American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-12, Issue-2, pp-146-160
www.ajer.org
Research Paper Open Access

Heat Transfer Analysis of Magnetohydrodynamics fluid flow past an Infinite vertical porous plate in the presence of Suction: The Adomian Decomposition Approach

Liberty Ebiwareme
Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.

Kubugha Wilcox Bunonyo
Department of Mathematics and Statistics, Federal University Otuoke, Nigeria.

Onengiyeofori Anthony Davies
Department of Physics, Rivers State University, Port Harcourt, Nigeria.

ABSTRACT: *This paper presents a semi-analytical study of the heat transfer analysis of an MHD fluid flowing past infinite vertical porous plate under the influence of variable suction and thermal radiation. The governing nonlinear equations of momentum and energy of the physical problem along with the imposed boundary conditions are nondimensionalized using nondimensional variables. Employing Adomian decomposition method (ADM), the velocity and temperature profiles are solved numerically. The proposed technique requires decomposing a given differential operator into linear and nonlinear operators, wherein the linear part is written as a decomposition series and the nonlinear part as Adomian polynomials. Successive approximates of the profiles are obtained using iterative algorithms which gives the solution as a converging series. Effects of magnetic field, suction, permeability, Eckert number, Grashof, Prandtl, radiation and porosity parameters on the temperature and velocity profiles are presented graphically and discussed. The result obtained showed the pertinent flow parameters have significant influence on the different distributions and validate existing results when compared with literature.*

KEYWORDS. *Adomian Decomposition method (ADM), Similarity Transformation, Velocity, Temperature profiles, Variable Suction, Porous plate, MHD, Permeability, Porosity*

Date of Submission: 13-02-2023 Date of acceptance: 26-02-2023

I. INTRODUCTION

In Science, Engineering and other technological endeavours, the problem of steady and unsteady natural convective flow of a viscous incompressible fluid in a vertical porous plate under the influence of variable suction and imposed magnetic field has been extensively studied. In practice, there are three modes of heat transfer namely, conduction, convection and radiation which is dependent on the medium of transfer. Similarly, convection can be classified into free and forced or natural convections. Natural or forced convection results when a buoyancy force occurs owing to temperature and density difference on the fluid thus making a hot and less fluid at the bottom of a fluid to rise and replace a cooler and more dense fluid, creating a pattern of convection current due to gravity. On the otherhand, when an external force such as fan or pump cause the flow of fluid, then the mode of heat transfer is called forced convection. Some of the numerous applications of convection heat transfer can be found in oceanic circulation, air cooling, steam turbines, propeller heating in aerodynamics, convection ovens, thawing of frozen materials [1].

Falodun and Fadugba (2017) considered the effects of heat transfer on an unsteady magnetohydrodynamics boundary layer flow of an incompressible fluid moving in a vertical plate. In this study, it was assumed the fluid is optically thick and yield to Roseland approximation. The governing physical equations of continuity, momentum and energy equations were non-dimensionalized using non-dimensional variables and solved using spectral relaxation method (SRM). The impact of the control parameters such as Grashof, Prandtl, radiation and magnetic field parameters on the velocity, temperature and as well as thermal boundary layer of the fluid are analysed graphically. The effect of Prandtl number is to decrease the velocity and temperature profiles, Grashof number increases the velocity profile, velocity and temperature profiles are increased by increase in thermal radiation while magnetic parameter decreases the velocity profile owing to the presence of transverse magnetic field.

Fagbade et al. (2016) have investigated the unsteady heat and mass transfer of a chemically reacting fluid past semi-infinite vertical porous plate under the influence of viscous dissipation, thermal radiation, Soret and Dufour effects. The fluid is modelled using Roseland approximation to account for the radiative heat loss and assumed to be optically thin. Using non-dimensional variables, the governing equations were reduced to nonlinear systems of coupled PDEs and solved using successive relaxation method (SRM). The study revealed, velocity and temperature profiles increased with increase in Eckert number. While increased thermal radiation reduced the temperature distribution in the fluid especially when its cooled

Hazarika and Jadav (2014) have analysed the effects of variable viscosity and thermal conductivity on a magnetohydrodynamics (MHD) free convection flow along a porous vertical plate immersed in a porous medium plate with viscous dissipation. Using similarity transformation, the governing boundary layer equations are transformed into ordinary differential equations and solved using Runge-Kutta shooting method. Parametric study of the effects of variable viscosity, variable thermal conductivity, Magnetic parameter, and Eckert number are analysed in table. The finding of this study revealed, viscosity parameter, thermal conductivity, and magnetic parameter decreased the velocity but increased the Eckert number. Also, the effect of viscosity parameter, magnetic field, and Eckert number is to increase the temperature, whereas thermal conductivity decreased the temperature of the system. Similarly, the concentration profiles are retarded in the presence of viscosity and thermal conductivity parameters while it is enhanced in the presence of Eckert and magnetic parameters.

The numerical study of the Influence of Soret on the unsteady MHD Kuvshinshiki fluid flow with heat and mass transfer past a vertical porous plate with variable suction has been carried out by Idowu et al. (2014). The study neglected induced magnetic field but incorporates the viscous-elastic parameter, Soret terms and permeability of the medium. The reduced non-dimensionalized governing equations are solved using combined implicit finite difference and Crank-Nicolson methods. The influence of various parameters on the velocity, temperature, coefficient of Skin-friction, Nusselt and Sherwood numbers are analysed and presented graphically.

Jana et al (2012) investigated the effects of radiation parameter on unsteady MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillatory vertical porous plate embedded in a porous medium with oscillatory heat flux under the influence of uniform transverse magnetic field. The reduced equations governing the physical flow are solved analytically. It was observed from the parametric study that, the presence of magnetic field significantly impacts the velocity. The fluid velocity near the plate decreases when the thermal radiation parameter increased and increase when the suction increases. The suction, radiation and Prandtl parameters cause the fluid velocity to increase near the plate and decrease away from it.

Mangathai et al. (2015) conducted an analytical study of the heat and mass transfer effects on a MHD free convection flow over an inclined plate embedded in a porous medium under the influence of thermal radiation and chemical reaction. The fluid is assumed to be viscous, incompressible, and electrically conducting. The dimensionless equations of mass conservation, momentum and energy are solved analytically with effects of physical parameters on velocity, temperature, and concentration fields as well as expressions for Skin friction, Nusselt numbers and Sherwood numbers are displayed graphically.

Using finite difference method analysis, Kishan et al. (2012) have examined the unsteady mixed convection flow past a semi-infinite vertical permeable moving plate with heat and mass transfer with radiation and viscous dissipation. The implicit finite- difference scheme and Crank-Nicolson methods which are unconditionally stable is employed to solve the dimensionless equations with graphically representations depicting the effects of model parameters.

Mohammed and Bhaskar (2013) studied the heat and mass transfer of natural convection over a moving vertical plate with internal heat generation and convective boundary conditions under the influence of chemical reaction, viscous dissipation and thermal radiation using similarity transformation method. The study assumed the hot fluid is in contact with the left surface whilst the cold fluid on the left contains a heat source which decayed exponentially. The governing equation are transformed using self-similar variable into systems of ordinary differential equations which are solved numerically using fourth order Runge-Kutta iteration technique. Variations of velocity, temperature, and concentrations profiles with Biot, Prandtl, buoyancy forces, internal heat generation, thermal radiation, Eckert, chemical reaction, and viscous dissipation numbers are presented graphically.

Reddy et al. (2014) used the combined shooting iteration technique and fourth order Runge-Kutta integration scheme to investigate the radiation effects on unsteady MHD free convective heat and mass transfer flow past a vertical porous plate embedded in a porous medium with viscous dissipation. The study assumed that the vertical porous plate is immersed in a porous medium with time dependent suction in the presence of viscous dissipation and induced magnetic field. The velocity, temperature and concentration profiles are presented graphically for different values of the physical parameters. Also, skin-friction coefficients, Nusselt number and Sherwood numbers are discussed for influence of various model parameters.

