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About Moving the Boat through A River

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ABSTRACT. *This work solves the well-known problem of ferrying a boat across the river. The problem is considered, in which the bow of the boat, while moving, is always directed to the pier located on the opposite bank of the river. Differential equations are obtained that describe the motion of the boat. These equations are solved numerically by the Euler method. MS Excel spreadsheets are used as a programming environment, the program is written in VBA (Visual Basic for Application).*

KEY WORDS: *ferrying a boat across a river, differential equations of motion, Euler's method.*

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I. INTRODUCTION

Nowadays, more than one serious scientific problem, in which it is necessary to obtain a specific numerical result, cannot be solved without the use of a computer. Computer modeling of physical processes and setting up computer experiments have become an integral part of the study of physics. At present, along with experimental and theoretical physics, a new name has even appeared "computer physics".

The solution of many physical problems is reduced to the solution of differential equations describing the considered physical process. Analytical solutions to these equations are often difficult to find. In many cases, it is easier to find numerical solutions to these equations using a computer. As our teaching experience shows, numerical modeling of physical processes develops a deeper understanding of physics. The sooner students start using the computer for solving physical problems, the more benefits it will bring in their future professional activities.

In this work, we solve a fairly simple problem, the analytical solution of which we could not find. Differential equations are obtained that describe the movement of a boat across the river. These equations are solved numerically by the Euler method [1, 2]. Spreadsheets MS Excel are used as a programming environment. The program is written in VBA (Visual Basic for Application). So, here is the problem statement.

II. THE TASK

The boat is crossed over the river in such a way that the bow of the boat is always directed to the pier, located exactly on the opposite bank of the river. At the initial moment, the boat starts perpendicular to the shore. River flow speed $u = 0.8$ м/с , boat speed $\upsilon = 3.2$ м/ c , river width $H = 540$ м . Determine how much gasoline the boat consumes for the crossing, if at the selected power it consumes 5 liters of gasoline per hour.

III. DECISION

Let's go to the frame of reference (FR) associated with the river. In this FR, the pier P moves uniformly against the direction of the current with a velocity u parallel to the x axis. In time t, the pier will cover the distance $|PP_1| = ut$. At this time, the boat will be at a point and its speed will be directed at an angle to the y-axis, as shown in Fig. 1.

Then, using the figure, we write the expression:

$$\operatorname{tg}\alpha = \frac{ut - x}{H - y}. \tag{1}$$

Differentiating both sides of expression (1) with respect to time t, we obtain the equation:

$$\frac{\dot{\alpha}}{\cos^2\alpha} = \frac{(u - \dot{x})(H - y) + \dot{y}(ut - x)}{(H - y)^2}. \tag{2}$$

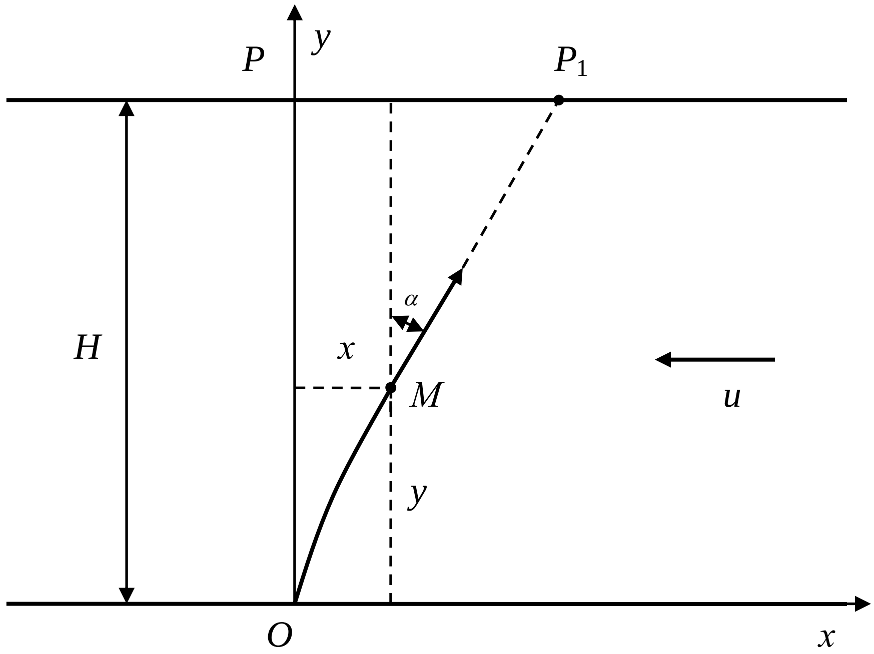

Fig. 1 – Movement of the boat in the frame of reference associated with the river

