., [13] stated that time-

dependent velocity of the moving sheet caused unsteadiness in the flow field. Chamkha and Khaled[14] investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption. Khan et al.[15] have analyzed the problem of magnetohydrodynamic radiative flow of a nanofluid past a stretching sheet. Gau et al. [16] conducted a study of mixed convection in rectangular cavities at various aspect ratios with moving isothermal side walls and constant vertical flux heat source on the bottom wall. Nasrin [17] performed an aspect ratio effect investigation on the problem of mixed magnetoconvection in a vertical lid driven chamber with a centered conducting solid. Shokouhmand and Sayehvand [18] performed numerical investigation of flow and heat transfer in a lid-driven square cavity. They discovered that at higher Reynolds numbers, there was an inviscid core region, though secondary eddies were still to be seen in the lower corners of the square all Reynolds numbers considered. Bhoite et al. [19] studied computationally mixed convection flow and heat transfer in a shallow enclosed domain with block-like multiple heat generating components for a range of Reynolds and Grashof numbers, and block-to-fluid thermal conductivity ratios slightly greater than unity. Al-Amiri et al. [20] investigated mixed convection heat transfer in lid-driven back cover with a sinusoidal wavy bottom surface. Raju et al. [21] investigated the ternary HN flow under various shaped forms and differing densities over contracting fluid-permeable walls. Their results were that, at low Richardson number for larger amplitude wavy surfaces, the attainment of a corrugated lid-driven cavity was an effective heat transfer mechanism.

This paper aims at examining the effects of variable permeability and thermal conductivity on the mixed convection boundary layer flow over a vertical plate in presence of magnetic field. A holistic parametric study was needed, and how the temperature and velocity profiles are portrayed graphically at some simple selected bars constitutes the main thrust of these diagrams. Modification of numerical results which are obtained from calculations is made. Computed numerical results are plotted and the characteristics of the flow and heat transfer are analyzed in detail.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the unsteady heat and mass transfer boundary layer flow of a viscous incompressible fluid through a porous medium bounded by continuously moving surface with uniform velocity U in presence uniform transverse magnetic field of strength B_0 . Taking magnetic Reynolds number as small, it is neglected that any induced magnetic field will be produced. By means of boundary layer approximation, and the usual notation, equations of continuity, momentum energy dependent variables temperature are written for the initial and unsteady-state.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left(k \frac{\partial u}{\partial y} \right) + g \beta^* (T - T_\infty) \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma_e B_0^2 u^2}{\rho c_p} \quad (3)$$

Where ρ is the density, μ be the coefficient of viscosity, σ_e the electrical conductivity, k be thermal conductivity of the fluid, β^* is the volumetric coefficient of thermal expansion, g is the acceleration due to gravity, T is the temperature, T_∞ is the free stream temperature and c_p the specific heat at constant pressure. The other symbols have their usual meanings.

The corresponding boundary conditions are:

$$u = U, v = 0; T = T_w \quad \text{at } y = 0 \quad (4)$$

$$\text{and } u = 0, T = T_\infty \quad \text{at } y = \infty; \quad (5)$$

The continuity equation (1) is satisfied by introducing the stream function $\psi(x, y)$,

$$\text{such that } u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

The momentum and energy equations (2) and (3) can be transformed to the corresponding ordinary differential equations by introducing the following similarity transformations:

$$\psi = \sqrt{\frac{U x v_\infty}{t}} f(\eta), \quad \eta = y \sqrt{\frac{U}{v_\infty x t}}, \quad \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)} \frac{u t}{U} = f', \quad x = \xi, \quad t = \tau \quad (6)$$

where v_∞ is a reference kinematic viscosity.

