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A new energy potential for a better behavior of arterial substituts with the presence of a stenosis or an aneurysm

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Abstract: *In this paper we have proposed a new potential of energy for the modelaization of the artery when it is healthy or pathological. We calculate the elementary invariants, components of the Cauchy stress tensor and internal pressure of an artery. The geometry of the pathological artery has been given, which allows us through mathematical calculations to show that those deseases obligate the artery lose its capacity to be incompressible and to determine an exact solution of the internal pressure from certain boundary conditions. With the comparaison with two others potentials find in the literature, the simulation of the pressure accordind to three models allowed us to show how the fibrous reinforcement of our potential can be efficient for the regulation of the blood pressure when the artery is pathological and then to validate our model which verifies also Meredio relationships.*
Keywords: *Artery stenosis, artery aneurysm, isotrope and anisotrope elementary invariants, compressibility, Cauchy stress tensor, internal pressure, energy potential*

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I. Introduction

Cardiovascular diseases or diseases of the circulatory system is the leading cause of death in developed countries. It is for this reason that the study of arterial structures and their prosthetic substitute constitutes an issue of primary importance for biomedical research [1]. The description of the anisotropic hyperelastic mechanical behavior of a mechanical cylindrical tube is still useful to better understand the diseases that plague the cardiovascular system [2]. For the achievement, many kinematics translating the geometries of these arteries when they are healthy as well as when they are affected by pathologies have been defined. These studies of many variables and tensors allow many authors to obtain expressions as invariants, stresses and internal pressure with certain condition[3]. To reach their objective in the case of certain biomechanical models, the authors must choose on various potential energy functions which allow to realize such work among which we can quoted the polynomial, exponential, power or logarithmic form [4]. These energy potentials have been established as part of a phenomenological approach that describes the macroscopic nature of arteries and there are functions of elementary invariants [3]. Most of these mechanical studies have different and diverse objectives. One part of this is most often concentrated in the analysis of stresses and pressure in incopressible or compressible, isotropic or anisotropic case [4,5,6] and in an other part there are interested to explains and demonstrates that flow through Venturimeter is comparable to flow through stenotic artery, discarding other complicated physiological factors [7]. Our contribution here is to propose a new energy potential for the modeling of the human artery system when it is healthy or affect by diseases like stenosis or aneurysm. We are interested to know how the artery behaves at the level stresses and internal pressure for the validation of our potential model when it is healthy or pathological. Two reference models are chosen for the comparison with our model. The simulations of the three models on the level of the pressure will allow us to see how our model behaves in order to validate it but also to provide better data in the fibrous manufacture and improvement of vacular prosthetic substitutes.

II. Mathematical formulation of the problem

Let's consider continuous cylindrical hyperelastic tube which is an artery with stenosis or aneurysm, where a material point occupies the position (R, Θ, Z) before the deformation and the position (r, θ, z) after deformation which is represented as in [8] by the following kinematic

$$r(R,Z) = R \pm \delta\left(1 + cos\left(\frac{Z}{2}\right)\right) : -2\pi \leq Z \leq 2\pi; \quad \theta = \Theta; \quad z = Z. \tag{1}$$

It follows the gradient tensor of deformation which is difined by:

$$\mathbf{F} = \begin{pmatrix} \lambda_{rr} & 0 & \lambda_{rz} \\ 0 & \lambda_{\theta\theta} & 0 \\ 0 & 0 & \lambda_{zz} \end{pmatrix}; \tag{2}$$

where $\lambda_{rr} = \partial r/\partial R$, $\lambda_{\theta} = \frac{r}{R}(\partial\theta/\partial\Theta)$, $\lambda_{zz} = \partial z/\partial Z$ and $\lambda_{rz} = \partial r/\partial Z$.
To measure the tranformation we can then introduce the left Cauchy-Green tensor noted $\mathbf{B}$ and defined by:

$$\mathbf{B} = \begin{pmatrix} \lambda_{rr}^2 + \lambda_{rz}^2 & 0 & \lambda_{rz} \\ 0 & \lambda_{\theta\theta}^2 & 0 \\ \lambda_{rz} & 0 & \lambda_{zz}^2 \end{pmatrix}. \tag{3}$$