Sharma et al. (2014) conducted a numerical study of viscous dissipation and mass transfer effects on unsteady MHD free convective flow along a moving vertical porous plate in the presence of internal heat generation and variable suction. It was assumed in this study that the fluid is viscous, incompressible, electrically conducting and the plate is non-isothermal and conducting. The effect of the model parameters on the different fields are presented in tables and graphs.

The Adomian decomposition method (ADM) originally developed by George Adomian [11-13]isa semi-analytical technique that can be employed to solve wide class of problems whose mathematical models involves algebraic, differential, delay differential, integral, integro-differential equations as well another higher ODEs and PDEs. In this method, a given differential operator is split into linear and nonlinear operators where the linear operator assumed invertible is written as a decomposition series and the nonlinear part as Adomian polynomial. The solution to the problem is obtained as the limiting sum of the approximations obtained from the recursive scheme. Comprehensive account of Adomian decomposition method and its application to practical problems can be found in [14-34]. Subsequent studies have shown that the method is convergent and yield exact solution if its exist and an approximate solution when closed form solution is difficult to obtained. [35-37]. The principal advantage of this method over semi-analytical methods is that it capable of reducing the computational work without affecting the accuracy of the numerical solution.

The motivation of this present paper is to undertake a semi-analytical approach using Adomian decomposition method approach which has not been considered by any of the preceding literatures we have considered. The rest of this study is organized as follows: Chapter one gives an in-depth introduction involving relevant literatures involving variable suction. In chapter two, the physical problem is formulated along with the governing equations of continuity, momentum, and energy. The fundamentals of the Adomian decomposition method are exhaustively discussed in Chapter three. Chapter four presents the application of the solution technique to the reduced equation. The results and discussion of the pertinent parameters and their influence on the velocity and temperature profiles are depicted graphically and discussed in Chapter five, while the conclusion is drawn in Chapter six.

II. FORMULATION OF THE PROBLEM

A two-dimensional steady, viscous, laminar, and incompressible fluid with heat source flowing past an infinite porous plate under the influence of thermal diffusion and thermal radiation is presented. The x^* is taken in the vertically upward direction along the plate and y^* is taken normal to the plane. The fluid and plate are initially at the same temperature when $t = 0$, while at $t > 0$, the temperature of the plate is raised to T_w. Let u^* and v^* be the components of the velocity along the x^* and y^* directions respectively. Since the plate is large, the radiative heat fluid is neglected, whereas that along y^* is considered.

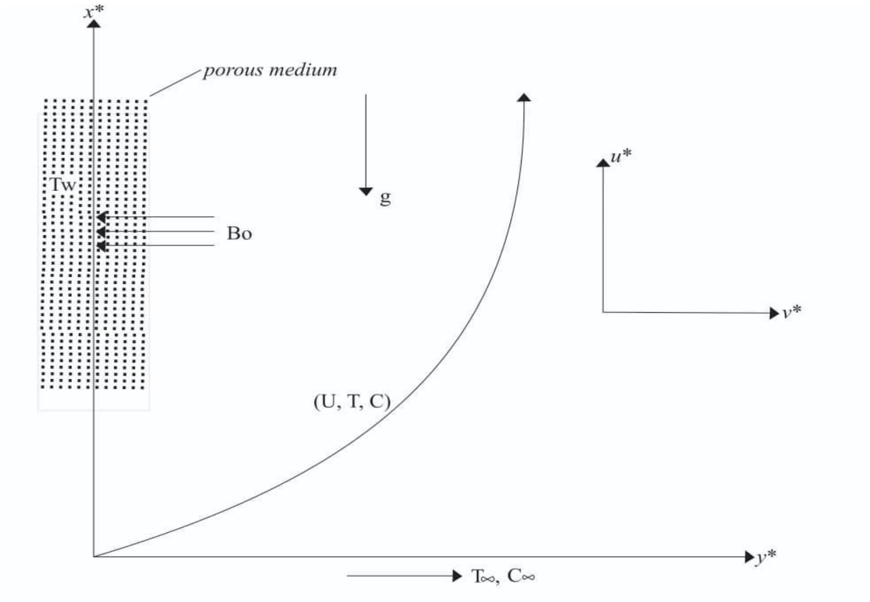

Figure 1. Physical Configuration of the system

III. ASSUMPTIONS OF THE STUDY

The mathematical equations describing the physical model are based upon the following assumptions

- The size of the porous plate is large
- The fluid is electrically conducting
- The induced magnetic field is negligible
- Applied magnetics Reynold number is very small
- There is no applied voltage
- The radiative heat fluid along the normal direction is negligible
- Boussinesq approximation is valid

IV. GOVERNING EQUATIONS

Following [38], the governing equations of the MHD fluid through an infinite vertical porous plate with suction is given by

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{v}{1+\lambda_1}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\bar{\beta}(\bar{T} - T_\infty) - \frac{\sigma B_0^2 \bar{u}}{\rho} - v\bar{u}\bar{k} \quad (1)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) + \frac{1}{\rho c_p}\frac{\partial q_r}{\partial \bar{y}} + \frac{Q_0}{\rho c_p}(\bar{T} - T_\infty) \quad (2)$$

The boundary conditions for the temperature and velocity fields as

$$\bar{t} = 0, \bar{u} = 0, \bar{T} \to \bar{T}_\infty \text{ for all } \bar{y}$$
$$\bar{t} > 0, \bar{u} = 0, \bar{T} \to \bar{T}_w \text{ as } \bar{y} = 0 \quad (3)$$
$$\bar{u} = 0, \bar{T} \to \bar{T}_\infty \text{ as } \bar{y} \to \infty$$

Upon Non-dimensionalization, Eqs. (1)- (2) reduced to the form

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - (M + K)u + Gr\theta \quad (4)$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{Pr}(R - S)\theta \quad (5)$$

Subject to the corresponding boundary condition

$$u = 0, \theta = 1, \text{ for } y = 0$$
$$u = 0, \theta = 0, \text{ as } y \to \infty \quad (6)$$

V. ADOMIAN DECOMPOSITION METHOD (ADM)

Suppose a general nonlinear differential equation in operator form as follows

$$D[y(x)] = f(x) \quad (7)$$

where D is a nonlinear differential operator comprising both the linear and nonlinear terms, while $f(x)$ is adifferentiable function of x

Decomposing the linear term in Eq. (7) into the form $L + R$, where L is the highest order derivative that is invertible and R is the remainder of the linear term.

Rewriting Eq. (7) in operator form, we have

$$L[y(x)] + R[y(x)] + N[y(x)] = f(x)$$

$$L[y(x)] = f(x) - Ry[(x)] - N[y(x)] \tag{8}$$

While $N[(y(x))]$ is a nonlinear term and $f(x)$ is the source term.

Applying the inverse operator L^{-1} of both sides of Eq. (8), we obtain

$$L^{-1}(L[y(x)]) = L^{-1}\big(f(x)\big) - L^{-1}(R[y(x)]) - L^{-1}(N[y(x)])$$

Where $L^{-1}(.) = \int_0^x \int_0^x (.)dxdx$

$$y(x) = \phi_0(x) + g(x) - L^{-1}R([y(x)]) - L^{-1}N([y(x)]) \tag{9}$$

Where g(x) is the term obtained from integrating the source termand ϕ_0 from the given conditions

Now rewriting the solution and nonlinear terms as decomposition series of the form

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \text{and } N\big(y(x)\big) = \sum_{n=0}^{\infty} A_n(x), \tag{10}$$

where the $A_n^{\prime s}$ are the Adomian polynomials obtained using the formula

$$A_k = \frac{1}{k!}\frac{\partial^k}{\partial\lambda^k}[N(\textstyle\sum_{n=0}^{\infty} y_{n\lambda^n})]_{\lambda=0},\ k = 0,1,2 \dots \tag{11}$$

The solution of the problem in Eq. (1) is obtain as limit of the decomposing series

$$y(x) = \lim_{n\to\infty} u_n(x) \tag{12}$$

Similarly, the nonlinear term can be determined by an infinite series of the Adomian polynomials. That is,

$$N(y_0, y_1, y_2, \dots, y_n) = \sum_{n=0}^{\infty} A_n \tag{13}$$

Then the $A_n^{\prime s}$ are obtained from the formula

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\textstyle\sum_{k=0}^{\infty} \lambda^k y_k)]_{\lambda=0},\ n = 0,1,2,3 \tag{14}$$