The momentum and energy equations (2) – (3) after some simplifications, reduce to the following forms:

$$f''' + P_r(ff'' + \eta f'' + \frac{1}{2}f' + \frac{1}{2}\lambda\theta) = 0 \quad (7)$$

$$\theta'' + AP_r(\eta\theta' + f\theta') + E_c\left(\frac{1}{2}MP_rf + Af''^2\right) = 0 \quad (8)$$

where $P_r = \frac{\rho\xi v_\infty}{2kU}$ (Modified Prandtl number), $\lambda(=J) = \frac{g\beta^*(T_w - T_\infty)\tau^2}{U}$ (Mixed convection parameter),

$M = \frac{\sigma_e \tau^2 B_0^2}{\rho U}$ (Magnetic Parameter),

$E_c = \frac{U^2}{c_p(T_w - T_\infty)\tau^2}$ (Modified Eckert number), $A = \frac{c_p}{U}$ (Unsteadiness Parameter)

The corresponding boundary conditions are:

$$\begin{aligned} f = 0; f' = 1; \theta = 1 & \quad \text{as } \eta = 0 \\ f' = 0; \theta = 0 & \quad \text{as } \eta = \infty \end{aligned} \quad (9)$$

Where the prime (') denotes differentiation with respect to η

III. Methodology and Solution of the problem:

The parameter free method was applied to solve the governing partial differential equations and Similarity Solutions were found. The free parameter method is a way of finding 'Similarity Solution' for PDE. You assume that in the equations, dependent variables can be written as functions of these f, g , etc. known as "similarity parameters" which then lead to specific Singularity solutions. In this way, Similarity Variables are set up so that the number of independent variables present in a group of equations is reduced by one or more (at least). The governing boundary layer equations (2)-(3) are solved numerically subject to boundary conditions (4) and (5) by means of the shooting method. Building. First, the higher order non-linear differential equations (2)-(3) are converted into simultaneous linear differential equations of first order. Then, the latter systems are transformed into initial value problems by applying the shooting technique with Runge-kutta-Gill method, a special way of design for this kind of problems which is very stable. The velocity and temperature profiles are given in figs.

IV. RESULTS AND DISCUSSION

In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs.1-10, to illustrate the influence of physical parameters viz., magnetic parameter M , Prandtl number P_r , Eckert number E_c , Mixed convection parameter $\lambda(=J)$ and Unsteadiness parameter, A on the velocity f' and temperature θ . The profiles for velocity and temperature have shown in Fig.1 to fig. 10.

Velocity Profiles:

The profiles for velocity are shown in fig.1 to Fig.5.

Eckert numbers are physical parameters. The impact of local Eckert number on fluid velocity is shown in Fig.1. It can be seen from the figure that velocity distribution curve increases with Eckert Numbers.)

The velocity of the conductor will increase with an increasing magnetic field, as shown in Fig. 2. This fact is known to have an objective reason that electrical conducting fluid, such as is exposed to the cross magnet field at an experiment will exert a body force called Lorenz force. It is to be expected that as the magnetic field, which moves with free stream has a tendency to create motive power, this will increase speed of fluid and thus make surface friction force more severe.

Fig. 3 presents nondimensional velocity profiles with x (see lines for Pr ; and Pr , respectively) and y (also see lines for Pr ; Pr , respectively) directions, as a function of the effects of Prandtl number Pr . (The Weibull type curve for each direction may be associated to a different physical effect.) The increased velocity of fluid flow can be detected above the surface at each point along both x -axis and y -axis line resonant with wavelength--and thus change in direction of Local wave-propagation vector

The variations of the dimensionless velocity profiles for mixed convection parameter J at the boundary are described in Figs. 4. The velocity of the boundary layer inside the plate increases with mixed convection parameter J in the buoyancy aiding flow ($J > 0$). It follows that as J increases and its effects become favorable for buoyant flow, heat transfer from any given node should be enhanced. From Fig. 4, this is explicitly shown by the velocity profiles decreasing where parameter J grows.

Fig. 5 shows the dimensionless velocity distribution for a given value of Ec , M , J , Ta but at several value of Dimensionless Velocity Parameter. The profiles of velocity are reduced as the Unsteady Parameter A , increases.