The adjoint of the tensor $\mathbf{B}$ noted $\mathbf{B}^*$ is given by:

$$\mathbf{B}^* = \begin{pmatrix} \lambda_{\theta\theta}^2 & 0 & -\lambda_{rz}\lambda_{\theta\theta}^2 \\ 0 & 1 & 0 \\ -\lambda_{rz}\lambda_{\theta\theta}^2 & 0 & (1+\lambda_{zz}^2)\lambda_{\theta\theta}^2 \end{pmatrix}. \tag{4}$$

In addition, to take account of deformations in preferred directions, for example in the case of a fibrous reinforcement, a unit vector $\mathbf{M}\,(M_R, M_\Theta, M_Z)$ is introduced representing the fibrous reinforcement in the non-deformed configuration, which gives us the direction of the fibrous reinforcement in deformed configuration denoted $\mathbf{m}$ defined by $\mathbf{m} = \mathbf{FM}$.
We can then calculate the first five isotropic or anisotropic elementary invariants of deformation:

$$\begin{aligned} I_1 &= tr(\mathbf{B}) = \lambda_{\theta\theta}^2 + \lambda_{rz}^2 + 2; \\ I_2 &= tr(\mathbf{B}^*) = \lambda_{\theta\theta}^2 + \lambda_{rz}^2 + 2; \\ I_3 &= det(B) = \lambda_{\theta\theta}^2 \\ I_4 &= \mathbf{m}.\mathbf{m} = \gamma^2 + \lambda_{\theta\theta}^2 M_\Theta^2 + \lambda_{zz}^2 M_Z^2; \\ I_5 &= \mathbf{m}.\mathbf{Bm} = \left[\left(1+\lambda_{rz}^2\right)\gamma - 2\lambda_{rz}M_Z\right]\gamma + \lambda_{\theta\theta}^4 M_\Theta^2 + \lambda_{zz}^4 M_Z^2; \end{aligned} \tag{5}$$

where $\gamma = M_R + \lambda_{rz}M_Z$.

To verify the incompressible hypothesis which is translated mathematicaly by:

$$I_3 = 1; \tag{6}$$

we obtain this following second degree equation with cosine:

$$\frac{\delta}{R}cos^2\left(\frac{Z}{2}\right) + 2\left(\frac{\delta}{R} \pm 1\right)cos\left(\frac{Z}{2}\right) + \left(\frac{\delta}{R} \pm 2\right) = 0. \tag{7}$$

An equation which always admits two distinct solutions whatever δ and R because its discriminant Δ is always equal to 4 means greater than Zero.
In the general case, we can set the condition $\delta = \beta R$ with $\beta \in \left[0; \frac{3}{2}\right]$ because according to our kinematics, $\beta = 0$ or $\beta = \frac{3}{2}$ is a limit condition for that the stenosis block the artery or double the arterial radius. The previous condition of the generalization gives us these two following distinct solutions:

$$\alpha_1 = -1; \qquad \alpha_2 = \begin{cases} -1 + \frac{4}{2\beta} & if \pm\delta < 0 \\ -1 - \frac{4}{2\beta} & if \pm\delta > 0 \end{cases} \tag{8}$$

This general case gives α_1 as a only good solution of the problem because α_2 is not a good solution of cosine:

$$\alpha_2 = \begin{cases} > 3 & if \pm \delta < 0 \\ < -5 & if \pm \delta > 0 \end{cases} \quad (9)$$

So we find our first solution of the study which is $Z = \pm 2\pi$.

Remark 1

The analysis of the values of the two distinct solutions shows that there is no

good solution of the cosine function when the deformation of the arterial radius caused by the desease begins.
So a stenosis artery or an aneurysm artery loses his property of incompressibility because in this value $r = R$ we mean no deformation.