Using Eq. (14), the first seven Adomian polynomials are given as

$$A_0 = N(y_0)$$

$$A_1 = y_1 N'(y_0)$$

$$A_2 = y_2 N'(y_0) + \frac{1}{2!} y_1^2 N''(y_0)$$

$$A_3 = y_3 N'(y_0) + y_1 y_2 N''(y_0) + \frac{1}{3!} y_1^3 N'''(y_0)$$

$$A_4 = y_4 N'(y_0) + \frac{1}{2} N''(y_0)(2y_1y_3 + y_2^2) + \frac{1}{2} N'''(y_0) y_1^2 y_2 + \frac{1}{4!} N^{(iv)}(y_0) y_1^4$$

$$A_5 = y_5 N'(y_0) + \frac{1}{2} N''(y_0)(2y_1y_4 + 2y_2y_3) + \frac{1}{3!} N'''(y_0)(3y_1^2y_3 + 3y_1y_2^2) + \frac{4}{4!} N^{(iv)}(y_0)(y_1^3y_2) + \frac{1}{5!} N^{(v)}(y_0) y_1^5$$

$$A_6 = y_6 N'(y_0) + \frac{1}{2!} N''(y_0)(2y_1y_5 + 2y_1y_4 + y_3^2) + \frac{1}{3!} N'''(y_0)(3y_1^2y_4 + y_2^3 + 6y_1y_2y_3) + \frac{1}{4!} N^{(iv)}(y_0)(4y_1^3y_3 + 6y_1^2y_2^2) + \frac{5}{5!} N^{(v)}(y_0) y_1^4 y_2 + \frac{1}{6!} N^{(vi)}(y_0) y_1^6$$

$$A_7 = y_7 N'(y_0) + \frac{1}{2!} N''(y_0)(2y_1y_6 + 2y_2y_5 + 2y_3y_4) + \frac{1}{3!} N'''(y_0)(3y_1^2y_5 + 3y_1y_3^2 + 3y_3y_2^2 + 6y_1y_2y_4) + \frac{1}{4!} N^{(iv)}(y_0)(4y_1^3y_4 + 12y_1^2y_2y_3 + 4y_1y_2^3) + \frac{1}{5!} N^{(v)}(y_0)(5y_1^4y_3 + 10y_1^3y_2^2) + \frac{1}{6!} N^{(vi)}(y_0) y_1^5 y_2 + \frac{1}{7!} N^{(vii)}(y_0) y_1^7$$

$$A_8 = y_8N'(y_0) + \frac{1}{2!}N''(y_0)(2y_1y_7 + 2y_2y_6 + 2y_3y_5 + y_4^2) + \frac{1}{3!}N'''(y_0)(3y_1^2y_6 + 3y_4y_2^2 + 3y_2y_3^2 + 6y_1y_2y_5 + 6y_1y_3y_4) + \frac{1}{4!}N^{(iv)}(y_0)(4y_1^3y_5 + 12y_1^2y_2y_4 + 12y_1y_2^2y_3 + 6y_1^2y_3^2 + y_2^4) + \frac{1}{5!}N^{(v)}(y_0)(5y_1^4y_4 + 20y_1^3y_2y_3 + 10y_1^2y_2^3) + \frac{1}{6!}N^{(vi)}(y_0)(y_1^5y_3 + 15y_1^4y_2^2) + \frac{7}{7!}N^{(vii)}(y_0)y_1^6y_2 + \frac{1}{8!}N^{(viii)}(y_0)y_1^8$$

Putting Eq. (10) into Eq. (9), we obtain the solution as decomposition series of the form.

$$\sum_{n=0}^{\infty} y_n(x) = y(x) = \varphi_0(x) + g(x) - L^{-1}R\left(\sum_{n=0}^{\infty} y_n(x)\right) - L^{-1}N\left(\sum_{n=0}^{\infty} A_n(x)\right) \quad (15)$$

Where $y_0(x) = \phi_0(x) + g(x)$ is the zeroth component of $y_n(x)$

The subsequent members of the series are obtained recursively using

$$y_{k+1} = -L^{-1}R\big(y_k(x)\big) - L^{-1}(A_k(x)), \qquad k \geq 0 \quad (16)$$

Then exact solution of the problem is the limit of the recursive relation

$$y(x) = \lim_{n\to\infty} \sum_{k=0}^{n} y_k(x) \quad (17)$$

VI. SOLUTION PROCEDURE VIA ADM

Rearranging Eqs. (4) and (5) subject to the appropriate boundary conditions, we get

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + (M+K)u - Gr\theta \quad (18)$$

$$\frac{\partial^2 \theta}{\partial y^2} = Pr\left[\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} - Ec\left(\frac{\partial u}{\partial y}\right)^2\right] + (R-S)\theta \quad (19)$$

Writing Eqs. (7) and (8) in operator form gives

$$L_1u = L_tu + vL_y + (M+K)u - Gr\theta \quad (20)$$

$$L_2\theta = Pr\big[L_t\theta + vL_y\theta - EcNu\big] + (R-S)\theta \quad (21)$$

Where the differential operators L_1, L_2, L_t and L_y are defined as follows

$$L_1(.) = L_2(.) = \frac{\partial^2}{\partial y^2}, L_t = \frac{\partial}{\partial t}, L_y = \frac{\partial}{\partial y} \quad (22)$$

Assuming the inverse operators, $L_i(i = 1,2), L_t$ and L_y exists and defined by

$$L_1^{-1}(.) = L_2^{-1}(.) = \int_0^y \int_0^y (.)dydy, L_t^{-1}(.) = \int_0^t (.)dt, L_y^{-1}(.) = \int_0^y (.)dy \quad (23)$$

Applying the operators, L_1^{-1} and L_2^{-1} on Eqs. (9) and (10)

$$u(y,t) = u(0,t) + yu'(0,t) + L_1^{-1}\big[L_tu + vL_yu + (M+k)u - Gr\theta\big] \quad (24)$$

$$\theta(y,t) = \theta(0,t) + y\theta'(0,t) + PrL_2^{-1}\big[L_t\theta + vL_y\theta - EcNu\big] + L_2^{-1}(R-S)\theta \quad (25)$$

Where $Nu = \sum_{n=0}^{\infty} A_n$ is called a nonlinear term and A_n are the special Adomian polynomials. Subsequent terms of this polynomials can be obtained using the formula.

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N\left(\sum_{k=0}^{n} \lambda^k u_k\right)\right)\right]_{\lambda=0}, n = 0,1,2,3 \quad (26)$$

By Adomian decomposition, we decompose the independent variables as an infinite series

$$u = \sum_{n=0}^{\infty} u_n(y,t)$$

$$\theta = \sum_{n=0}^{\infty} \theta_n(y,t) \quad (27)$$

Applying Eq. (15) into Eqs. (24) and (25), and exerting the boundary conditions, we have the equivalent form

$$\sum_{n=0}^{\infty} u_n(y,t) = \alpha y + L_1^{-1}\left[\left(L_t + vL_y + (M+k)\right)\sum_{n=0}^{\infty} u_n(y,t) - Gr\sum_{n=0}^{\infty}\theta_n(y,t)\right] \quad (28)$$

$$\sum_{n=0}^{\infty} \theta_n(y,t) = 1 + \beta y + PrL_2^{-1}\left[\left(L_t + vL_y\right)\sum_{n=0}^{\infty}\theta_n(y,t) - EcNu\right] + L_2^{-1}(R-S)\sum_{n=0}^{\infty}\theta_n(y,t) \quad (29)$$

Matching both sides of Eqs. (28) and (29), we obtain

Where $u_0(y,t) = \alpha y$

$$\theta_0(y,t) = 1 + \beta y \quad (30)$$

The recursive algorithms for both equations become

$$u_{k+1}(y,t) = L_1^{-1}\left[\left(L_t + vL_y + (M+k)\right)u_k - Gr\theta_k\right] \tag{31}$$

$$\theta_{k+1}(y,t) = PrL_2^{-1}\left[(L_t + vL_y)\theta_k - EcA_k\right] + L_2^{-1}(R-S)\theta_k \tag{32}$$