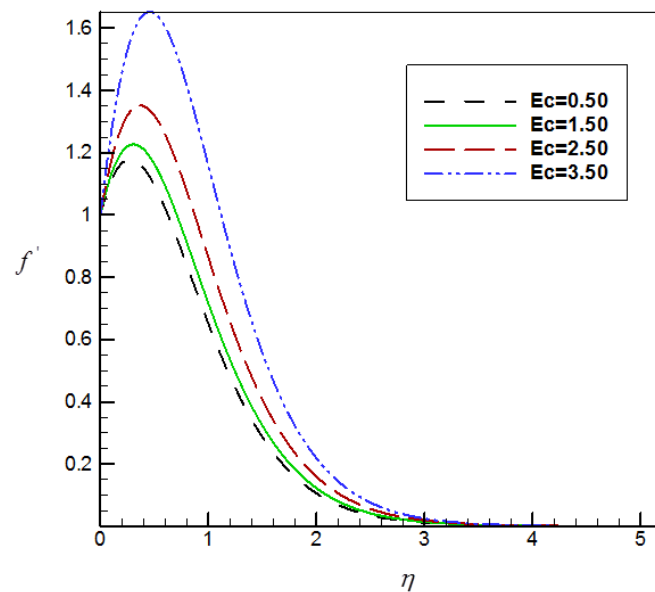


Fig1. Effect of dimensionless velocity profiles across the boundary layer for different values of Ec and for $Pr=0.71$, $M=2.00$, $J=0.20$, $A=0.50$

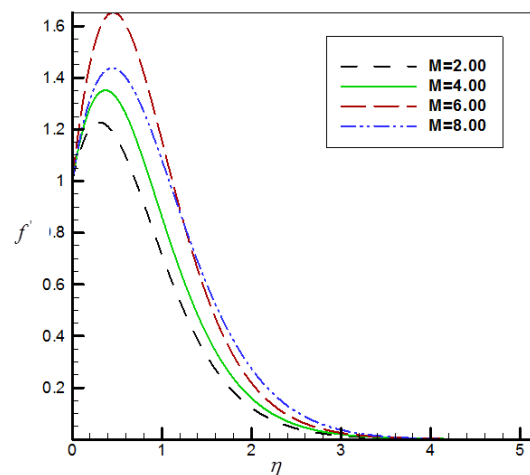


Fig2. Effect of dimensionless velocity profiles across the boundary layer for different values of M and for $Pr=0.71$, $Ec=1.50$, $J=0.20$, $A=0.50$

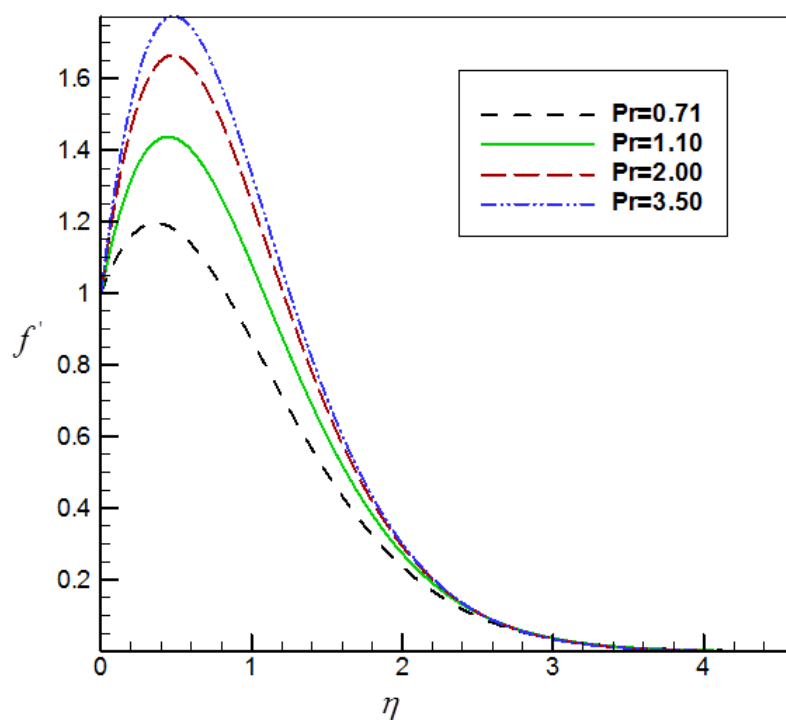


Fig3. Effect of dimensionless velocity profiles across the boundary layer for different values of Pr and for $M=2.00$, $Ec=1.50$, $J=0.20$, $A=0.50$