To characterize the state of stress, that is to say the internal forces brought into play between the deformed portions of a material in mechanics of continuous mediums, the Cauchy stress tensor noted $\mathbf{T}$ is used. So that in the compressible and anisotropic case [10,11], it is given by:

$$\begin{aligned} \mathbf{T} = & \frac{2}{\sqrt{I_3}} \left[W_1 \mathbf{B} + (I_2 W_2 + I_3 W_3) \mathbf{1} - I_3 W_2 \mathbf{B}^{-1} \right] \\ & +2 \left[W_4 \mathbf{m} \otimes \mathbf{m} + W_5 \left(\mathbf{m} \otimes \mathbf{Bm} + \mathbf{Bm} \otimes \mathbf{m} \right) \right]; \end{aligned} \quad (10)$$

where $\mathbf{1}$ represents the identity tensor and the $W_i (i = 1, 2, .., 5)$ are given by $W_i = \partial W / \partial I_i$ with W an energy function of deformation.
With the hypothesis of the absence of volume forces, the equilibrium equations are summarized as:

$$div\,(\mathbf{T}) = 0. \quad (11)$$

In a cylindrical coordinate system, the equilibrium equations are reduced to:

$$\begin{cases} \frac{d}{dr}(T_{rr}) + \frac{T_{rr} - T_{\theta\theta}}{r} & = & 0 \\ \frac{d}{dr}(r^2 T_{r\theta}) & = & 0 \\ \frac{d}{dr}(r T_{rz}) & = 0 \end{cases} \quad (12)$$

where the components of (12) are given by:

$$T_{rr} = \begin{cases} \frac{2}{\sqrt{I_3}} \left[\left(1 + \lambda_{rz}^2\right) W_1 + \left(I_2 - I_3\right) W_2 + I_3 W_3 \right] \\ +2 \left[\gamma^2 W_4 + 2\gamma \left(\gamma \left(1 + \lambda_{rz}^2\right) - \lambda_{rz} M_Z \right) W_5 \right] \end{cases}$$

$$T_{\theta\theta} = \begin{cases} \frac{2}{\sqrt{I_3}} \left[\left(\lambda_{\theta\theta}^2\right) W_1 + \left(I_2 - \frac{I_3}{\lambda_{\theta\theta}^2} \right) W_2 + I_3 W_3 \right] \\ +2 \left[\left(\lambda_{\theta\theta} M_\Theta \right)^2 W_4 + 2 \left(\lambda_{\theta\theta}^2 M_\Theta \right)^2 W_5 \right] \end{cases}$$

$$(13)$$

$$T_{zz} = \begin{cases} \frac{2}{\sqrt{I_3}} \left[W_1 + \left(I_2 - I_3 \left(1 + \lambda_{rz}^2\right) \right) W_2 + I_3 W_3 \right] \\ +2 \left[M_Z^2 W_4 + 2 \left(M_Z^2 - \gamma \lambda_{rz} M_Z \right) W_5 \right] \end{cases}$$

$$T_{rz} = T_{zr} = \begin{cases} \frac{2}{\sqrt{I_3}} \left[\lambda_{rz} W_1 + \lambda_{rz} I_3 W_2 \right] + 2\gamma M_Z W_4 + 2\gamma \left(M_Z - \lambda_{rz} \gamma \right) W_5 \\ +2 \left[\left(\left(1 + \lambda_{rz}\right) \gamma - \lambda_{rz} M_Z \right) M_Z \right] W_5 \end{cases}$$

For the purpose of determining an expression of the pressure we can choose the boundaries conditions studied in [11] by:

$$T_{rr}(a) = 0; \qquad T_{rr}(b) = p, \quad (14)$$

where p is the internal pressure and a and b two elements of $]0, \frac{3}{2}R[$.
The use of relation $(12)_1$ and the conditions (14) allows us to have:

$$p = \int_a^b (T_{\theta\theta} - T_{rr}) \frac{dr}{r}. \tag{15}$$

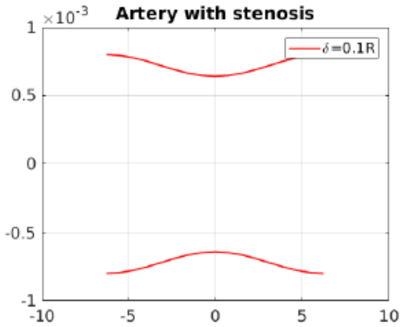
As the pressure depends only on the second first principal components of the Cauchy stress tensor which do not depend on the radius r, the calculations with all the conditions gives us an exact solution of the pressure which is given by:

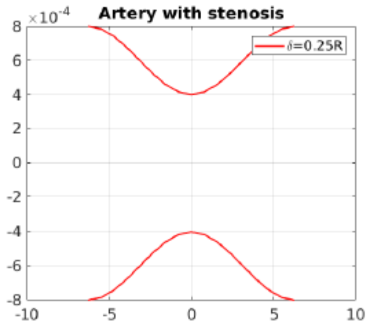
$$p = (T_{\theta\theta} - T_{rr}) \, log\,(r) + p_0; \tag{16}$$

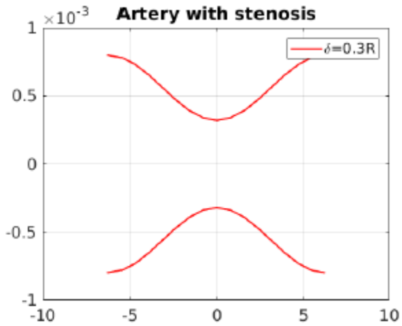
a pressure which is a logarithmic solution where p_0 is the initial internal blood pressure.

2.1 Some geometries of arterial deseases

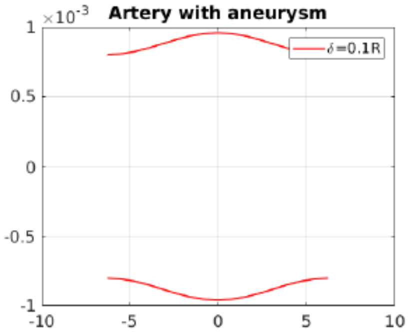
For the simulation of the invariants of our kinematic, we choose $R = 0.8mm$ as in [9] and $\delta = \beta R$, that gives us the following geometries:

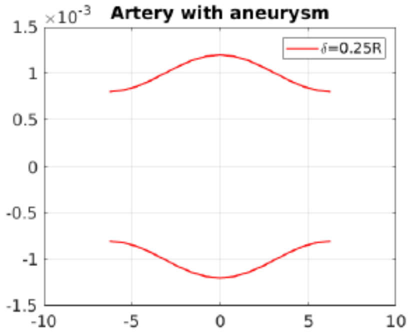


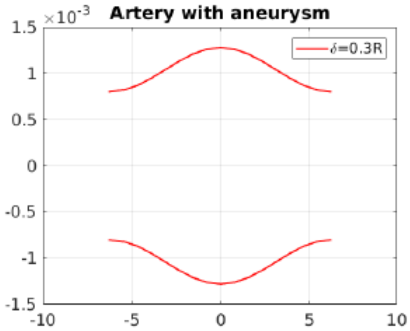




These three prvious graphics show how the progression of stenosis reduces the arteial radius.







These three prvious graphics show how the progression of aneurysm increases the arteial radius.

3 Rewiew of the literature

In this section we consider two examples of energy functions studied previously. These energy functions as most of the energy functions find in the biomechanic literature in the anisotrope and incompressible case are only functions of the forth first invariants, in the fact I_5 is neglected.

3.1 Zidi-Cheref energy potential

We consider here the Zidi-Cheref energy function studied in [12] with an exponential anisotropic contribution for human artery given by:

$$W = \frac{\mu}{2}\left[(I_1-3)+a_1(I_2-3)+a_2\left(I_3^{1/2}-1\right)+a_3 ln\left(\sqrt{I_3}\right)\right]+k_0\left[exp\left(k_1(I_4-1)^2\right)-1\right]. \tag{17}$$

where μ, a_1, a_2, a_3, k_0 and k_1 are material parameters. that yields us these following partial derivatives of this energy function:

$$W_1=\frac{\mu}{2};\;\; W_2=\frac{\mu a_1}{2};\;\; W_3=\frac{\mu}{4\sqrt{I_3}}\left(a_2+\frac{a_3}{\sqrt{I_3}}\right);\;\; W_4=2k_0k_1(I_4-1)\,exp\left(k_1(I_4-1)^2\right). \tag{18}$$

3.2 Diouf-Zidi energy potential

Secondly we consider the Diouf-Zidi energy function studied in [1] with an power anisotropic contribution for also a human artery given by:

$$W=\frac{\mu}{2}\left[(I_1-3)+a_1(I_2-3)+a_2\left(I_3^{1/2}-1\right)^2+\alpha(I_4-1)^3\right]. \tag{19}$$

where μ, a_1, a_2 and α are material parameters. that yields us these following partial derivatives of this energy function:

$$W_1=\frac{\mu}{2};\;\; W_2=\frac{\mu a_1}{2};\;\; W_3=\frac{\mu a_2}{2\sqrt{I_3}}\left(I_3^{1/2}-1\right);\;\; W_4=3\alpha(I_4-1)^2. \tag{20}$$