$N_{k+1} = L_2^{-1}(\sum_{n=0}^{\infty} A_k)$ for $k \geq 0$

$$u_1(y,t) = L_1^{-1}\left[\left(L_t + vL_y + (M+K)\right)u_0 - Gr\theta_0\right] \tag{33}$$

$$\theta_1(y,t) = PrL_2^{-1}\left[(L_t + vL_y)\theta_0 - EcA_0\right] + L_2^{-1}(R-S)\theta_0 \tag{34}$$

$N_1 = L_2^{-1}(\sum_{n=0}^{\infty} A_0)$, for $k \geq 0$

Evaluating Eqs. (33) and (34) using (30), the result in explicit form become

$$u_1(y,t) = \frac{1}{2}y^2(-\mathrm{Gr} + v\alpha) + \frac{1}{3}y^3\left(\frac{K\alpha}{2} + \frac{M\alpha}{2} - \frac{\mathrm{Gr}\beta}{2}\right) \tag{35}$$

$$\theta_1(y,t) = \frac{1}{2}\mathrm{Pr}y^2(-\mathrm{Ec}\alpha^2 + V\beta) + (R-S)\left(\frac{y^2}{2} + \frac{y^3\beta}{3}\right) \tag{36}$$

Similarly, the second approximates of the profiles gives

$$u_2(y,t) = \frac{1}{2}Vy^2\alpha + y^5\left(\frac{K^2\alpha}{15} + \frac{2KM\alpha}{15} + \frac{M^2\alpha}{15} - \frac{\mathrm{Gr}K\beta}{15} - \frac{\mathrm{Gr}M\beta}{15} - \frac{\mathrm{Gr}R\beta}{15} + \frac{\mathrm{Gr}S\beta}{15}\right) + y^4\left(-\frac{\mathrm{Gr}K}{8} - \frac{\mathrm{Gr}M}{8} - \frac{\mathrm{Gr}R}{8} + \frac{\mathrm{Gr}S}{8} + \frac{KV\alpha}{8} + \frac{MV\alpha}{8} + \frac{1}{8}\mathrm{EcGrPr}\alpha^2 - \frac{1}{8}\mathrm{GrPr}V\beta\right) \tag{37}$$

$$\theta_2(y,t) = y^7\left(-\frac{1}{105}\mathrm{Ec}K^2\mathrm{Pr}\alpha - \frac{2}{105}\mathrm{Ec}KM\mathrm{Pr}\alpha - \frac{1}{105}\mathrm{Ec}M^2\mathrm{Pr}\alpha + \frac{1}{105}\mathrm{EcGr}K\mathrm{Pr}\beta + \frac{1}{105}\mathrm{EcGr}M\mathrm{Pr}\beta + \frac{1}{105}\mathrm{EcGrPr}R\beta - \frac{1}{105}\mathrm{EcGrPr}S\beta\right) + y^6\left(\frac{1}{48}\mathrm{EcGr}K\mathrm{Pr} + \frac{1}{48}\mathrm{EcGr}M\mathrm{Pr} + \frac{1}{48}\mathrm{EcGrPr}R - \frac{1}{48}\mathrm{EcGrPr}S - \frac{1}{48}\mathrm{Ec}K\mathrm{Pr}V\alpha - \frac{1}{48}\mathrm{Ec}M\mathrm{Pr}V\alpha - \frac{1}{8}\mathrm{Ec}^2\mathrm{GrPr}^2\alpha^2 + \frac{1}{48}\mathrm{EcGrPr}^2V\beta\right) + y^5\left(\frac{1}{15}R(R-S)\beta + \frac{1}{15}S(-R+S)\beta + \frac{1}{15}\mathrm{Pr}RV\beta - \frac{1}{15}\mathrm{Pr}SV\beta\right) + y^4\left(\frac{1}{8}R(R-S) + \frac{1}{8}S(-R+S) + \frac{\mathrm{Pr}RV}{8} - \frac{\mathrm{Pr}SV}{8} - \frac{1}{8}\mathrm{EcPr}V\alpha - \frac{1}{8}\mathrm{EcPr}(R-S)\alpha^2 - \frac{1}{8}\mathrm{EcPr}^2V\alpha^2 + \frac{1}{8}\mathrm{Pr}(R-S)V\beta + \frac{1}{8}\mathrm{Pr}^2V^2\beta\right) \tag{38}$$

The three-term approximation is obtained using the partial sum

$$u(y,t) = \sum_{k=0}^{3} u_n(y,t), \text{ and} \theta(y,t) = \sum_{k=0}^{3} \theta_n(y,t) \tag{39}$$

$$u(y,t) = y\alpha + \frac{1}{2}Vy^2\alpha + y^2\left(-\frac{\mathrm{Gr}}{2} + \frac{V\alpha}{2}\right) + y^3\left(\frac{K\alpha}{3} + \frac{M\alpha}{3} - \frac{\mathrm{Gr}\beta}{3}\right) + y^5\left(\frac{K^2\alpha}{15} + \frac{2KM\alpha}{15} + \frac{M^2\alpha}{15} - \frac{\mathrm{Gr}K\beta}{15} - \frac{\mathrm{Gr}M\beta}{15} - \frac{\mathrm{Gr}R\beta}{15} + \frac{\mathrm{Gr}S\beta}{15}\right) + y^4\left(-\frac{\mathrm{Gr}K}{8} - \frac{\mathrm{GrM}}{8} - \frac{\mathrm{GrR}}{8} + \frac{\mathrm{Gr}S}{8} + \frac{KV\alpha}{8} + \frac{MV\alpha}{8} + \frac{1}{8}\mathrm{EcGrPr}\alpha^2 - \frac{1}{8}\mathrm{GrPr}V\beta\right) \tag{40}$$

$$\theta(y,t) = 1 + y\beta + \frac{1}{2}\mathrm{Pr}y^2(-\mathrm{Ec}\alpha^2 + V\beta) + (R-S)\left(\frac{y^2}{2} + \frac{y^3\beta}{3}\right) + (R-S)\left(\frac{Ry^4}{8} - \frac{Sy^4}{8} - \frac{1}{8}\mathrm{EcPr}y^4\alpha^2 + \frac{1}{8}\mathrm{Pr}Vy^4\beta + \frac{1}{15}Ry^5\beta - \frac{1}{15}Sy^5\beta\right) + \mathrm{Pr}\left(\frac{1}{8}RVy^4 - \frac{1}{8}SVy^4 + \frac{1}{48}\mathrm{EcGr}Ky^6 + \frac{1}{48}\mathrm{EcGr}My^6 + \frac{1}{48}\mathrm{EcGr}Ry^6 - \frac{1}{48}\mathrm{EcGr}Sy^6 - \frac{1}{8}\mathrm{Ec}Vy^4\alpha - \frac{1}{48}\mathrm{Ec}KVy^6\alpha - \frac{1}{48}\mathrm{Ec}MVy^6\alpha - \frac{1}{105}\mathrm{Ec}K^2y^7\alpha - \frac{2}{105}\mathrm{Ec}KMy^7\alpha - \frac{1}{105}\mathrm{Ec}M^2y^7\alpha - \frac{1}{8}\mathrm{EcPr}Vy^4\alpha^2 - \frac{1}{48}\mathrm{Ec}^2\mathrm{GrPr}y^6\alpha^2 + \frac{1}{8}\mathrm{Pr}V^2y^4\beta + \frac{1}{15}RVy^5\beta - \frac{1}{15}SVy^5\beta + \frac{1}{48}\mathrm{EcGrPr}Vy^6\beta + \frac{1}{105}\mathrm{EcGr}Ky^7\beta + \frac{1}{105}\mathrm{EcGr}My^7\beta + \frac{1}{105}\mathrm{EcGr}Ry^7\beta - \frac{1}{105}\mathrm{EcGr}Sy^7\beta\right) \tag{41}$$

Using the imposed boundary conditions, $u(0) = 0$ and $\theta(0) = 1$ at infinity we obtain the values of α and β. Putting the obtain values into Eqs. (39) and (40), we get the distributions of the fluid flow.