Hereinafter, the points denote time derivatives. Taking into account that, $\dot{x} = \upsilon \sin\alpha$ and $\dot{y} = \upsilon\cos\alpha$, we rewrite the last equation in the following form:

$$\dot{\alpha} = \frac{\cos^2\alpha}{H - y}\left(u - \upsilon\sin\alpha + \upsilon\cos\alpha \,\mathrm{tg}\,\alpha\right) = \frac{\cos^2\alpha}{H - y}u. \tag{3}$$

It is convenient to introduce a new variable:

$$p = \tan\alpha. \tag{4}$$

Now, solving the problem is reduced to solving the following system of differential equations:

$$\dot{p} = \frac{u}{H - y}, \tag{5}$$

$$\dot{y} = \frac{\upsilon}{\sqrt{1 + p^2}}, \tag{6}$$

$$\dot{x} = \frac{\upsilon p}{\sqrt{1 + p^2}}. \tag{7}$$

Let us numerically integrate equations (5) – (7) in time t with a step h. Using Euler's method, we write the following formulas describing the solution algorithm:

$$t_n = nh \quad (n = 0, 1, 2, 3, \ldots), \tag{8}$$

$$p_{n+1} = p_n + \dot{p}_n h, \tag{9}$$

$$y_{n+1} = y_n + \dot{y}_n h, \tag{10}$$

$$x_{n+1} = x_n + \dot{x}_n h. \tag{11}$$

On the basis of the given algorithm, a computational program was written in the VBA language and the trajectories of the boat were calculated, shown in Fig. 2. The boat continues to move over time $T \cong 180$ c. During this time, it consumes 0.25 liters of fuel.

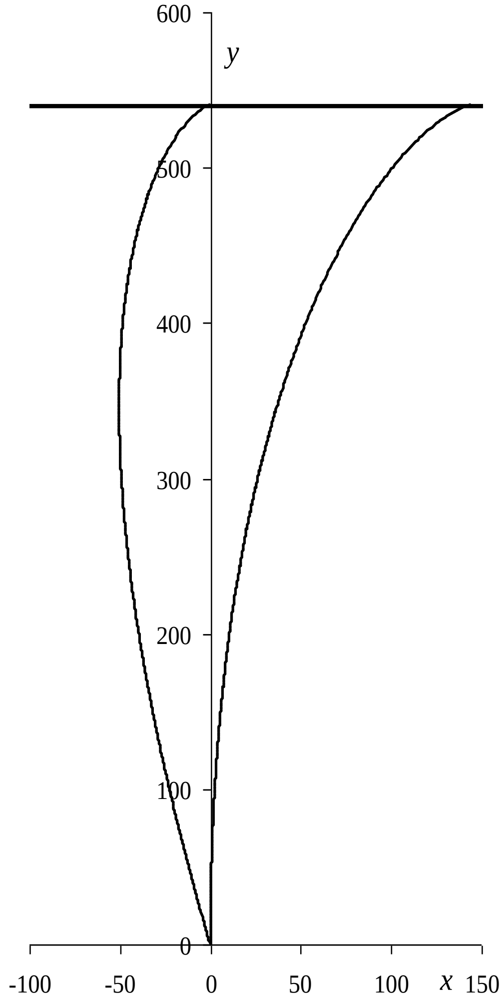

Fig. 2 – Trajectories of boat movement in CO, connected with the river and with the coast

A more traditional solution is when the boat is crossing the river, moving in a straight line along the axis y, so that it is not carried away by the current. In this case, the boat must move at an angle β to the axis y, here

$$\sin\beta = \frac{u}{\upsilon}. \tag{12}$$

In this case, he will spend the minimum time on the crossing:

$$T_m = \frac{H}{\upsilon\cos\beta} = \frac{H}{\upsilon\sqrt{1-\frac{u^2}{\upsilon^2}}}. \tag{13}$$

Substituting numerical data into formula (13), we find $T_m \cong 174.3$ c.
In Fig. 3 shows the launch sheet of the Boat1 program. Below is a listing of the program "Boat1", according to which the boat movement is calculated and the graphs are plotted in Fig. 2.