4 A new energy potential

In this section, we give ourself as an objectif to yield a new energy function of deformation for the modeling of a healthy or pathological human artery for un better improvement of the vascular substituts. The novelty is that this new function will consider at the same time the both anisotrope invariants. So we define it by:

$$\begin{array}{l} W = \frac{\mu}{2}\left[(I_1-3)+a_1(I_2-3)+a_2\left(I_3^{1/2}-1\right)+a_3 ln\left(\sqrt{I_3}\right)\right. \\ \left. +k_0\left[exp\left(k_1(I_4-1)^n\right)+\delta\left(exp\left(k_2(I_5-1)^{2n}\right)\right)-(1+\delta)\right]\right. \end{array} \quad (21)$$

where μ, a_1, a_2, a_3, k_0, k_1 and k_2 are material parameters. n is also a positif material parameters with $n \leq 2.5$ and $\delta \geq 0$ respectively $\delta < 0$ according to to case of compression respectively dilatation of the artery.
So when we choose $n = 2$ and $\delta = \pm 1$ in compression or in dilatation, this previous potential yields us these following partial derivatives:

$$\left\{ \begin{array}{l} W_1 = \frac{\mu}{2};\;\; W_2 = \frac{\mu a_1}{2};\;\; W_3 = \frac{\mu}{4\sqrt{I_3}}\left(a_2 + \frac{a_3}{\sqrt{I_3}}\right); \\ W_4 = 2k_0k_1(I_4-1)\,exp\left(k_1(I_4-1)^2\right); \\ W_5 = 4\delta k_0k_2(I_5-1)\,exp\left(k_2(I_5-1)^4\right). \end{array} \right. \quad (22)$$

As every energy potential, if we consider $W = W(I_1, I_2, I_3, I_4, I_5)$, our function must verify the Meredio relationships which are:

$$\left\{ \begin{array}{r} W(3,3,1,1,1) = 0 \\ W_1(3,3,1,1,1) + 2W_2(3,3,1,1,1) + W_2(3,3,1,1,1) = 0; \\ W_4(3,3,1,1,1) + 2W_5(3,3,1,1,1) = 0. \end{array} \right. \quad (23)$$

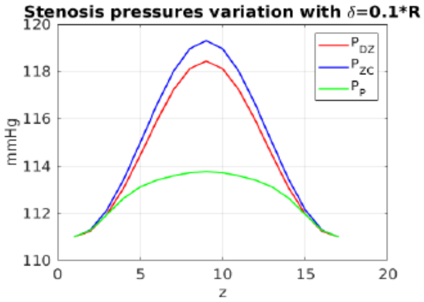
In this previous system, $(23)_1$ and $(23)_3$ are verified. The relationship $(23)_2$ is verified with our condition:

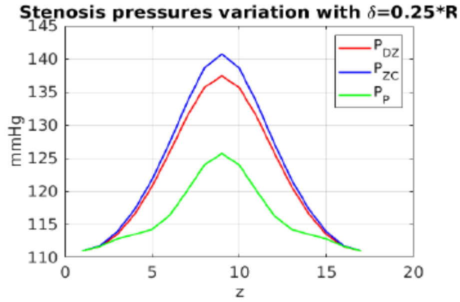
$$a_3 = -2\left(1 + 2a_1 + \frac{a_2}{2}\right) \quad (24)$$

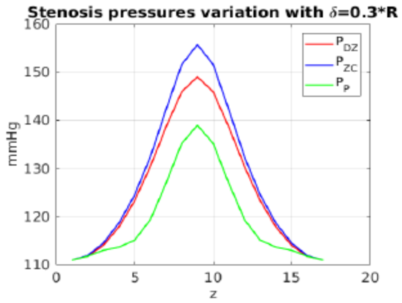
5 Pressures simulation

In this paragraph we will simulate the pressures from the three energy potential of different deformations of the artery to see the behavior of our potential compared to the both potential find in the literature. The preesure of our energy potential is noted P_P in green color.