VII. RESULTS

In this section, we present the graphical results of the numerical simulations carried out on the fluid distributions using Maple 22.

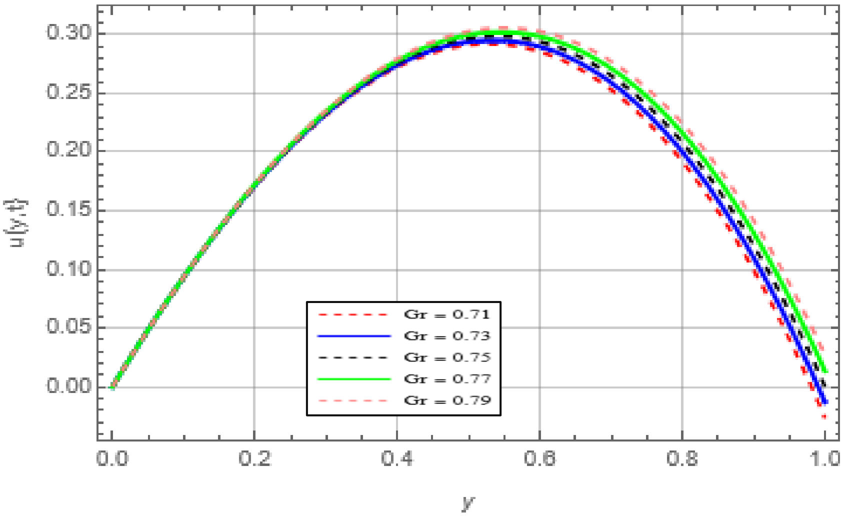

Figure 1. Effect of varying Grashof number (Gr) on the velocity profile and constant values of $V = 2, M = 2, K = 1$

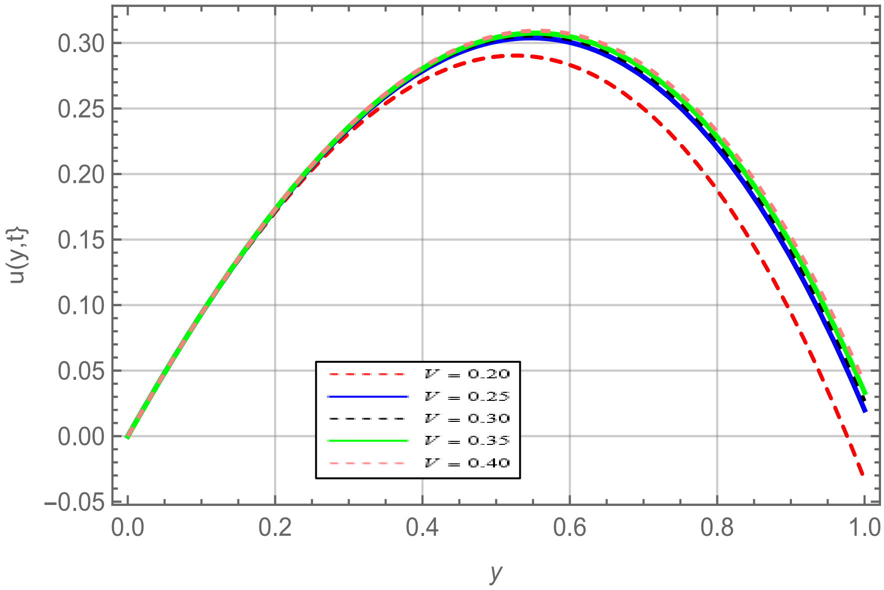

Figure 2. Effect of suction (V) on velocity profile and other values of $Gr = 0.71, M = 0, K = 1$

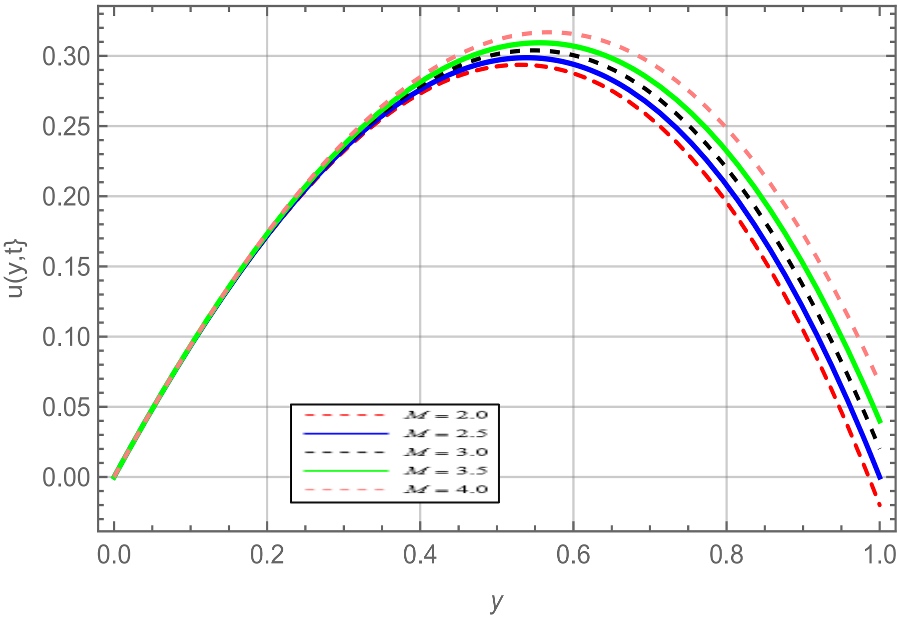

Figure 3. Effect of variation of Magnetic parameter (M) on velocity profile for constant values of $V = 2, Gr = 0.71, K = 1$

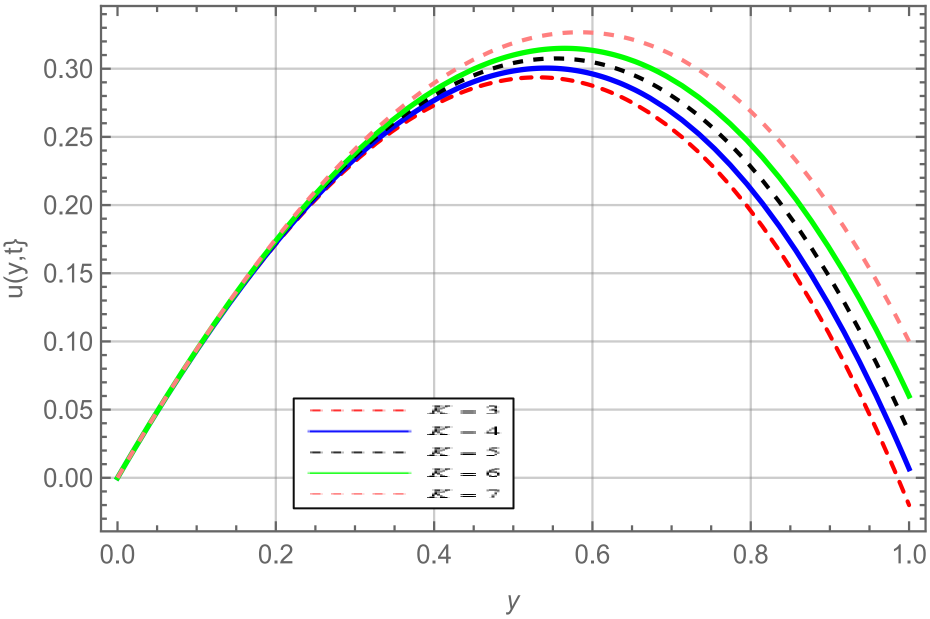

Figure 4. Effect of permeability parameter (K) on velocity profile for Constant values of $V = 2, Gr = 0.71, M = 2$