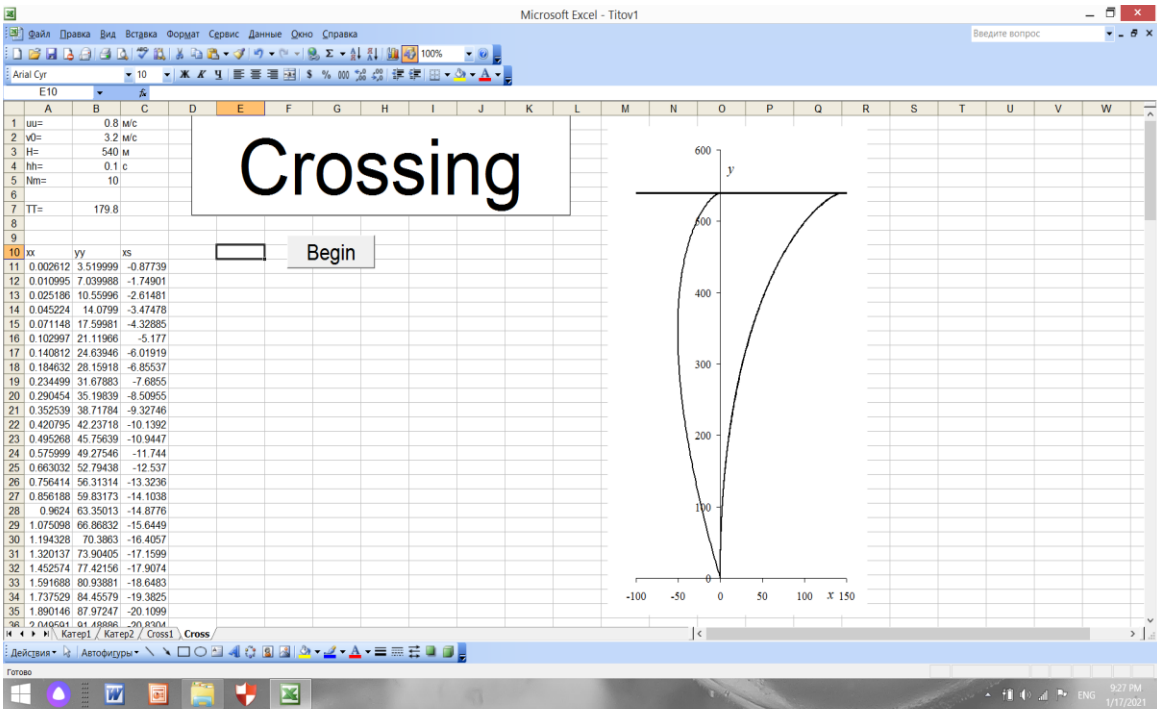

Fig. 3 – Program «Boat1» – launch list

IV. LISTING OF THE PROGRAM «BOAT1»

```
Sub Boat1()

' Crossing river
    Sheets("Cross").Select
    uu = Cells(1, 2)
    vv = Cells(2, 2)
    SH = Cells(3, 2)
    hh = Cells(4, 2)
    Nm = Cells(5, 2)
    tt = 0
    xx = 0
    yy = 0
    pp = 0
    ii = 0
    mm = 0

AA:
    tt = tt + hh
    pp1 = uu / (SH - yy)
    kk = Sqr(1 + pp * pp)
    yy1 = vv / kk
    xx1 = yy1 * pp
    mm = mm + 1

    pp = pp + pp1 * hh
    xx = xx + xx1 * hh
    yy = yy + yy1 * hh
    If mm > Nm Then
    mm = 0
    ii = ii + 1
    Cells(10 + ii, 1) = xx
    Cells(10 + ii, 2) = yy
    Cells(10 + ii, 3) = xx - uu * tt
```

```
    End If

    If yy < SH Then
    GoTo AA
    End If

    Cells(7, 2) = tt
    ii = ii + 1
    Cells(10 + ii, 1) = xx
    Cells(10 + ii, 2) = yy
    Cells(10 + ii, 3) = xx - uu * tt
End Sub
```

REFERENCES

[1]. Gould H., Tobochnik J. An Introduction to Computer Simulation Methods Applications to Physical Systems. – Part 1, Addisson-Vessley, Publishing Company, 1988.

[2]. Feynman R.P., Leighton R.B., Sands M. The Feynman Lectures on Physics. – Vol. 1, Addisson-Vessley, 1963.

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