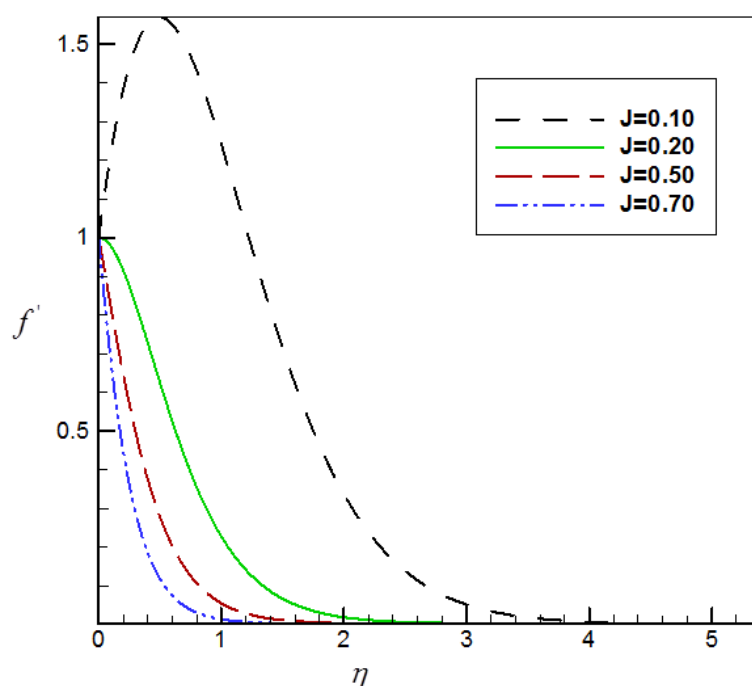


Fig4. Effect of dimensionless velocity profiles across the boundary layer for different values of J and for $M=2.00$, $Ec=1.50$, $Pr=0.71$, $A=0.50$

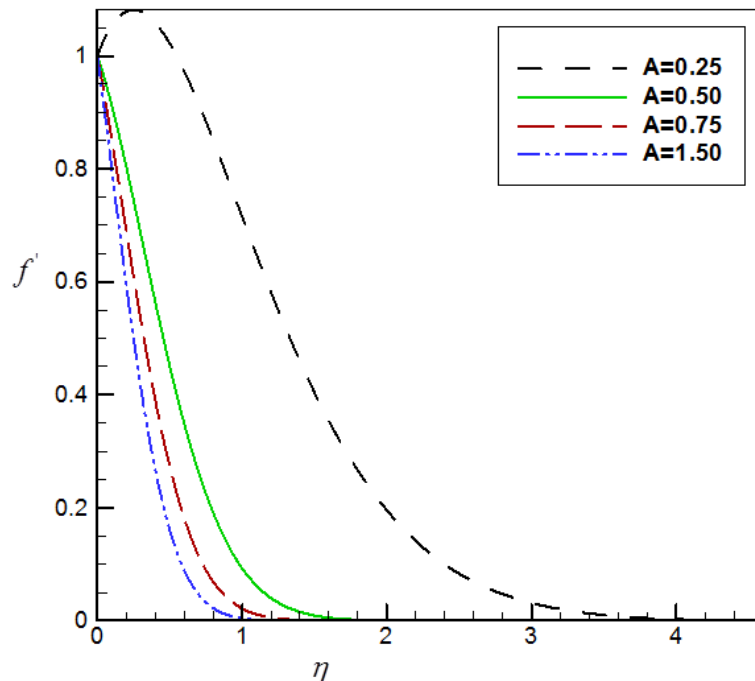


Fig5. Effect of dimensionless velocity profiles across the boundary layer for different values of A and for $M=2.00$, $Ec=1.50$, $Pr=0.71$, $J=0.20$

Temperature Profiles:

Figures 6 through 10 plot the dimensionless temperature profiles.

In Fig. 6, the effects experimental result of Eckert number (Ec) on temperature profile to be shown. As can be seen from this plot, an increase in Eckert number leads to a decrease of temperature distribution.

It can be seen from Fig.7 that as the magnetic field parameter increases, temperature distribution is slightly decreased. This indicates that a magnetic field could be utilized to regulate the flow, as well as the rotation of micro-constituents and heat transportation.

Fig. 8 presents temperature profiles for various values of the Prandtl number, Pr . The Prandtl number is a dimensionless number that defines the ratio of momentum diffusivity to thermal diffusivity.

From Fig. 9, the following can be seen: False T. The temperature profiles of a nanofluid are essentially the same as those for a quiescent state in common fluids because water also tends to have certain high characteristics. An increase in Prandtl number results in increase of fluid viscosity which leads to decrease in flow velocity. This phenomenon is similar to that for temperature profile.