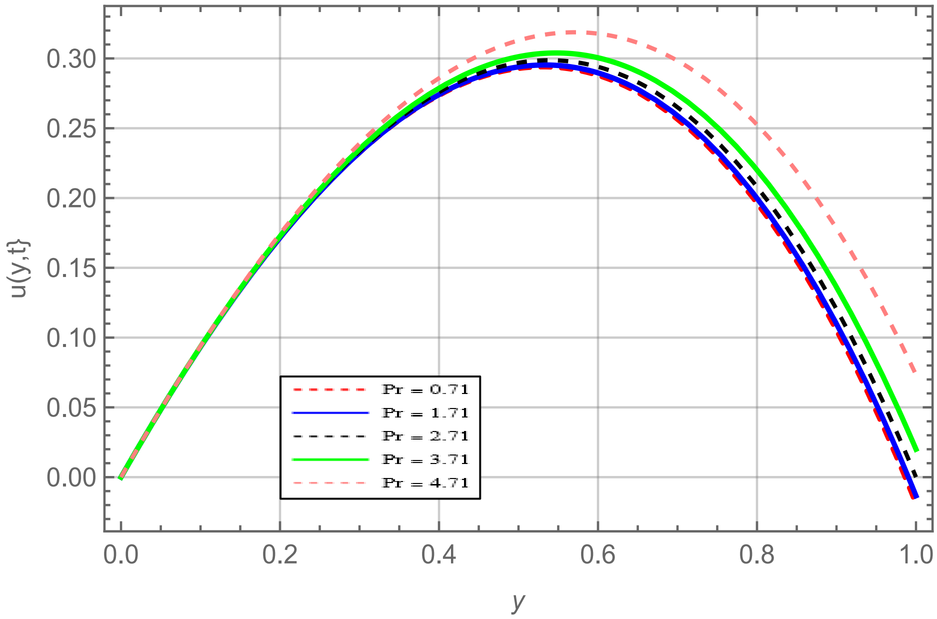

Figure 5. Influence of Prandtl number,(Pr) on the velocity profile for Constant values of $Ec = 0.1, R = 3, S = 1, V = 0.3$

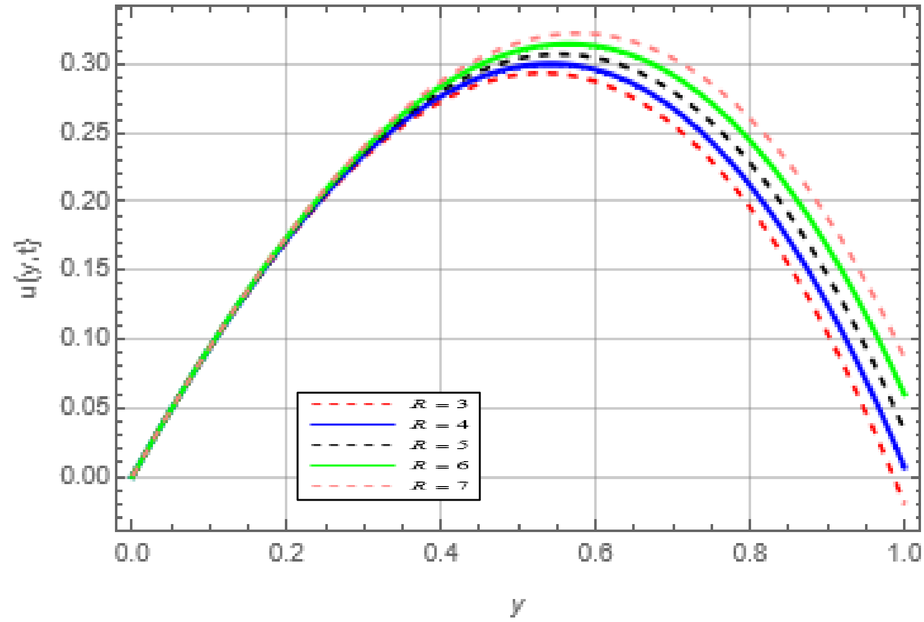

Figure 6. Effect of Radiation parameter, (R)on velocity profile for constant values of $Pr = 0.71, R = 3, S = 1, V = 0.3$

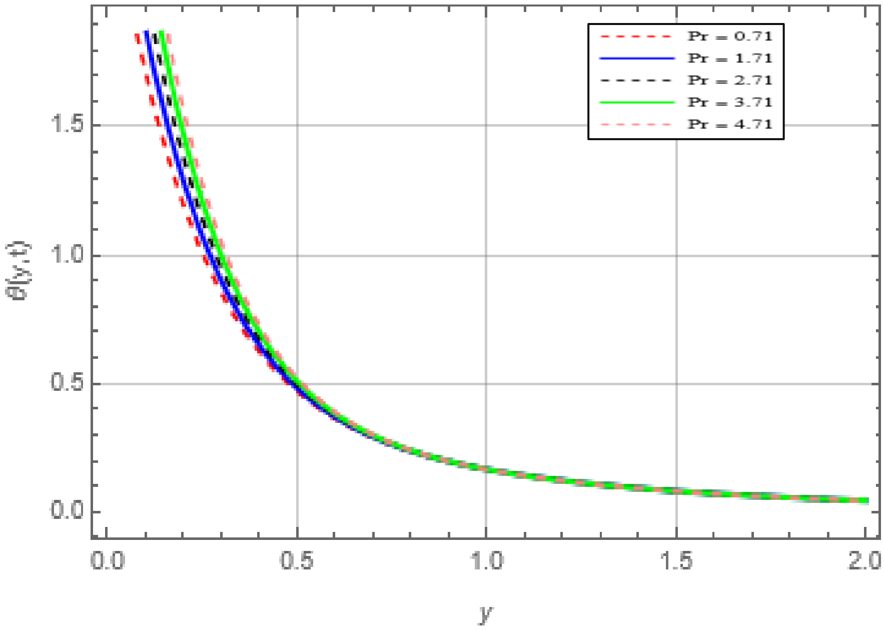

Figure 7. Influence of Prandtl number, (Pr) on temperature profile for constant values of $Pr = 0.71, Ec = 0.1, S = 2, V = 1$

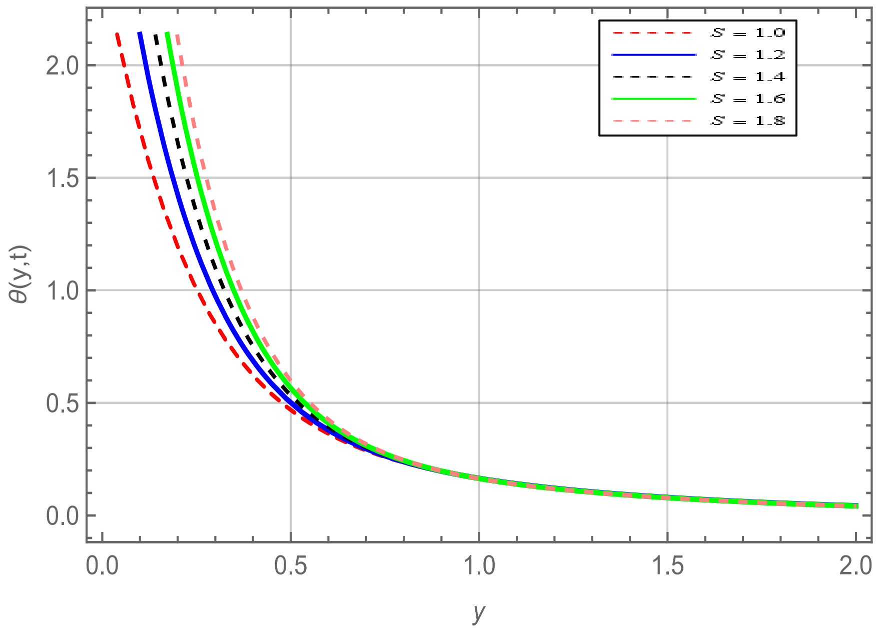

Figure 8. Influence of heat source, (S)on temperature profile for constant values of $Pr = 0.71, Ec = 0.1, R = 3, V = 0.3$