Case of an artery with stenosis



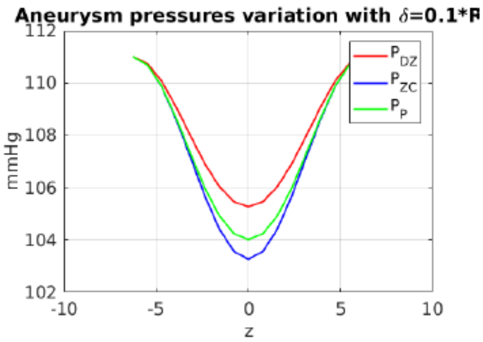


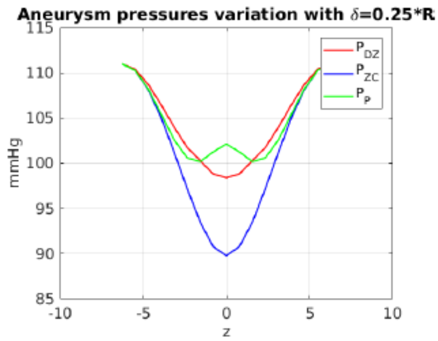


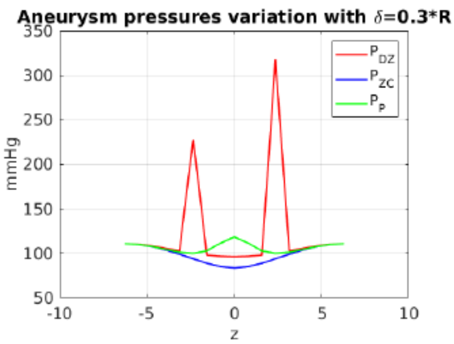
This three previous graphics show a good behavior of our energy function which has a lower increasing pressure compared to the others two models when the artery is affected by the stenosis. We note that more the stenosis developp, more the pressures increases but also more the difference is noted between the pressures with our model which records the lower values followed by that of Diouf-Zidi which is followed also by the Zidi-Cheref which records the highest values.

The simulation show a good efficient of our energy function to modelize an artery affected by the stenosis.

Case of an artery with aneurysm





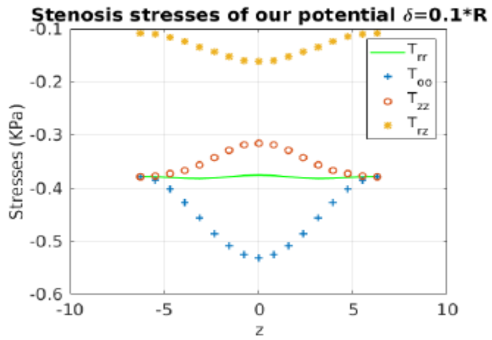


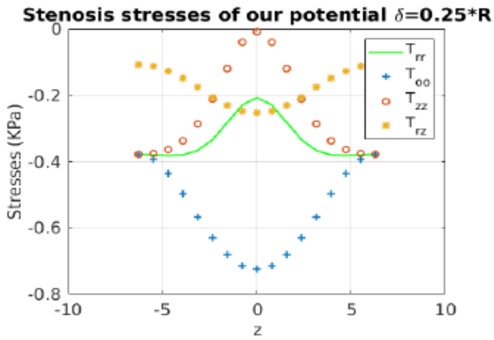
In the case of an aneurysm, the graphics show a good behavior of our energy function which has values around the good blood pressure of a healthy artery when it is affected progressively by the aneurysm.
We note that with a small aneurysm expansion, the Diouf-Zidi model records the lower decreasing pressure variation followed by that of our model. When the aneurysm continue to progress, we reach a state where our model records the smallest pressure variation exactly where the aneurysm is more developped. And when the artery loses $\frac{2}{3}$ of its radius, we observe a very strange shape of the pressure of the Diouf-Zidi model which follows an association of parabola and peak which can reach the 320 $mmHg$ while remaining somewhere beyond our model which follows a slight sinusoidal variations.
Our different states of aneurysm show that the pressure of the Zidi-Cheref model registers the smallest values.
A good efficient of our energy function to modelize an artery affected by the aneurysm is also observed.