Figures 6 through 10 plot the dimensionless temperature profiles.

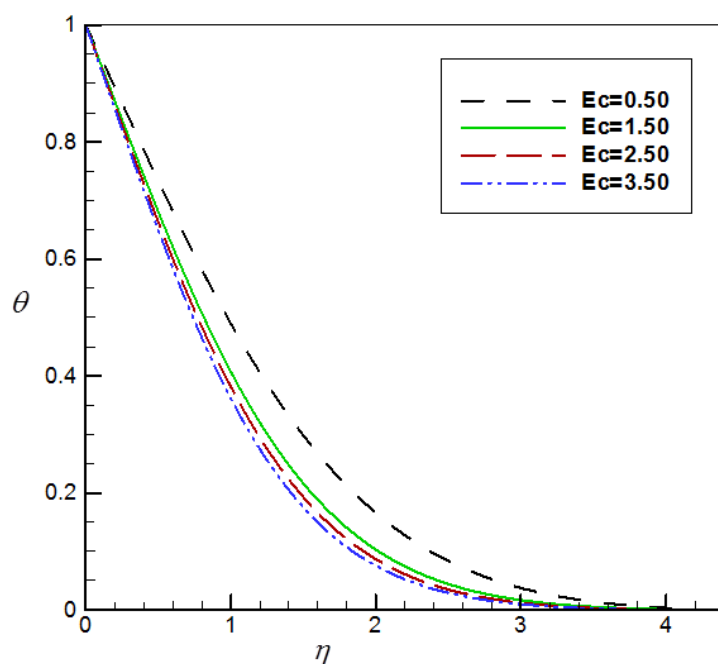


Fig06. Effect of dimensionless temperature profiles across the boundary layer for different values Ec and for $Pr=0.71, M=2.00, J=0.20, A=0.50$

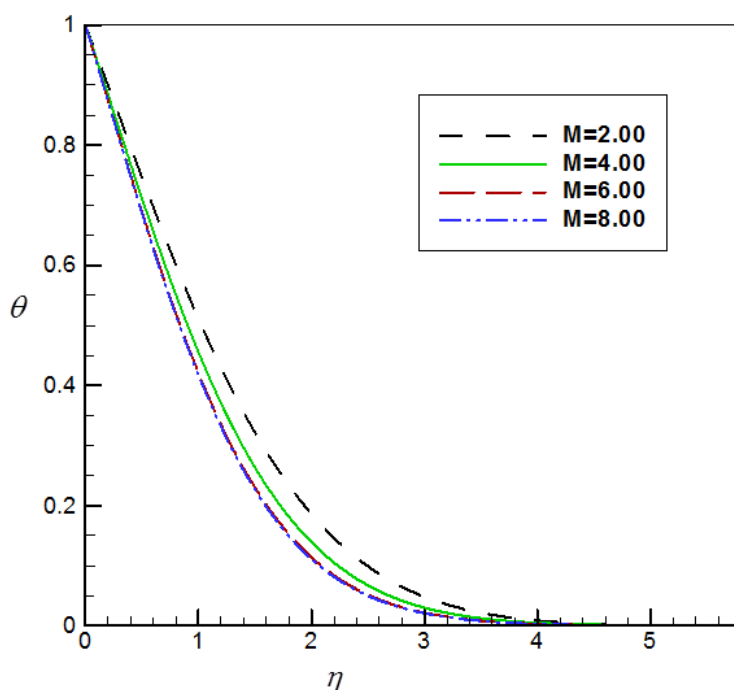


Fig07. Effect of dimensionless temperature profiles across the boundary layer for different values of M and for $Pr=0.71, Ec=1.50, J=0.20, A=0.50$