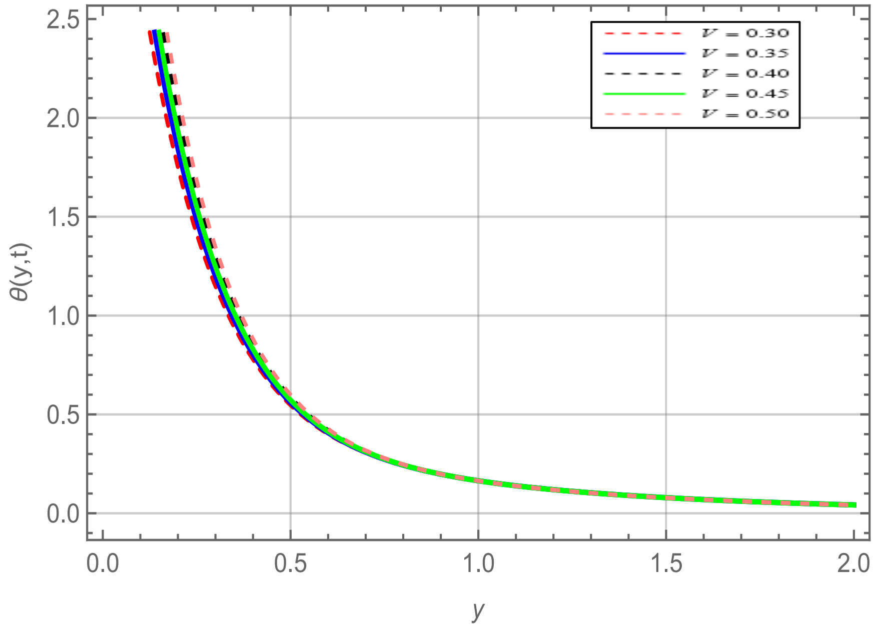

Figure 9. Effect of suction parameter, (V) on temperature profile for constant values of $Pr = 0.71, Ec = 0.1, R = 3, S = 1$

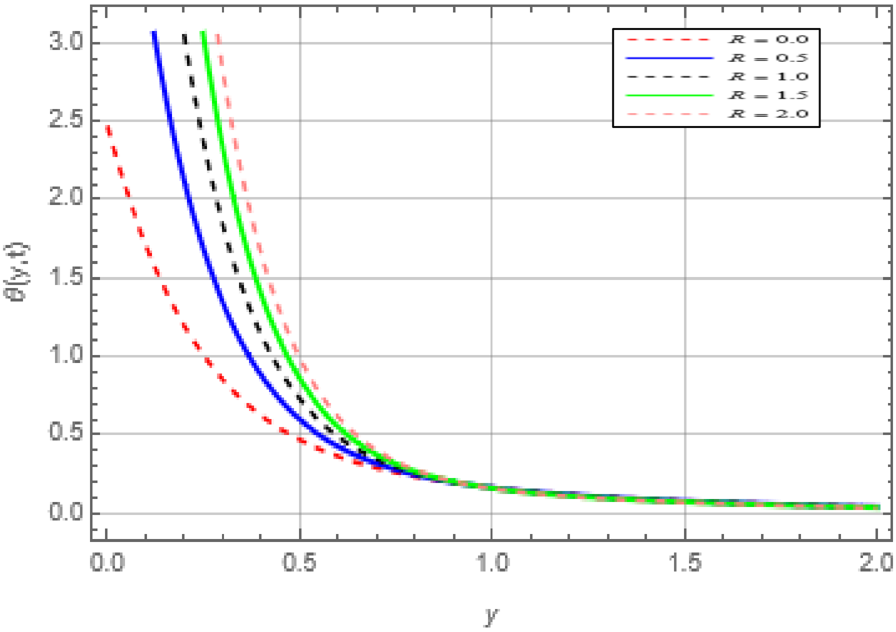

Figure 10. Effect of radiation parameter, (R) on temperature profile and constant values of $Pr = 0.71, Ec = 0.1, V = 4, S = 1$

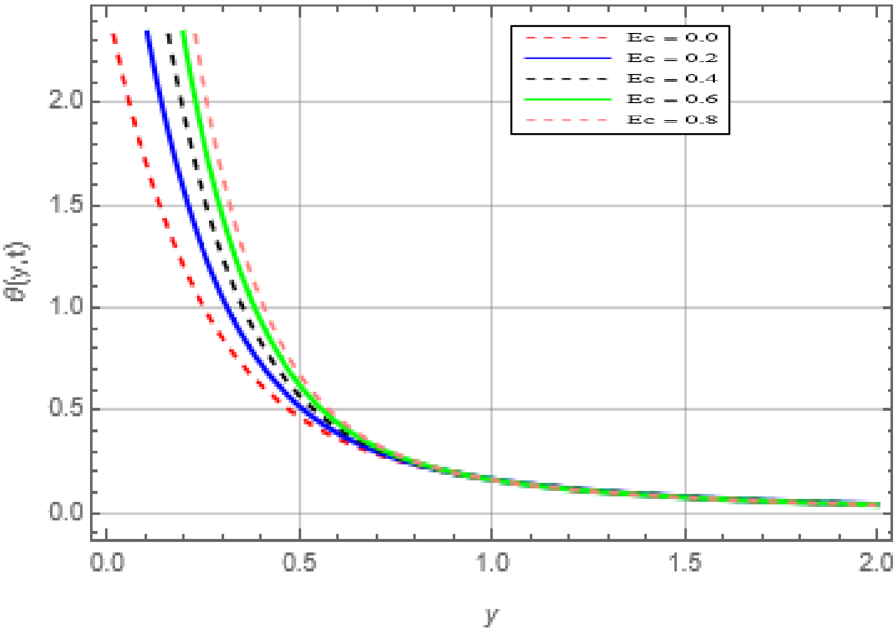

Figure 11. Effect of Eckert number, (Ec) on temperature profile and Constant values of $Pr = 0.71, R = 3, V = 4, S = 1$

VIII. DISCUSSION OF RESULTS.

Figure 1. depicts the variation of Grashof number,Gr with the velocity profile. The findings shows that both the velocity and temperature increase with an increase in the Grashof number. Also, the peak point of the curve increases rapidly near the centre of the porous plate as Grashof increases and finally decay to the free stream velocity.

Figure 2. shows the influence of the suction parameter on the velocity profile. It is observed, increase in suction parameter leads to both increase in the velocity and temperature profiles of the fluid. The effect of Hartmann number on the velocity profile I displayed in figure 3, the result showed, increase in magnetic field parameter lead to a decrease in the velocity of the fluid owing to the presence of Lorentz force acting on the fluid.

Figure 4 display the impact of the permeability parameter on the velocity profile. It is observed that, due to resistance of the porous medium, increase in permeability lead to a decrease in the velocity distribution of the fluid. The variation of Prandtl number, Pr on the velocity profile is depicted in figure 5. The result indicates that, increase in Prandtl number, Pr cause a decrease in the velocity of the fluid and vice versa.

Figure 6. illustrates the influence of the Eckert number, Ec on the temperature profile of the fluid. The finding showed, increase in Eckert number increases both velocity and temperature of the fluid across the boundary. The depiction of the effect on velocity profile by the radiation parameter is presented in figure 7. It is observed that, the temperature profile is decreased when the radiation parameter is increased.

Figure 8 show the variation of heat source parameter on temperature profile. The study show, increase in heat source parameter increases both the velocity and temperature profiles of the fluid. The influence of suction parameter on the temperature distribution is illustrated in figure 9. The result reveal that, both the velocity and temperature profiles increase with increase in the suction parameter.

IX. CONCLUSION

In this article, an investigation is carried out on the heat transfer analysis of MHD fluid flowing past a vertical porous plate in the presence of variable suction using Adomian decomposition method. The approximate solutions of the velocity and temperature distributions was obtained using the third term approximation. The findings of the study are summarized as follows.

1. Increase in Prandtl number decreases the velocity and temperature distribution of the fluid.
2. Positive increase in the Grashof number increases the velocity but decreases the temperature of the fluid
3. The velocity and temperature profiles decrease with increase in the magnetic or Hartmann number.
4. The presence of Suction, heat source and Eckert numbers is to increase both velocity and temperature profiles of the fluid.
5. Increase in permeability parameter lead to decrease in the velocity profile of the fluid.

REFERENCES

[1] Tritton, D.J. (1988). Physical Fluid Dynamics. Second Edition, Oxford University Press, New York, USA.