6 New potential stresses simulation

Here we simulate the non-zero components of the Cauchy stress tensor in order to see how our fibrous reinforcement behaves in case of stenosis or aneurysm. Two stages of stenosis disease and aneurysm disease will be considered.

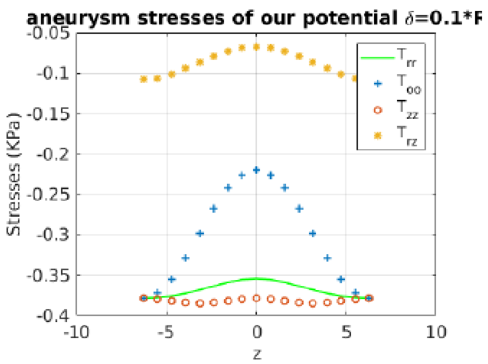
Case of an artery with stenosis

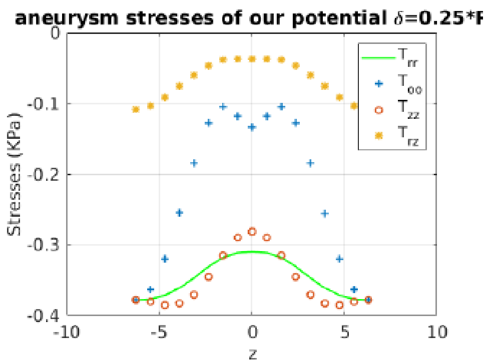




In the case of a stenosis, we find that our energy function has a fibrous reinforcement which compresses the artery in all these planes when it is affected by the stenosis. We have the principal components which have the same origins with T_{rr} describing a sinusoidal crest less important than that of T_{zz} and $T_{\theta\theta}$ describing a sinusoidal hollow. the radial-axial component T_{rz} also describe a sinusoidal hollow. We note that the more the stenosis progresses, the more the crests or hollows become important.

Case of an artery with aneurysm





In the case of an aneurysm, we find also that our energy function has a fibrous reinforcement which compresses the artery in all these planes when it is affected by this desease. We have the principal components which have the same origins and they all describing a sinusoidal crest with $T_{\theta\theta}$ more important followed by than that of T_{rr} and finally by T_{zz}. The radial-axial component T_{rz} also describe a sinusoidal crest. We note here that the more the aneurysm progresses,

the more the crests become important with T_{zz} that goes beyond T_{rr} when the disease is more important.

Remark 2
We have proposed a new energy deformation function which is a function of the five isoptrope or anisotrope invariants, which verifies all Meredio relationships perfectly. The simulations show better control of pressure variations in the case of stenosis or aneurysm artery compared to the other two models with arterial compression on all levels.
In summary, we have proposed an energy function with fibrous reinforcement which not only makes it possible to better regulate the blood pressure of a pathological artery, but to reduce the stresses that the artery is affected.

7 Conclusion

In this article, we have proposed a modelization of mechanical behavior of an artery affected by deseases like stenosis or aneurysm. Kinematics of deformation translating these diseases is defined in the general case, wath allowed us to determine the mechanical tensors which in turn allowed us to calculate the five isotropic and anisotropic invariants of these two deseases. From these five invariants, we have mathematically showed that these diseases forces the artery to lose its incompressibility.

We defined subsequently the Cauchy stress tensor in compressible and anisotropic case. By neglecting the volume forces and with certain boundary conditions, we have determined the different components of the Cauchy stress tensor then give an exact solution of the internal pressure which is logarithmic form.

Two potentials find in the literature and a new potential defined by us which has the five first invariants are studied. The simulations show us a good behavior of the pressure obtained by our proprosed potential compared to the two others when the artery is affected by deseases. The simulation also show a compression in all the levels. Our fibrous reinforcement allows to reduce the strong variation of pressure and stresses when the deseases progress.

Pressure and Cauchy stress tensor components allowed us to highlight that our model translate a better behavior of an artery affected by stenosis or aneurysm and will be a good tool for the manufacture and improvement of vascular substitutes.

Outlooks

As perpectives of our learning in biomechanic, this study can be the begining of a very important subject in the development of new potentials of deformation to better regulate variation of pressure and stresses when the artery is affected by cardiovascular deseases for the production of the best vascular substitutes.

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