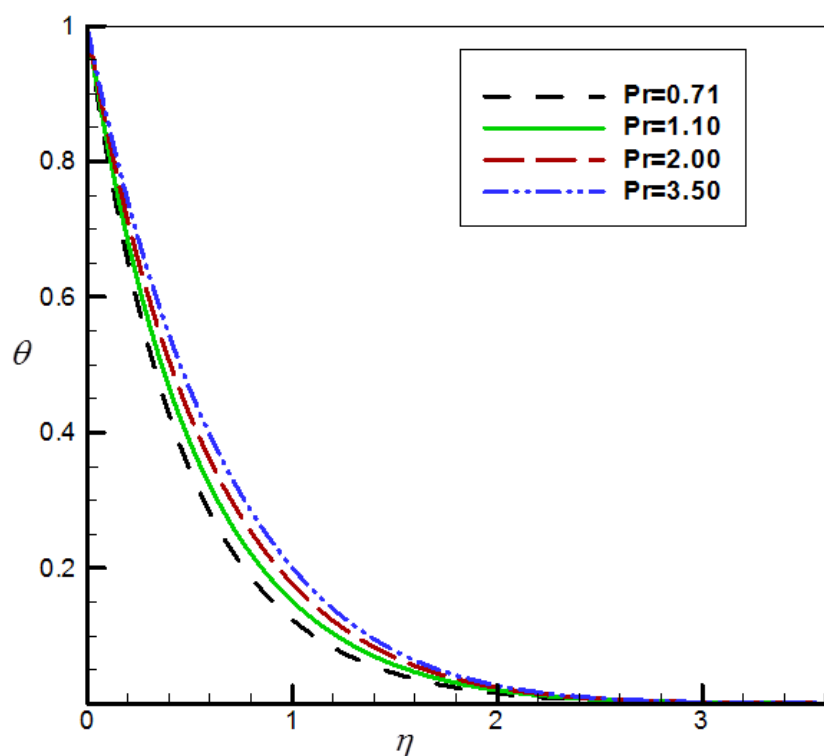


Fig08. Effect of dimensionless temperature profiles across the boundary layer for different values of Pr and for $M=2.00$, $Ec=1.50$, $J=0.20$, $A=0.50$

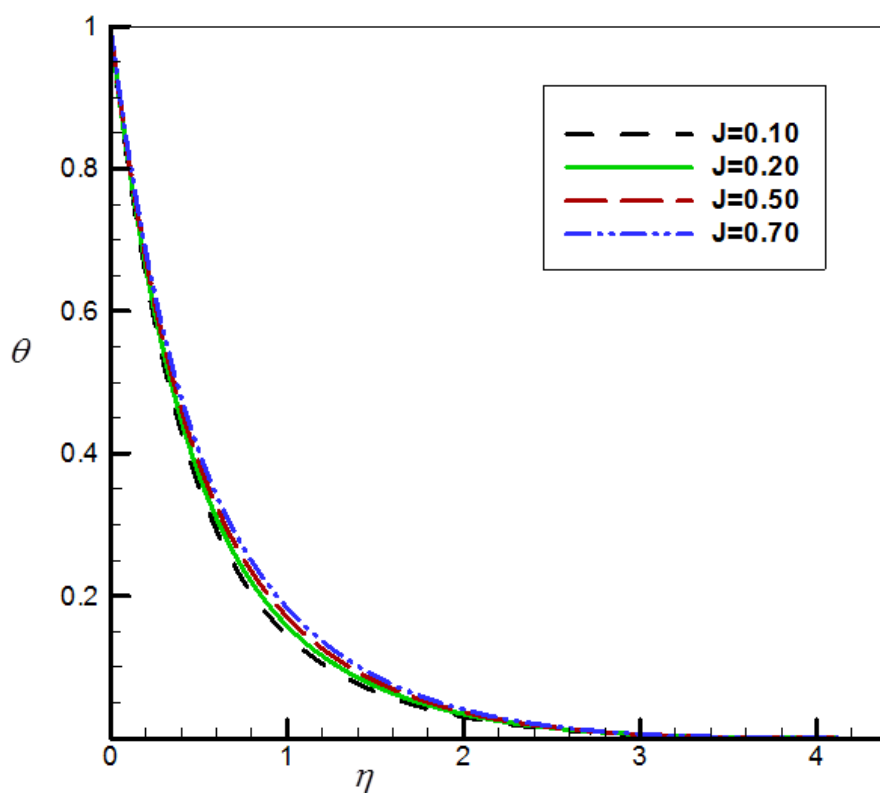


Fig09. Effect of dimensionless temperature profiles across the boundary layer for different values of J and for $M=2.00$, $Ec=1.50$, $Pr=0.71$, $A=0.50$