[2] Alao, F.I., Fagbade, A.I.., Falodun, B. O. (2016). Effects of thermal radiation, Soret and Dufour on an unsteady heat and mass transfer flow of a chemically reacting fluid past a semi-infinite vertical plate with viscous dissipation. Journal of the Nigerian Mathematical society, 35(12), 142-158.

[3] Hazarika, G.C., Konch, J. (2014). Effects of variable viscosity and thermal conductivity on MHD free convection flow along a vertical porous plate with viscous dissipation. International Journal of Mathematics Trends and Technology, 15(1), 70-85.

[4] Jimoh, A., Idowu, A.S, Titiloye, K. (2014). Influence of Soret on unsteady MHD flow of Kuvshinshiki fluid flow with heat and mass transfer past vertical porous plate with variable suction. International Journal of Recent Innovation Trends in Computing and communication, 2(9), 25999-2611.

[5] Manna, S.S., Das, S., Jana, R.N. (2012). Effects of Radiation on unsteadyMagnetohydrodynamics free convective flow past an oscillatory vertical porous plate embedded in a porous medium with oscillatory heat flux. Advances in Applied Science Research, 3(6), 3722-3736.

[6] Mangathai, P., Reddy Ramana, G.V., Reddy Rami, B. (2015). Heat and mass transfer effects of a MHD free convection flow over an inclined plate embedded in a porous medium. Journal of Advanced Computer and Mathematical Sciences, 7(1), 1-12.

[7] Kishan, N., Haritha, C., Madhuri, D.L. (2012). Finite difference analysis of an unsteady mixed convection flow past a semi-infinite vertical permeable moving plate with heat and mass transfer with radiation and viscous dissipation. Advances in AppliedScience Research, 3(4), 2266-2279.

[8] Mohammed, L., Bhaskar, P. (2013). Similarity solution of heat and mass transfer for natural convection over a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation, viscousdissipation, and chemical reaction. Hidawi publishing Corporation, ISRN thermodynamics.

[9] Mohammed, I.S., Reddy, I.S., Roja, P. (2014). Radiation effects on unsteady MHD free convective heat and mass transfer flow past a vertical porous plate embedded in a porous medium with viscous dissipation. International Journal of Innovative Research in Science, Engineering and Technology, 3(1), 17181-17184.

[10] Sharma, P.R., Sharma, M., Yadav, R.S. (2014). Viscous dissipationand mass transfer effects on unsteady MHD free convective flow along a moving vertical porous plate in the presence of internal heat generation and variable suction. International Journal of Scientific and Research publication, 4(9), 1-9.

[11] Adomian, G. (1998). A review of Decomposition method in Applied Mathematics, Journal of Analytical Applied, (135), 501-544.

[12] Adomian, G. (1990). A review of the decomposition method and some recent results for nonlinear equations. Mathematical and Computer Modelling, Vol, 13, pp. 17-43.

[13] Adomian, G. (1994). Solving Frontier problems of Physics. The Adomian decomposition method, Kluwer, Dordrecht.

[14] Adomian, G. and Roch, R. (1986). On the solution of Nonlinear differential equation with convolution product nonlinearities. Journal of Applied Analytical Mathematics, 114, 171-175.

[15] Al-Jawary, M.A. and Al-Razaq, S.G. (2016). A Semi Analytical Iteration Technique for solving Duffing Equations. International Journal of Pure and Applied Mathematics, Vol. 18, pp. 871-885.

[16] Farshad, E., Hadi, A., Farshad, E, F., and Rohoallah, M. (2013). An Iterative Method for solving Partial Differential Equations and solution of Korteweg-Devries Equations for Showing the Capability of the Iterative Method. World Applied Programming, Vol. 3, pp. 320-327.

[17] Ebiwareme, L., Kormane, F.A, Odok, E.O. (2022). Simulation of unsteady MHD flow of incompressible fluid between two parallel plates using Laplace-Adomian decomposition method. World Journal of Applied Research and Reviews, 2022, 14(03), 136-145

[18] Ebiwareme, L., Da-WaribokoY.A. (2021). Modified Adomian decomposition method and Pade approximant for the Numerical approximation of the deterrence model in society. The International Journal of Engineering Science, Volume 16, Issue 7, Series 1, pages 01-12, 2021.

[19] Ebiwareme, L. (2022). A comparative study between semi-analytical iterative schemes for the reliable treatment of coupled nonlinear partial differential equations. International Journal of Innovation Engineering and Science Research. Volume 6, Issue 1, January-February 2021

[20] Ebiwareme, L. (2022). Analytical solution of the dynamics of atmospheric C02 using LADM-Pade approximation approach, International Journal of Trend in Scientific Research and Development, Volume 6, Issue 2, January-February 2022.

[21] Ebiwareme, L. (2022). Application of semi-analytical iteration techniques for the numerical solution of linear and nonlinear differential equations. International Journal of Mathematics Trends and Technology. Volume 67, Issue 2, 146-158, February 2021.

[22] Ebiwareme, L., Akpodee, R. E., Ndu, R.I. (2022). An application of LADM-Pade approximation for the analytical solution of the SIR infectious Disease Model. International Journal of Innovation Engineering and Science Research. Volume 6, Issue 2, March-April 2022.

[23] Ebiwareme, L. (2022). Analytical study of the Hepatitis E Virus Model (HEV) vis Hybrid semi-analytical Laplace Transformation Adomian decomposition method. Volume 5, Issue 1, Jan-Feb 2022.

[24] Bakodah, H.O., Mohamed, A.D. (2012). On discrete Adomian Decomposition method with Chebyshev Abscissa for Nonlinear Integral Equations. Advances in Pure Mathematics, 2(5), 310-313 .

[25] Darania, P., Ebadin, A. (2007). A method for the Numerical Solution of the Integro-differential Equations. Applied Mathematics and Computation. 188(1), 657-668.

[26] Dogan, N. (2012). Solution of the systems of ordinary Differential Equation by Combined Laplace Transform-Adomian Decomposition Method. Mathematical and Computational Applications. 17(3), 203-212.

[27] Holmquist, S.M. (2007). An Examination of the Effectiveness of the Adomian Decomposition Method in Fluid Dynamics Application. Doctor of Philosophy, University of Central Florida, Orlando, Florida.

[28] Khan, M., Hussain, M. (2011). Application of Laplace Decomposition Method on Semi-infinite Domain. Numerical Algorithm, 56(2), 211-218.

[29] Wang, Y. (2009). Adomian Method Applied to Navier-Stokes Equation with Fractional Order. Proceedings of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference August 30 - September 2, San Diego, California, USA.

[30] Hasan, Y. Q. (2012). Modified Adomian decomposition method for second order singular initial value problems, Advances in Computational Mathematics, and its applications, 1(2), 201-218.

[31] Wazwaz, A.M. (2005). Adomian decomposition method for a reliable treatment of the Emden- Fowler equation, Applied Mathematics and Computation, Vol. 161, pp 543-560.

[32] Biazar. J. (2006). Solution of the epidemic model by Adomian decomposition method. Applied Mathematics and Computation, Vol. 173, 1101–1106.

[33] Bulut, H., Erg¨ut, M., Asil, V., Bokor, R. H. (2004). Numerical solution of a viscous incompressible flow problem through an orifice by Adomian decomposition method. Applied Mathematics and Computation, Vol. 153, 733–741.

[34] Singh N., Kumar M. (2011). Adomian Decomposition Method for Solving Higher Order Boundary Value Problems, Mathematical Theory and Modeling, Vol 2, No. 1, 12-24.

[35] Adomian, G. (1984). Convergent Series solution of Nonlinear equations, Journal of Computational Applied Mathematics, 11, 225-230.

[36] Adomian, G. (1984). On the Convergence region for the Decomposition solutions. Journal of Computational Applied Mathematics, 11, 379-380.

[37] Cherruault, Y., and Adomian, G. (1993). Decomposition Method: A new proof of convergence. Mathematics Computational Modelling, 18(12),103-106.

[38] Durojaye, M.O., Okonofua, A.N. (2019). Influence of Heat transfer on magnetohydrodynamics fluid past an infinite vertical porous plate with suction. Abacus Mathematics and Science Series, Vol. 46, N0. 1, 573-583.