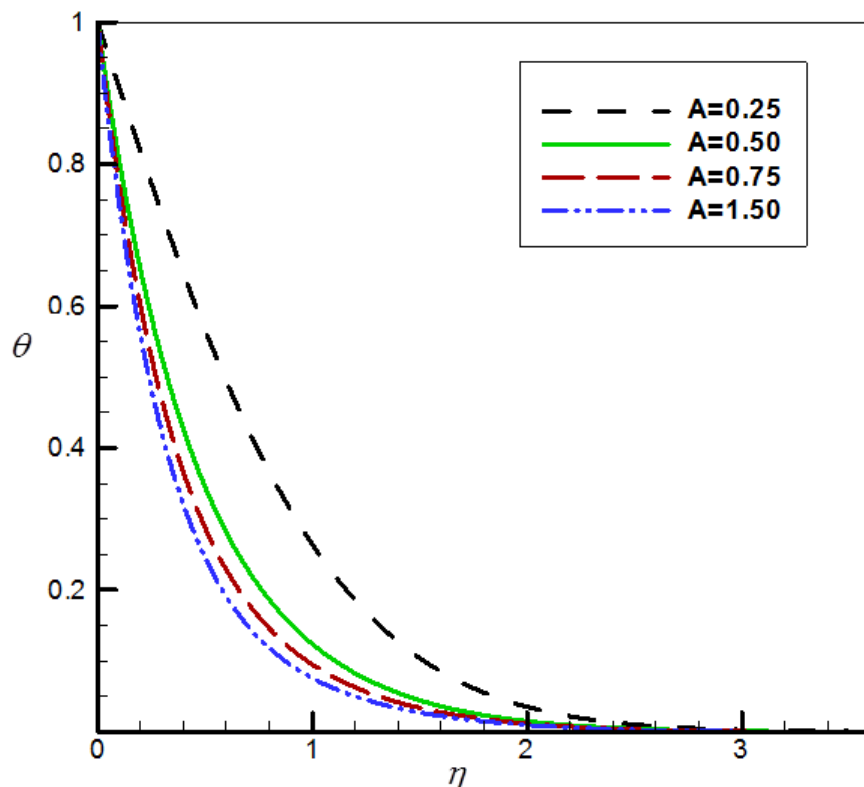


Fig10. Effect of dimensionless temperature profiles across the boundary layer for different values of A and for $M=2.00$, $Ec=1.50$, $Pr=0.71$, $J=0.20$

V. CONCLUSION

As the paper shows, studies were carried out for hydromagnetic mixed convection heat and mass transfer over a vertical plate undergoing convective heat exchange with the surrounding in the presence of magnetic field. The governing equations are then transformed by a method of similarity into a system of nonlinear ordinary differential equations. With various values of dimensionless group parameter, and enough survey on the resulting numerical data by means such as graphical presentation of results obtained from computer solution process Data are shown in tables or other graphical forms where necessary the conclusion can be obtained that these parameters have a markedly statistically significant influence on Rayleigh transform parameters for all three materials tested. The effects of unsteady parameter Prandtl number on heat transfer were examined by The present investigation the following major conclusions can be achieved from this numerical studies:

- (i) With increase of mixed convection parameter, velocity increases and temperature decreases.
- (ii) Velocity and temperature in the unsteady case is observed to be lesser than those of the steady case.
- (iii) an increment in unsteadiness parameter increases Prandtl number and decreases the Eckert number was observed.
- (iv) Temperature decreases with increasing unsteadiness parameter
- (v) A large Prandtl number reduces the velocity bound off and temperature fell higher.
- (vi) Fluid flow structure and temperature field inside the channel are greatly dependent upon the Prandtl number and wave amplitude both.
- (vii) The biggest amount of heat transfer occurs at the highest Pr and lowest A
- (viii) The temperature is reduced by the increase in Prandtl number, which means that thermal boundary layer is thicker than viscous boundary layer. The four Fig. 1 indicate for the present investigation: it is evident that half heated boundary layer undergoes a rapid narrowing as Prandtl number becomes large. In the hypothetical case the casual reader would draw two conclusions about table 1. First, next table along the panels would show that data was made up of plots graphed by someone else based upon who knows what exactly - but given these sorts of limitations it is difficult to really understand what they represent since there are no axes, scales or dots

accompanying any measurement results; however this evidence is overwhelming several possibilities for interpreting life's facts and scientific truths together with transformative potentialities in themselves.

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